Probing the quantum nature of the neutrino with two-particle interferometry

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Two-particle interferometry, a second-order interference effect, is explored as another possible tool to distinguish between massive Dirac and Majorana neutrinos. A simple theoretical framework is discussed as well as some experimental considerations. While the method can in principle provide both the mass scale and the quantum nature of the neutrino for a certain class of incoherent left handed source currents, the detector requirements are currently beyond what is technically possible.

Two contemporary problems in neutrino physics are determining the absolute mass of the neutrino and discovering if the neutrino is its own antiparticle [1]; that is, is the neutrino a massive Majorana or Dirac fermion? The existence of neutrino mass has been established through oscillation experiments such as Super-Kamiokande, SNO, and KamLAND [2–4], which have successfully extracted the differences of the masses between the energy eignstates. Various experimental approaches, such as tritium decay [5, 6] and cosmological background studies [7], are capable of extracting the absolute mass scale of the electron neutrino. While these challenging experiments have been able to put an ever-improving upper limit on the neutrino mass, they tell us nothing about the neutrino's nature.

Currently, the only experimental approach used to determine the quantum nature of the neutrino is neutrinoless double beta decay ($\beta\beta(0\nu)$) [8]. The decay rate is proportional to the effective mass of the neutrino and only proceeds if the neutrino is a Majorana particle. While a claim of $\beta\beta(0\nu)$ discovery in ^{76}Ge has been made [9], this result has not been independently confirmed. A host of next generation experiments are poised to verify the current claim and, if the claim proves false, further explore ever-smaller mass scales using this exotic decay as a tool [10–12]. This discovery approach has clear advantages because if the neutrino is a Majorana particle this approach will, with enough patience, confirm it. The strategic disadvantage to this approach is equally clear. If the neutrino is a Dirac fermion one is left staring at background for decades with no knowledge that the decay will not proceed.

There are other problems with the approach. While the existence of the decay unambiguously identifies the Majorana character of the electron neutrino, extracting a precise value of the mass is model-dependent. The decay rate is proportional to the square of the effective neutrino mass times the square of nuclear matrix elements that govern the process. The effective mass is the magnitude of a sum of the mass eigenvalues weighted by unknown phases. Because of these unknown phases and the intrinsic difficultly in calculating the matrix elements, the effective mass extracted for a measured $\beta\beta(0\nu)$ rate is highly model and isotope dependent. The masses for a given lifetime limit and isotope are often quoted as a range of masses based on a growing, and not entirely universally standardized, body of theoretical work. Lastly, if the current claim is refuted, the most favored theoretical neutrino mass region, the direct hierarchy, will only be potentially accessible to *next*-next generation $\beta\beta(0\nu)$ experiments whose technological feasibility has not been fully assessed or determined.

Another experimental technique that provides information about the mass and nature of the neutrino, as will be shown below, is two-particle intensity interferometry. This form of interferometry has been used extensively in many areas of physics and has served to cross-pollinate ideas in different sub-fields for over fifty years. It is natural to wonder what role this technology might play in neutrino physics.

Intensity interferometry was originally developed by Robert Hanbury Brown and Richard Twiss as an alternative to Michelson interferometry to measure the angular sizes of stars in radio astronomy [13]. This and related methods are often called HBT after the original co-founders. By correlating intensities rather than adding amplitudes, the measurement is insensitive to high frequency noise fluctuations that would normally make first order interferometry prohibitive.

The ideas of intensity interferometry were eventually quantum mechanically applied to photons rather than classical waves, instigating a revolution in modern quantum optics. Two- and multi-photon effects are routinely studied, often in the time domain. The technology was independently applied in momentum space to final state particles in elementary particle physics and is sometimes called the Goldhaber-Goldhaber-Lee-Pais (GGLP) effect [14–18] in that context. The modern incarnation of GGLP, "femtscopy" [19], is routinely used to study the space-time dynamics of incoherent particle sources in high energy and heavy ion collisions. The effect has also been studied in fermionic systems such as with neutrons and protons [20, 21] (also see [15]). Two-electron HBT in 2D condensed matter systems has also been reported [22].

The essential observation in a two-particle interferometry is that pairs of incoherently generated indistinguishable bosons tend to clump while close in phase space while similarly generated fermions tend to anti-clump. What "close" means exactly depends on the scale and geometry of the problem and in what space one is performing the measurement. The sensitivity to the quantum statistics obeyed by measured pairs, in particular the tendency for fermions to anti-clump in phase space, is of interest in an attempt to measure the quantum nature of the neutrino.

The physical-observable in intensity interferometry is the two-particle correlation function, C_2, which is a measure of the degree of independence between a joint measurement of two particle events in some variable of interest such as momentum, space, or time. The two-particle correlation function can be written

$$C_2 = \frac{P(1,2)}{P(1)P(2)} \sim \frac{\mathrm{Tr}[\hat{\rho}\hat{a}_k^\dagger \hat{a}_q^\dagger \hat{a}_k \hat{a}_q]}{\mathrm{Tr}[\rho \hat{a}_k^\dagger \hat{a}_k]\mathrm{Tr}[\rho \hat{a}_q^\dagger \hat{a}_q]} \tag{1}$$

where $P(1,2)$ represents the joint probability of measuring two events while $P(i)$ represents the individual probabilities of events $i = 1,2$ and can be naturally generalized to higher order correlations. The explicit momentum space form of C_2 on the right highlights the essential physical components of the correlation function. The density matrix is $\hat{\rho}$ while $\hat{a}^\dagger$ and $\hat{a}$ are the creation and annihilation operators for the quanta of momenta q and k associated with the appropriate fields of interest. When normalized to the single particle distributions as shown, C_2 is proportional to the relative probability for a joint two-particle measurement as compared to two single-particle measurements. If the measurements are independent, then C_2=1. If the measurements are correlated, C_2 deviates from unity.

As can be seen in Eq. (1), the correlation function is sensitive to several interesting physics effects. First, it probes the quantum statistics obeyed by the pairs being studied (e.g. boson, fermion, anyon [23], etc.) as determined by the (anti-)commutation relations between $\hat{a}$ and $\hat{a}^\dagger$. The form of the density matrix will determine the quantum field configuration (e.g. thermal states, coherent states, Fock states, etc.). Tacitly contained in the density matrix, when projected as a Wigner function, are the space-time geometry of the source, the source dynamics (e.g. space-momentum correlations such as explosions, flow, jets, conservation laws etc.), and any pairwise interactions (Coulomb, strong, weak, etc.). As mentioned, although counterintuative, the quantum mechanical aspect of this interference effect is maximal for incoherent sources, making it a powerful tool to study such systems. Coherent sources of non-interacting bosons, such as laser light well above the lasing threshold, give a constant $C_2 = 1$, reflecting their Poissonian, and thus independent, nature. For a complex source such as in a heavy ion collision, disentangling all of the effects described above is a challenging task. However, for static, incoherent sources of non-interacting fermions, as will approximately be the case with neutrinos, the correlation function will only be sensitive to the spatial source size and the quantum statistics. Therefore, by performing a two particle correlation measurement on neutrino pairs, one can obtain direct information about the quantum statistics obeyed by such particles.

As Eq. (1) implies, there are many possible approaches one can use to obtain an explicit expression for the correlation function. They all give essentially the same final results – although some methods illuminate relevant details more clearly than others. A particularly simple form for Eq. (1) that illustrates the essential physics is given by the Koonin-Pratt equation [15, 24]

$$C_2 = \int d^3R |\psi(\vec{x}_1, \vec{x}_2)|^2 \rho(\vec{R}). \tag{2}$$

The equation assumes an incoherent emission of a pair of particles from a normalized source pair distribution $\rho(\vec{R})$ where $\vec{R}$ is the vector separation between the source pairs. For simplicity, and without loss of generality, time has been implicitly integrated out of Eq. (2). However, the formalism can easily be expanded to include such correlations. The two-particle wave function, $\psi(\vec{x}_1, \vec{x}_2)$, contains information about the quantum statistics and any pairwise interactions. Working in natural units ($c = \hbar = 1$), if we consider a pair of free identical fermions in any specific triplet spin configuration, the spatial part of the wave function will be antisymmetric upon label exchange and given by the usual plane wave solution

$$\psi(\vec{x}_1, \vec{x}_2) = \frac{1}{\sqrt{2}}(e^{-i\vec{p}_a\cdot\vec{x}_1} e^{-i\vec{p}_b\cdot\vec{x}_2} - e^{-i\vec{p}_a\cdot\vec{x}_2} e^{-i\vec{p}_b\cdot\vec{x}_1}). \tag{3}$$

One interprets this two-particle wave function to be the amplitude for particles emitted at points $\vec{x}_1$ and $\vec{x}_2$ to be measured with momenta $\vec{p}_a$ and $\vec{p}_b$. For free particles, C_2 is simply related to the cosine transform of the incoherent pairwise source distribution, $\rho(\vec{R})$.

If we assume two identical free fermions are emitted from exactly two point sources separated by $\vec{R}$ ($\rho(\vec{R}) \sim \delta(\vec{R})$), Eq. (2) can be written

$$C_2(\vec{Q}) = 1 - \xi \cos(\vec{Q} \cdot \vec{\delta x}) \tag{4}$$

where $\vec{Q} = \vec{p}_a - \vec{p}_b$ and $\delta\vec{x} = \vec{x}_1 - \vec{x}_2$. The parameter $\xi = 1$ for triplet states and -1 for singlet states. If the system is spin-averaged, then $\xi = \frac{1}{2}$. Notice in the triplet case $C_2(Q = 0) = 0$ and the fermions are anticorrelated if in the same momentum state. For identical spinless bosons under the same kinematic conditions, $C_2(0) = 2$. Because the emission is incoherent and there are no interactions, the correlations arise only from the quantum statistics obeyed by the particles. The scale of the correlation is set by the source size. It is instructive to note that for non-identical particles, where the wave function has no particular symmetry, $C_2 = 1$ for all $\vec{Q}$. A coherent source of bosons (which is technically not accurately represented by Eq. (2) but rather by Eq. (1) using the appropriate density matrix generated from coherent states) would give the same result.

Let's examine a useful limit of Eq. (4) we will use later for a series of gedanken experiments. Consider two point sources of fermions separated by a distance $\vec{R}$ and measured by a pair of distant detectors separated by $\vec{d}$. The source and detector are a distance L from each other such that $L \gg R \gg d$. That is, we have well-separated sources far away from a relatively close pair of detectors. We also assume a pair of single-mode fermions ($p = |\vec{p}_a| = |\vec{p}_b|$ but $\vec{p}_a$ is not necessarily equal to $\vec{p}_b$). In this limit Eq. (4) becomes

$$C_2(d) = 1 - \xi \cos(\Delta\theta d/\lambda). \tag{5}$$

This is similar to the original HBT experiment used to measure the angular size of stars. The correlation function is measured at different detector separations, d, for waves of known wavelength, λ. From the shape of $C_2(d)$, the angular size, $\Delta\theta$, can be extracted. As before the correlation strength depends on various factors including how extra degrees of freedom, like spin, are handled.

For all the examples below, with no change in the final conclusions, one can also use the femtoscopic limit of Eq. (4) ($L \gg d \gg R$). In that limit, one might use neutrinos and antineutrinos generated from muon or Z_0 decays. Also, the method could be applied as a antibunching counting experiment in the time domain performed on a beam, similar to what is done in quantum optics. For illustrative purposes we will proceed with the macroscopic limit as described by Eq. (5).

We can imagine not knowing *a priori* the quantum nature of the particles we are measuring, but instead knowing some other information such as the source geometry (i.e. the angle subtended by the source from the detectors). In that case, using Eq. (5), one would fix the angular size and wavelength but then look for a correlation (for bosons) or anticorrelation (for fermions) as the distance between detectors approached zero to determine the quantum statistics obeyed by the particles of interest.

Can two-particle interferometry, in one of its many incarnations, be applied to neutrinos to determine if they are Dirac of Majorana particles? Let's examine four variations, labeled A through D below, of a simple gedanken experiment to answer this question. A summary of the relevant formulae and the ability of the four cases to resolve the neutrino mass and nature are outlined in Table I. For all cases we will only consider the macroscopic limit for point sources as described by Eq. (5) using only one neutrino flavor. However, the formalism can easily be extended to include any source geometry, including continuous ones. It should also be noted that, unlike $\beta\beta(0\nu)$, the formalism applies to all individual neutrino flavors not just electron neutrinos.

It will be helpful to remember for the cases below that although Majorana neutrinos are their own antiparticle, that is, the field operators transform to themselves under a charge conjugation operation, the left handed weak source currents creating them will generate final state particles with a handedness as if they were Dirac fermions [8]. However, the extra lepton number label "anti" is removed for Majorana particles because the handedness represents different states of the same object. So a source of what is normally called "right-handed Dirac antineutrinos" might instead generate right-handed Majorana *neutrinos*. Similarly, a source current that normally creates "left-handed Dirac neutrinos" could create left-handed Majorana neutrinos.

TABLE I: The two-particle correlation function for Dirac, $C_2^{\mathrm{Dir}}(d)$, and Majorana, $C_2^{\mathrm{Maj}}(d)$, neutrinos are shown for various situations. Where ξ alone is quoted, use Eq. (5). The helicity column indicates if detectors are filtering on same, opposite, or averaged final state helicities. The final rows provide an overview of the case-by-case physics capability to determine the neutrino mass or discover the neutrino nature. Case A: $m = 0$, identical sources; Case B: $m = 0$, distinguishable sources; Case C: $m \neq 0$, identical sources; Case D: $m \neq 0$, distinguishable sources. See the text for a detailed case-by-case discussion.

	helicity	Gedanken Cases A	B	C	D
$C_2^{\mathrm{Dir}}(d)$	same	$\xi = 1$	n/a	$\xi = 1$	$C_2 = 1$
	opp	n/a	$C_2 = 1$	$C_2 = 1$	$C_2 = 1$
	ave	$\xi = 1$	$C_2 = 1$	$\xi = 1 - m^2/E^2$	$C_2 = 1$
$C_2^{\mathrm{Maj}}(d)$	same	$\xi = 1$	n/a	$\xi = 1$	$\xi = 1$
	opp	n/a	$C_2 = 1$	$C_2 = 1$	$C_2 = 1$
	ave	$\xi = 1$	$C_2 = 1$	$\xi = 1 - m^2/E^2$	$\xi = m^2/E^2$
Mass?		no	no	yes	yes
Nature?		no	no	no	yes

First, in case A, we consider a massless neutrino and a geometric setup like that describing Eq. (5): well-separated sources far away from close detectors. Imagine two reactors acting as incoherent point sources of distinguishable particles we would normally call Dirac antineutrinos (this case will work equally well for a pair of point Dirac neutrino sources, like two small suns). Far away we place two ideal detectors separated by a distance d filtering on low energy monoenergetic antineutrinos. Relative to the detectors, the reactor pair subtends a known angle $\Delta\theta$. By measuring $C_2(d)$ can we distinguish between two scenarios: one where we have identical sources of right-handed massless Dirac antineutrinos and another where we have sources of right-handed massless Majorana neutrinos? In this case, the answer is no. The measured correlation function will be equal to that in Eq. (5) with $\xi = 1$ and will give the same result for both the Dirac and Majorana cases. This is because quantum indistinguishability applies equally well for the two situations and the two-particle wave function will be identical in both cases. Indeed, this is a sanity check because in the massless limit we do not expect to be able to distinguish between Dirac and Majorana particles based on the Practical Dirac-Majorana Confusion Theorem [25].

Next, for case B we have massless neutrinos with a similar geometric source-detector setup as above except with one of the reactor sources being replaced by a "small sun". That is, we have two sources of *distinguishable* objects: one an incoherent point source of what we would normally call Dirac neutrinos (a small sun) and another that would again be of Dirac antineutrinos (a reactor). We ask the question: do we have one source of Dirac neutrinos and another of Dirac antineutrinos or do we have a pair of sources spitting out Majorana neutrinos of opposite handedness? Again, in this case, we have no way of knowing. The correlation function $C_2(d) = 1$ for both situations. This is because the two-particle wave function for either situation has no special symmetry. That is, it factorizes and the particles are not entangled at the detector. From Eq. (2) if the normalized wave function factorizes the correlation function becomes unity.

For cases C and D let's consider the above two situations again but this time give the neutrino a mass that is small compared to its energy. The presence of mass complicates the situation because chirality ("handedness"), is no longer the same as helicity. Left-handed (in the chiral sense) weak source currents can now create massive neutrinos and antineutrinos of the "wrong" helicity with an amplitude that goes like m/E when $m \ll E$.

With this in mind, consider case C where the source-detector geometry with two reactors is the same as case A. However, this time each reactor is the source of either Dirac antineutrinos of mixed helicity or Majorana particles of mixed helicity. The mixed helicity is because the handedness of the source current and the helicity of the emitted neutrino are now somewhat decoupled, as discussed above. For $m \ll E$, the helicity mixture will be mostly $\Lambda = +1$ with some $\Lambda = -1$ in both the Dirac and Majorana cases. With this configuration, can C_2 distinguish between Dirac and Majorana particles? For this exercise, we will consider ideal detectors that are capable of filtering on the neutrino helicity. If the detectors filter on identical helicities in the final state, C_2 will be Eq. (5), the same as case A. Particles of opposite helicity are quantum mechanically distinguishable so if the detectors filter on opposite helicities then $C_2 = 1$, as in case B. However, if the detectors helicity-average particles in the final state, the mixed helicty of the source has the effect of introducting a helicity "contamination" at the detector and there will only be quantum distinguishably for a fraction of the measurements. This contamination will have the effect of diluting the correlation function by a factor $\vartheta(m^2/E^2)$ so we use Eq. (2) but with $\xi \sim (1 - m^2/E^2)$ for both Dirac and Majorana particles. Again, we cannot distinguish between Dirac and Majorana neutrinos, but a careful helicity-averaged measurement of $C_2(d)$ could in principle extract the mass by measuring the strength of the anticorrelation.

Finally, in case D, we revisit the non-identical sources of sun-reactor geometry of case B extending it to the massive neutrino case. The essential quantum distinguishability arguments are the same as B, however, there are more combinatorics for the Dirac particles because of the extra lepton quantum number. Nevertheless, like the sun-reactor case in the massless case, the Dirac particles are always distinguishable at the detector either by helicity or by lepton number. No matter how one filters on the final state, the Dirac particles are distinguishable so $C_2 = 1$.

If the neutrino is a Majorana particle, however, case D will be different. The reactor source will be emitting primarily Majorana neutrinos with $\Lambda = +1$ with a small component of $\Lambda = -1$. The sun source will be emitting Majorana neutrinos of the opposite degree of contamination. That is, mostly $\Lambda = -1$ with a small $\Lambda = +1$ mixture. Because these particles have no lepton quantum number, with a judicious choice of filtering at the detector, one can detect a distinct signal from the Dirac case. For example, if the detectors filter on opposite final state helicity, $C_2(d) = 1$ because the particles are distinguishable. But if the detectors filter on the same helicity use Eq. (5) with $\xi = 1$. If one performs a helicty-average in the final state, this introduces contamination (more severe than case C) that will reduce the correlation strength. The probability of measuring two equal helicity states with open final state helicity filters scales like m^2/E^2 so we use Eq. (2) with $\xi \sim m^2/E^2$.

Let us entertain some experimental considerations. The primary concern is data rate, detector efficiency, and energy resolution. The above discussion assumed infinite energy resolution to resolve neutrinos of an arbitrary wavelength with no loss of fidelity or smearing. This assumption, using Heisenberg's Uncertainty Principle, permits infinitely slow counting statistics, allowing quantum mechanically coherent data to arrive over infinitely long time scales. This is an unrealistic assumption since the neutrino production rate roughly sets the quantum coherence time scale. The data rates for current experiments such as KamLAND and SNO are about one event per day. Even ignoring production rate limits, to perform the measurements (perhaps using the neutrino pair vertex distribution in their fiducial volume, rather than multiple detectors, as a distance variable), even assuming copious statistics, the ability to measure neutrinos of arbitrary energy, and very fine vertex resolution, these experiments would require an otherworldly energy resolution to see the effect as described. But what are the numbers required? Using $\Delta E \Delta t \sim$ (eV)(fs) as our guide, we can see that with extremely good, but still physical, energy resolutions on the order of eV or keV an experiment needs to measure neutrino pairs separated by times on the order of femto- to attoseconds – a rate approaching weak-charge micro-Amperes of neutrinos. One can also consider the femtoscopic limit where there are very small sources like those generated in a high energy physics collision. To measure this effect in that context, an experiment needs to be able to measure two or more identified inclusive neutrinos per event (exclusive measurements usually have severe phase space constraints such as energy-momentum conservation that mask any other correlations) with an energy resolution of roughly MeV.

Based on the above discussion and reviewing Table I we see the rather promising result that, with the right sources and filters, two-particle interferometry can theoretically obtain both the mass and the nature of the neutrino of any flavor using a single physical-observable C_2. However, experimental requirements currently render the method prohibitive.

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