

Exercise 1.1.1.1a: Given locality, causality, Lorentz invariance, and known physical data since 1860, show that the Lagrangian describing all observed physical processes (sans gravity) can be written:

$$\begin{aligned}
& -\frac{1}{2}\partial_\nu g_\mu^a \partial_\nu g_\mu^a - g_s f^{abc} \partial_\mu g_\nu^a g_\mu^b g_\nu^c - \frac{1}{4}g_s^2 f^{abc} f^{ade} g_\mu^b g_\nu^c g_\mu^d g_\nu^e + \\
& \frac{1}{2}ig_s^2(\bar{q}_i^\sigma \gamma^\mu q_j^\sigma)g_\mu^a + \bar{G}^a \partial^2 G^a + g_s f^{abc} \partial_\mu \bar{G}^a G^b g_\mu^c - \partial_\nu W_\mu^+ \partial_\nu W_\mu^- - \\
& M^2 W_\mu^+ W_\mu^- - \frac{1}{2}\partial_\nu Z_\mu^0 \partial_\nu Z_\mu^0 - \frac{1}{2c_w^2}M^2 Z_\mu^0 Z_\mu^0 - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - \frac{1}{2}\partial_\mu H \partial_\mu H - \\
& \frac{1}{2}m_h^2 H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - M^2 \phi^+ \phi^- - \frac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \frac{1}{2c_w^2}M\phi^0 \phi^0 - \beta_h[\frac{2M^2}{g^2} + \\
& \frac{2M}{g}H + \frac{1}{2}(H^2 + \phi^0 \phi^0 + 2\phi^+ \phi^-)] + \frac{2M^4}{g^2}\alpha_h - igc_w[\partial_\nu Z_\mu^0(W_\mu^+ W_\nu^- - \\
& W_\nu^+ W_\mu^-) - Z_\nu^0(W_\mu^+ \partial_\nu W_\mu^- - W_\mu^- \partial_\nu W_\mu^+) + Z_\mu^0(W_\nu^+ \partial_\nu W_\mu^- - \\
& W_\nu^- \partial_\nu W_\mu^+)] - ig s_w[\partial_\nu A_\mu(W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-) - A_\nu(W_\mu^+ \partial_\nu W_\mu^- - \\
& W_\mu^- \partial_\nu W_\mu^+) + A_\mu(W_\nu^+ \partial_\nu W_\mu^- - W_\nu^- \partial_\nu W_\mu^+)] - \frac{1}{2}g^2 W_\mu^+ W_\mu^- W_\nu^+ W_\nu^- + \\
& \frac{1}{2}g^2 W_\mu^+ W_\nu^- W_\mu^+ W_\nu^- + g^2 c_w^2(Z_\mu^0 W_\nu^+ Z_\nu^0 W_\mu^- - Z_\mu^0 Z_\nu^0 W_\nu^+ W_\mu^-) + \\
& g^2 s_w^2(A_\mu W_\nu^+ A_\nu W_\mu^- - A_\mu A_\nu W_\nu^+ W_\mu^-) + g^2 s_w c_w[A_\mu Z_\nu^0(W_\mu^+ W_\nu^- - \\
& W_\nu^+ W_\mu^-) - 2A_\mu Z_\mu^0 W_\nu^+ W_\nu^-] - g\alpha[H^3 + H\phi^0 \phi^0 + 2H\phi^+ \phi^-] - \\
& \frac{1}{8}g^2 \alpha_h[H^4 + (\phi^0)^4 + 4(\phi^+ \phi^-)^2 + 4(\phi^0)^2 \phi^+ \phi^- + 4H^2 \phi^+ \phi^- + 2(\phi^0)^2 H^2] - \\
& gM W_\mu^+ W_\mu^- H - \frac{1}{2}g\frac{M}{c_w^2}Z_\mu^0 Z_\mu^0 H - \frac{1}{2}ig[W_\mu^+(\phi^0 \partial_\mu \phi^- - \phi^- \partial_\mu \phi^0) - \\
& W_\mu^-(\phi^0 \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^0)] + \frac{1}{2}g[W_\mu^+(H \partial_\mu \phi^- - \phi^- \partial_\mu H) - W_\mu^-(H \partial_\mu \phi^+ - \\
& \phi^+ \partial_\mu H)] + \frac{1}{2}g\frac{1}{c_w}(Z_\mu^0(H \partial_\mu \phi^0 - \phi^0 \partial_\mu H) - ig\frac{s_w^2}{c_w}MZ_\mu^0(W_\mu^+ \phi^- - W_\mu^- \phi^+) + \\
& ig s_w M A_\mu(W_\mu^+ \phi^- - W_\mu^- \phi^+) - ig\frac{1-2c_w^2}{2c_w}Z_\mu^0(\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) + \\
& ig s_w A_\mu(\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) - \frac{1}{4}g^2 W_\mu^+ W_\mu^- [H^2 + (\phi^0)^2 + 2\phi^+ \phi^-] - \\
& \frac{1}{4}g^2 \frac{1}{c_w^2}Z_\mu^0 Z_\mu^0 [H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2 \phi^+ \phi^-] - \frac{1}{2}g^2 \frac{s_w^2}{c_w}Z_\mu^0 \phi^0 (W_\mu^+ \phi^- + \\
& W_\mu^- \phi^+) - \frac{1}{2}ig^2 \frac{s_w^2}{c_w}Z_\mu^0 H (W_\mu^+ \phi^- - W_\mu^- \phi^+) + \frac{1}{2}g^2 s_w A_\mu \phi^0 (W_\mu^+ \phi^- + \\
& W_\mu^- \phi^+) + \frac{1}{2}ig^2 s_w A_\mu H (W_\mu^+ \phi^- - W_\mu^- \phi^+) - g^2 \frac{s_w}{c_w}(2c_w^2 - 1)Z_\mu^0 A_\mu \phi^+ \phi^- - \\
& g^1 s_w^2 A_\mu A_\mu \phi^+ \phi^- - \bar{e}^\lambda (\gamma \partial + m_e^\lambda) e^\lambda - \bar{\nu}^\lambda \gamma \partial \nu^\lambda - \bar{u}_j^\lambda (\gamma \partial + m_u^\lambda) u_j^\lambda - \\
& \bar{d}_j^\lambda (\gamma \partial + m_d^\lambda) d_j^\lambda + ig s_w A_\mu [-(\bar{e}^\lambda \gamma^\mu e^\lambda) + \frac{2}{3}(\bar{u}_j^\lambda \gamma^\mu u_j^\lambda) - \frac{1}{3}(\bar{d}_j^\lambda \gamma^\mu d_j^\lambda)] + \\
& \frac{ig}{4c_w}Z_\mu^0 [(\bar{\nu}^\lambda \gamma^\mu (1 + \gamma^5) \nu^\lambda) + (\bar{e}^\lambda \gamma^\mu (4s_w^2 - 1 - \gamma^5) e^\lambda) + (\bar{u}_j^\lambda \gamma^\mu (\frac{4}{3}s_w^2 - \\
& 1 - \gamma^5) u_j^\lambda) + (\bar{d}_j^\lambda \gamma^\mu (1 - \frac{8}{3}s_w^2 - \gamma^5) d_j^\lambda)] + \frac{ig}{2\sqrt{2}}W_\mu^+ [(\bar{\nu}^\lambda \gamma^\mu (1 + \gamma^5) e^\lambda) + \\
& (\bar{u}_j^\lambda \gamma^\mu (1 + \gamma^5) C_{\lambda\kappa} d_j^\kappa)] + \frac{ig}{2\sqrt{2}}W_\mu^- [(\bar{e}^\lambda \gamma^\mu (1 + \gamma^5) \nu^\lambda) + (\bar{d}_j^\kappa C_{\lambda\kappa}^\dagger \gamma^\mu (1 + \\
& \gamma^5) u_j^\lambda)] + \frac{ig}{2\sqrt{2}}\frac{m_\lambda^\lambda}{M}[-\phi^+(\bar{\nu}^\lambda (1 - \gamma^5) e^\lambda) + \phi^-(\bar{e}^\lambda (1 + \gamma^5) \nu^\lambda)] - \\
& \frac{g}{2}\frac{m_e^\lambda}{M}[H(\bar{e}^\lambda e^\lambda) + i\phi^0(\bar{e}^\lambda \gamma^5 e^\lambda)] + \frac{ig}{2M\sqrt{2}}\phi^+[-m_d^\kappa(\bar{u}_j^\lambda C_{\lambda\kappa}(1 - \gamma^5) d_j^\kappa) + \\
& m_u^\lambda(\bar{u}_j^\lambda C_{\lambda\kappa}(1 + \gamma^5) d_j^\kappa) + \frac{ig}{2M\sqrt{2}}\phi^-[m_d^\lambda(\bar{d}_j^\lambda C_{\lambda\kappa}^\dagger(1 + \gamma^5) u_j^\kappa) - m_u^\kappa(\bar{d}_j^\lambda C_{\lambda\kappa}^\dagger(1 - \\
& \gamma^5) u_j^\kappa) - \frac{g}{2}\frac{m_d^\lambda}{M}H(\bar{u}_j^\lambda u_j^\lambda) - \frac{g}{2}\frac{m_d^\lambda}{M}H(\bar{d}_j^\lambda d_j^\lambda) + \frac{ig}{2}\frac{m_u^\lambda}{M}\phi^0(\bar{u}_j^\lambda \gamma^5 u_j^\lambda) - \\
& \frac{ig}{2}\frac{m_d^\lambda}{M}\phi^0(\bar{d}_j^\lambda \gamma^5 d_j^\lambda) + \bar{X}^+(\partial^2 - M^2)X^+ + \bar{X}^-(\partial^2 - M^2)X^- + \bar{X}^0(\partial^2 - \\
& \frac{M^2}{c_w^2})X^0 + \bar{Y}\partial^2 Y + igc_w W_\mu^+(\partial_\mu \bar{X}^0 X^- - \partial_\mu \bar{X}^+ X^0) + ig s_w W_\mu^+(\partial_\mu \bar{Y} X^- - \\
& \partial_\mu \bar{X}^+ Y) + igc_w W_\mu^-(\partial_\mu \bar{X}^- X^0 - \partial_\mu \bar{X}^0 X^+) + ig s_w W_\mu^-(\partial_\mu \bar{X}^- Y - \\
& \partial_\mu \bar{Y} X^+) + igc_w Z_\mu^0(\partial_\mu \bar{X}^+ X^+ - \partial_\mu \bar{X}^- X^-) + ig s_w A_\mu(\partial_\mu \bar{X}^+ X^+ - \\
& \partial_\mu \bar{X}^- X^-) - \frac{1}{2}gM[\bar{X}^+ X^+ H + \bar{X}^- X^- H + \frac{1}{c_w^2}\bar{X}^0 X^0 H] + \\
& \frac{1-2c_w^2}{2c_w}igM[\bar{X}^+ X^0 \phi^+ - \bar{X}^- X^0 \phi^-] + \frac{1}{2c_w}igM[\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-] + \\
& igMs_w[\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-] + \frac{1}{2}igM[\bar{X}^+ X^+ \phi^0 - \bar{X}^- X^- \phi^0]
\end{aligned}$$