Significance and Confidence of an $J/\psi$ Yield from pp Collisions at STAR using the $\text{CL}_S$ Method

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Abstract. The modification of heavy quarkonium states in heavy ion collisions can give valuable information on the state of matter after these collisions. In order to accurately measure the suppression of $J/\psi$ in high energy density matter, an accurate calculation of the $J/\psi$ invariant yield in pp collisions is necessary. The $\text{CL}_S$ method can allow a determination of the confidence and significance of a particular signal and background by comparing measured values to simulated pseudo-experiments.

INTRODUCTION

It has been predicted that high energy heavy ion collisions will result in the formation of a quark gluon plasma (QGP), where the quarks are in a deconfined state. Heavy quarkonium can give information on this state, as the dominant creation mechanism is hard parton-parton interactions where $Q \bar{Q}$ pairs are created.[1] $J/\psi$ suppression had been calculated with lattice QCD to be a signal of QGP formation, however experimental measurements at NA50 and at RHIC have shown this to be a simplistic view.[1][7] The NA50 Collaboration compared the $J/\psi$ production in Pb-Pb collisions, with a beam of 158 GeV/c per nucleon on a fixed target, to that of pp collisions. They observed a $J/\psi$ suppression of a factor of 2 with respect to the background Drell-Yan leptons.[2]

The NA50 Pb-Pb collisions were done at $\sqrt{s}$ of 17.4 GeV/c per nucleon. The Relativistic Heavy Ion Collider (RHIC) collides Au-Au at $\sqrt{s}$ of 200 GeV/c per nucleon. The Solenoidal Tracker at RHIC (STAR) has $4\pi$ coverage and can accurately measure the $e^+e^-$ decay channel of $J/\psi$ in order to measure the invariant yield. $J/\psi$ yields will be measured in pp, d+Au and Au-Au collisions so that a properly scaled ratio of the $J/\psi$ yield in pp to Au-Au collisions, with nuclear modification corrections can be measured.

FIGURE 1. Measured Signal + Background resulting from $e^+e^-$ invariant mass calculations
FIGURE 2. Measured Background resulting from combining the e+e+ and e-e- invariant calculations with the formula B = 2(N++ + N--)^{1/2}

PSEUDO-EXPERIMENTS AND THE $CL_S$ METHOD

The $CL_S$ method is a method for determining the confidence level of a signal. The indicator, $CL_S$ is defined as the ratio of $CL_{sb}/CL_b$, where $CL_{sb}$ is the confidence in the signal plus the background and $CL_b$ is the confidence in the background. This means that the significance of the test is $1 - CL_b$. In order to apply this method, a quantity $Q$, is defined as $Q = L(b+s)/L(b)$ where $L(b+s)$ is the likelihood that a particular data set is background plus signal and $L(b)$ is the likelihood that a particular data set is background only.

$L(\bar{\alpha}) = \prod_{i=1}^{N} P(y_i(x_i); \bar{y}(\bar{\alpha}))$

Where $y_i(x_i)$ is the set of measurements and $\bar{y}(\bar{\alpha})$ is a functional form that can be used to describe the data.[6] $Q' = -2\ln(Q)$ is calculated for a number of pseudo-experiments which are background plus signal, and a number that are background only. The resulting distribution of $Q'$ will involve two peaks. $Q'_{obs}$ is determined from the data, and then the values of $CL_{sb}$ and $CL_b$ are calculated by:[5]

$1 - CL_b \equiv \int^{Q'_{obs}}_{-\infty} P(Q'; b) dQ'$

$CL_{sb} \equiv \int^{\infty}_{Q'_{obs}} P(Q'; s+b) dQ'$

The data analyzed in this paper was from the RHIC 2006 pp run, with a trigger requirement detailed in reference[3]. In order to increase the efficiency of selecting $J/\psi$ e+e- decay products, two cuts were made on the data. First only particles with $p > 1.5$ GeV/c were used in order to increase the ratio of $J/\psi$ decay products to the background. To separate e+ and e- particles a cut on $\log(10^{6} dE/dx) > 1.25$ was used. Using the Bichsel curves and STAR TPC resolution, this resulted in a set of data with a calculated purity of 93.0% and a calculated efficiency of 85.6% with respect to e+e-.[4]

For each event the invariant masses of e+e-, e+e+, e-e- combinations were calculated and added to histograms labeled by $N_{++}$, $N_{++}$ and $N_{--}$. $N_{--}$ will contain both the signal, invariant masses which point to a $J/\psi$, and the background, invariant masses which point to other products decaying through this channel as well as the combinatoric background. A combination defined as $B = 2(N++ + N--)^{1/2}$ will contain the background.[2] The signal can be calculated using the formula $S = N_{--} - B$.

Simulations of $J/\psi$ background and background plus signal events were not available, so the data was used to generate Monte Carlo simulations of the background and the background plus signal events. It was assumed that
Poisson statistics governed the number of counts in each mass bin. An estimated value of $\mu = \text{number of counts in the measured mass bin}$ was used in the generation of the pseudo-experiment data sets. The average error in $\mu$ in the signal plus background sets is 19.8%, and for the background only sets the average error in $\mu$ is 20.4%. A generated data set was calculated by using a random number, $P(n)$, generated on a flat distribution between 0 and 1 for each bin. The number of counts in each bin was then calculated such that:

$$P(n) = \sum_{i} \frac{\mu^{i} e^{-\mu}}{i!}$$

For both the background and the signal plus background pseudo-experiments, this resulted in a Poisson distribution with a mean number of particles per data set within one standard deviation of the measured number of particles, and a standard deviation equal to the square root of the mean.

In order to calculate the likelihood of either a signal plus background or a background event, an assumed function for both the background and the signal is needed. For the signal function, a Gaussian normalized on the range 2 to 6
FIGURE 5. Gaussian fit on background histogram, B, using Root’s Minuit minimization routine.

FIGURE 6. Likelihood versus background and signal amplitudes for a generated background pseudo-experiment.

GeV was fit to S, giving a signal function of $\tilde{y}_s = 3.1941e^{-(x-3.068)^2/0.0312}$. For the background, a Gaussian normalized on the range of 2 to 6 GeV was fit to B given a background function of $\tilde{y}_b = 0.39905e^{-(x-3.081)^2/3.1651}$.

RESULTS

A set of 1500 generated background and 1500 background plus signal pseudo-experiments were used in this analysis. The Likelihood of a Poisson distributed number is:

$$L(\tilde{\alpha}) = \prod_i \frac{\mu_i e^{-\mu_i}}{y_i!}$$

where N = 20, the number of bins used in this analysis, $\mu_i$ is the estimated number of counts in the ith bin for a given signal and background amplitudes, and $y_i$ is the number of counts in the ith bin of the particular pseudo-experiment.
under examination. For each pseudo-experiment, the maximum likelihood versus the background amplitude was
computed and this value was used for $L(b)$. The maximum likelihood versus the background and signal amplitudes
was calculated and used for $L(s+b)$. $Q = L(s+b)/L(b)$ was put into a histogram for each pseudo-experiment, yielding
two different distributions for the signal plus background versus the background only pseudo-experiments.

The $Q' = -2\ln(Q)$ histogram was fit with a gaussian plus an exponential, and normalized to give a $Q'$ versus
probability density function. An exponential was chosen because the definitions of $L(b), L(b+s)$ and $Q$ led to a cut
off at $Q' = 0$. The experimental value of $Q'$ was determined to be $-26.3423$ with the maximum Likelihood at 665
background particles and 71 J/$\psi_s$. $CL_b$ was calculated to be 0.99991 and $CL_{sb}$ was calculated to be 0.264416, the ratio
of which gives a value of $CL_S = 0.26444$. This indicates that the confidence in the background is 1, the confidence in
the signal plus background is 26.4% and the test significance is only 0.00009.
FIGURE 9. Distribution of -2lnQ versus probability density with measured value of Q. The area in red is equal to $CL_{sb}$. The area under the blue curve to the left of $Q_{obs}$ is $1 - CL_b$

CONCLUSION

The $CL_S$ method is much better suited to discovery than it is to a measurement such as this. In order to more accurately use the $CL_S$ method for this type of analysis, several issues will need to be addressed. It is important to determine how the range in the functional form of the signal and background effects the result. A first order study could be done simply by repeating the analysis done in this paper while varying the parameters. The functional form of the signal is dependent on the detector resolution, so it should be possible to determine this form without directly measuring the data we wish to analyze. Another source of error was in the assumed average number of counts in each mass bin for both the signal plus background and the background only cases. This may be improved by simulating signal plus background and background events. At the very least this would allow improved systematics. Perhaps this method can be used in the attempt to understand the $J/\psi$ suppression in heavy ion collisions where there are more background effects which are of the same order as the calculated suppression due to QGP.

REFERENCES