

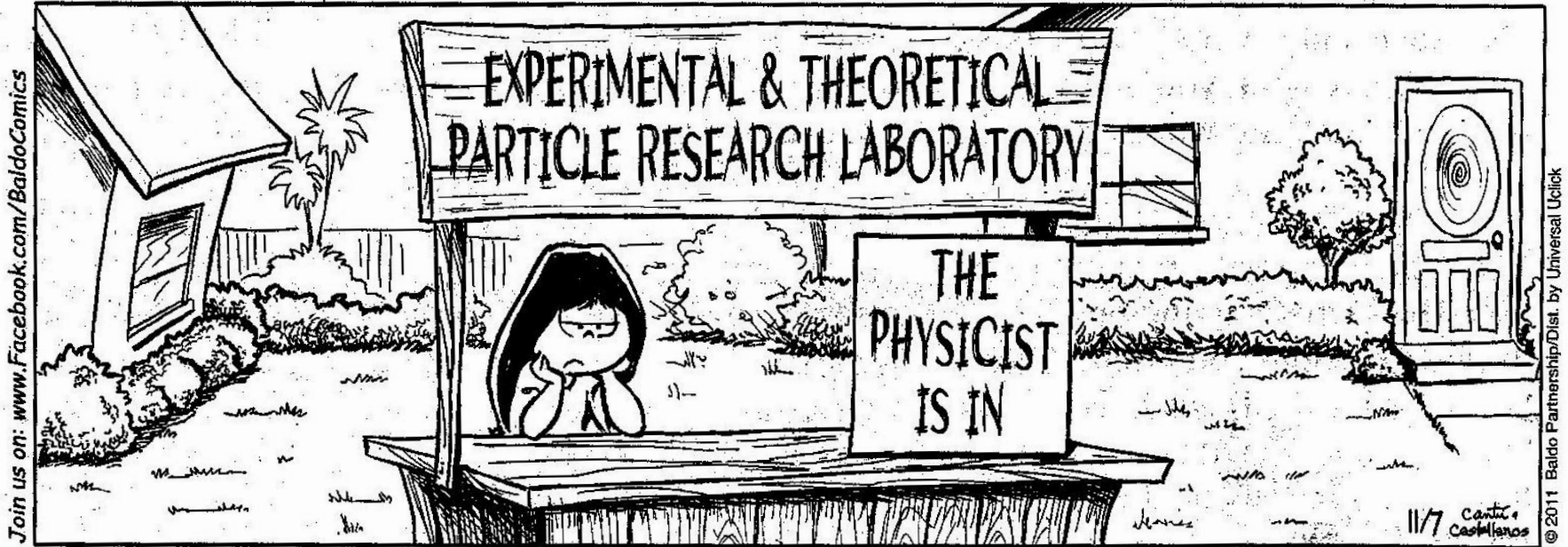
# Determining the Uncertainty on the Charm Cross Section and the Effect on the $J/\psi$ Cross Section

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31 Mar 2012

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# Baldo Cantú and Castellanos



# Outline

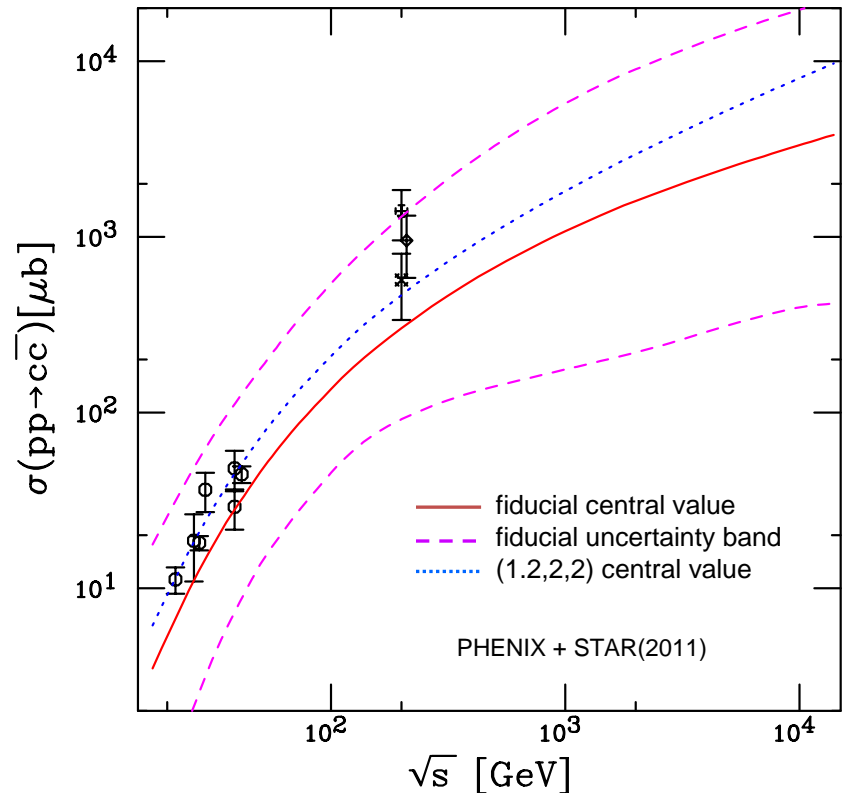
- Current approach to pQCD uncertainties
- Open heavy flavor theory
- Exploring parameter space
- “Best Fit” total charm cross section
- Applying the best fit parameters

# Motivation

- QCD calculation is OK, it's the inputs that need examination
- Heavy quark cross section uncertainty calculations need a more rigorous foundation
- “Central value” and uncertainties are somewhat ad hoc (need systematizing)
- “Invisible variables” (scale choice) should have no effect on final values

# An Uncertain World

- FONLL uses “fiducial” inputs
  - Central value:  
 $m_c = 1.5 \text{ GeV}/c^2$ ;  $\mu_F/m_c = \mu_R/m_c = 1$
  - Mass variance:  
 $m_c$  in  $[1.3, 1.5, 1.7] \text{ GeV}/c^2$  (@ central scale)
  - Scale variance:  
 $\mu_{[FR]}/m_c$  in  $[0.5, 1, 2]$  (@ central mass)
  - Uncertainty band combines mass/scale variances in quadrature
- Problems with this approach
  - Inputs have unknown distributions and means
  - Subjective choice of inputs
  - Can't assign a confidence level to results
- Clearly a more rigorous and defensible method is needed
  - Examine response surface for flat spots (relatively insensitive to scale choice)
  - Employ PDG mass value
  - Try fitting to experimental data



NLO uncertainty with FONLL inputs

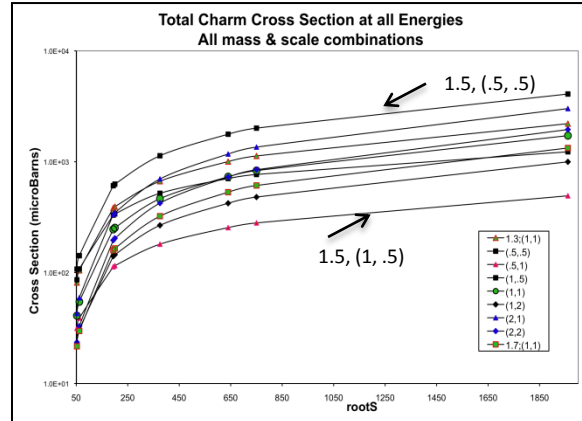
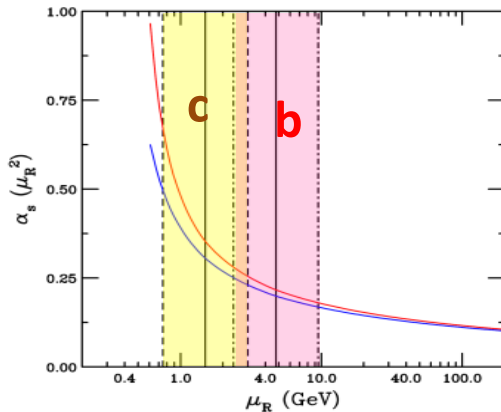
# Open Heavy Flavor Theory

$$\sigma_{pp}(s, m^2) = \sum_{i,j=q,\bar{q},g} \int dx_1 dx_2 f_i^p(x_1, \mu_F^2) f_j^p(x_2, \mu_F^2) \hat{\sigma}_{ij}(\hat{s}, m^2, \mu_F^2, \mu_R^2)$$

$$\text{where } \hat{\sigma}_{ij}(\hat{s}, m, \mu_F^2, \mu_R^2) = \frac{\alpha_s^2(\mu_R^2)}{m^2} \left\{ f_{ij}^{(0,0)}(\rho) + 4\pi\alpha_s(\mu_R^2) \left[ f_{ij}^{(1,0)}(\rho) + f_{ij}^{(1,1)}(\rho) \ln\left(\frac{\mu_F^2}{m^2}\right) \right] + \mathcal{O}(\alpha_s^2) \right\}$$

Examples of cross section sensitivity to fiducial values of quark mass and scales.

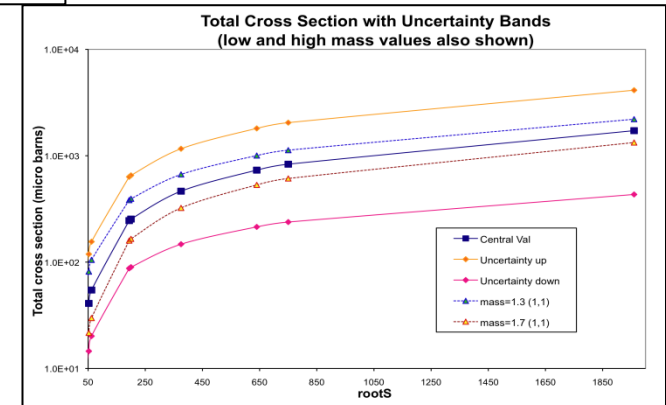
Running of  $\alpha_s$  with Renormalization Scale



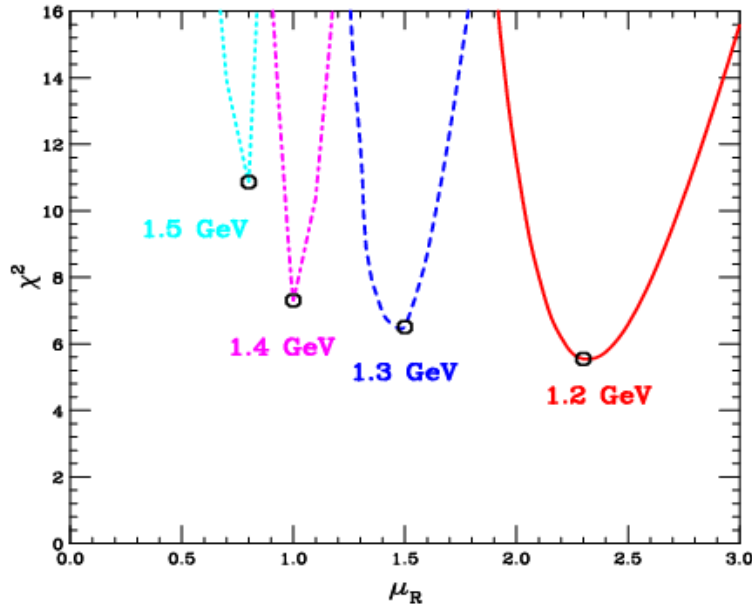
Example of variation in total XS span for the mass/scale combinations.

Mass	$(\mu_F, \mu_R) / m_T$
1.3	(1, 1)
1.5	(1, 1), (1, 2), (2, 1), (2, 2), (1, 0.5), (0.5, 1), (0.5, 0.5)
1.7	(1, 1)

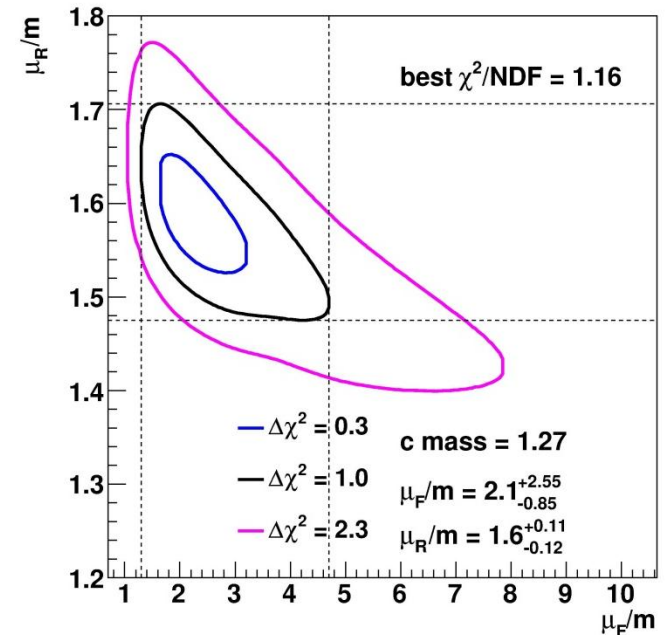
Variation in scale more significant than mass for fiducial scale choices



# Exploring Parameter Space to Improve Uncertainty



Early attempt to visualize the mass/scale parameter space. Here,  $\chi^2$  is plotted vs.  $\mu_R/m_T$  for fixed  $\mu_F/m_T$  at various heavy quark masses.



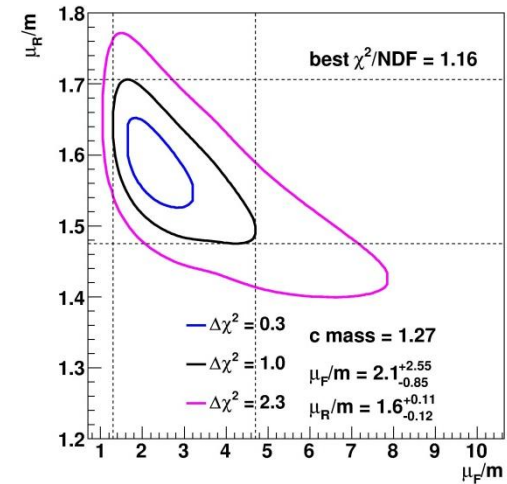
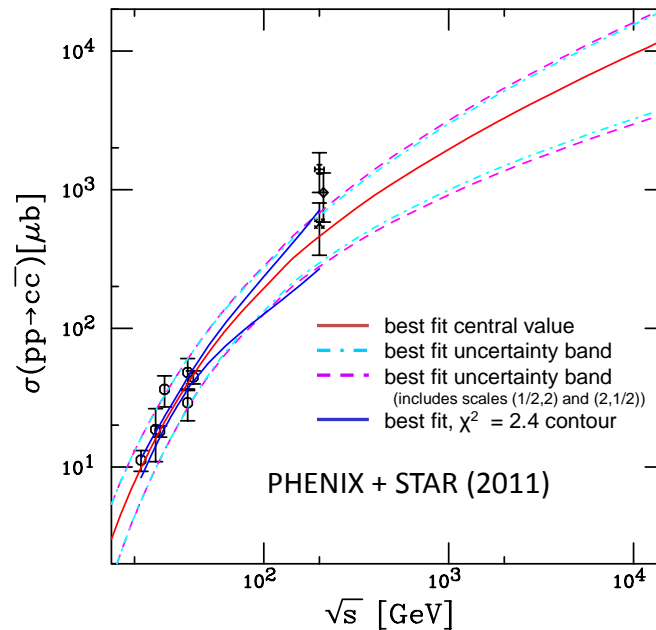
The  $\Delta\chi^2/\text{dof}$  contours for fixed target data, PHENIX 200 GeV cross section, STAR 2011 cross section (excluding STAR 2004). The “Best Fit” mass and scale values derived from the furthest extent of the  $\Delta\chi^2 = 1$  contours

# “Best Fit” Total Charm Cross Section

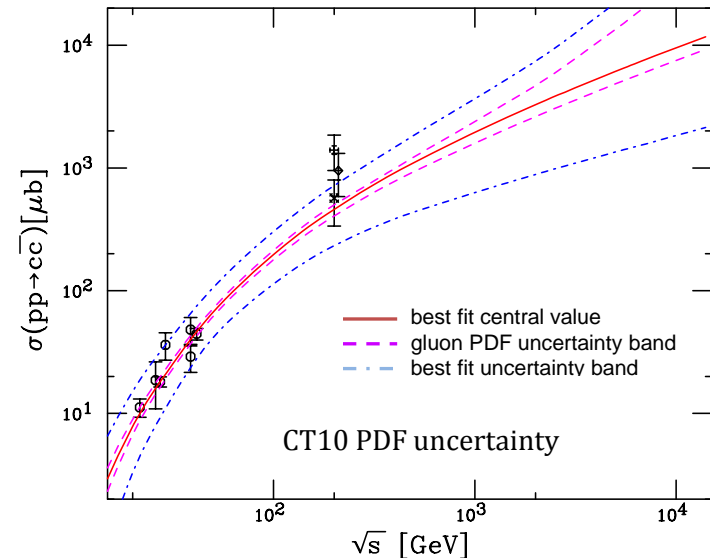
The total charm cross section central value using the PDG charm quark mass and “Best Fit” scales taken from the minimum of the  $\Delta\chi^2$  contours of the far right plot.

The outer dashed curves represent the uncertainty obtained from the PDG charm quark uncertainty and the furthest extent of the  $\Delta\chi^2 = 1$  contour.

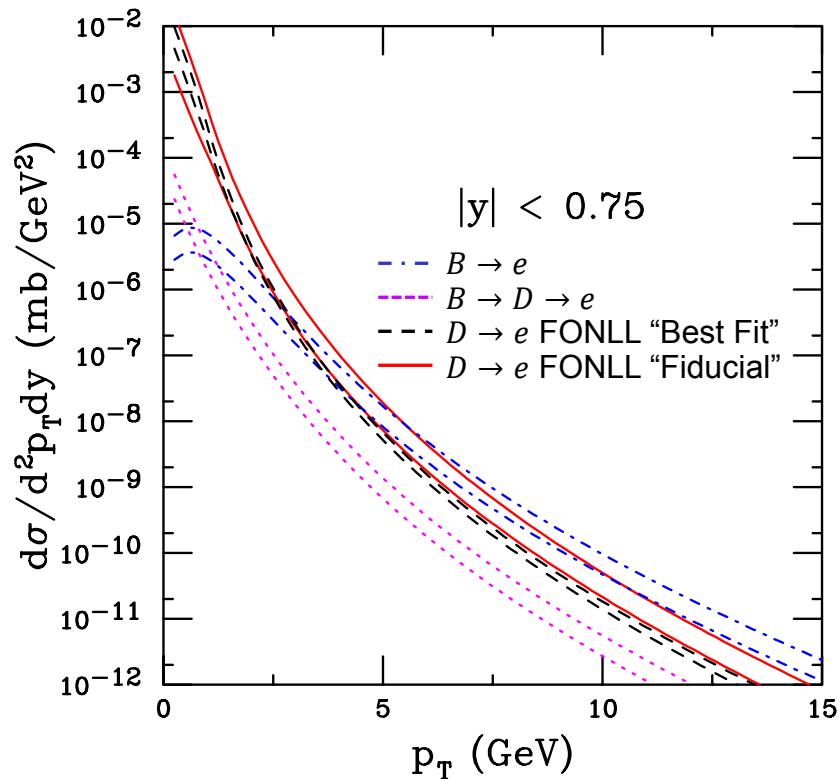
The solid blue curves ( $19.4 \leq \sqrt{s} \leq 200$  GeV) represent the uncertainties from the furthest extent of the  $\Delta\chi^2 = 2.3$  contour.



Contribution of CT10 gluon PDF uncertainty to total charm cross section is denoted by the dashed magenta lines. The total uncertainty due to the mass and scale uncertainty as well as the gluon uncertainty combined in quadrature is given by the blue curves.

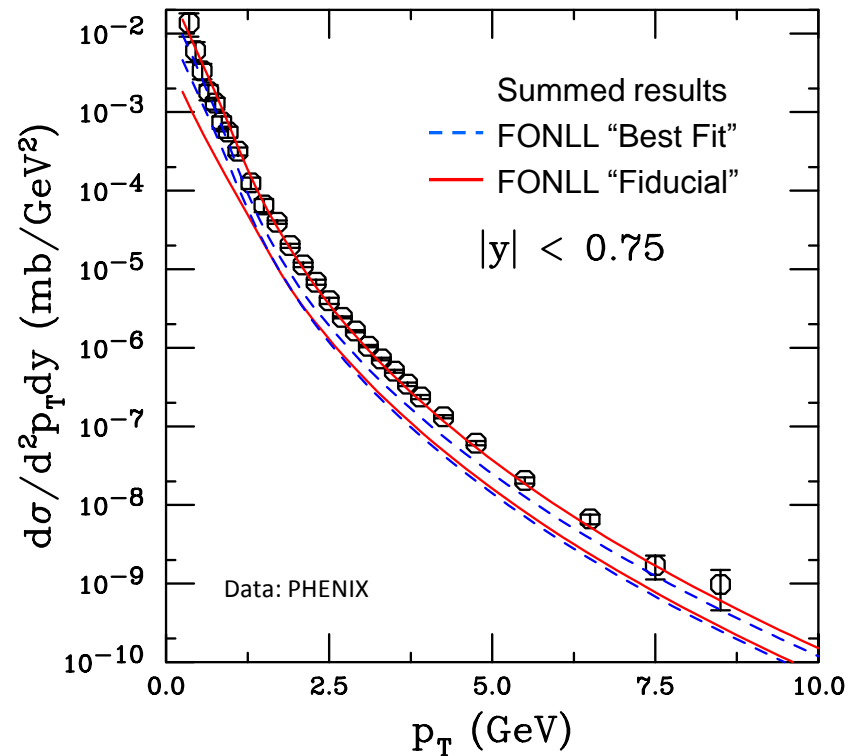


# The Non-Photonic Electron Spectrum



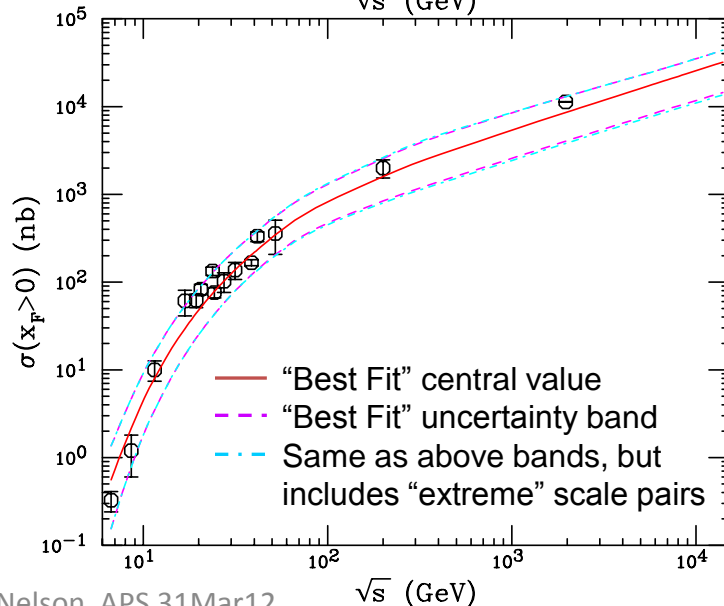
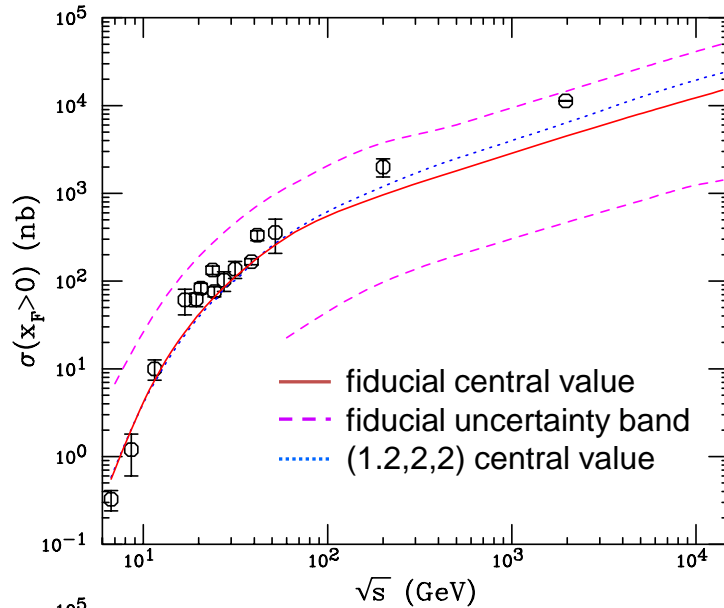
The calculated non-photonic electron spectrum for direct B,D  $\rightarrow$  e and B  $\rightarrow$  e feed down (B  $\rightarrow$  D  $\rightarrow$  e).

The D  $\rightarrow$  e uncertainty bands are shown using the FONLL fiducial and "Best Fit" mass and scales.



Results of summing the three components contributing to the electron spectrum, with the D  $\rightarrow$  e results from either the fiducial or best fit mass/scale parameter sets.

# $J/\psi$ Total Cross Section



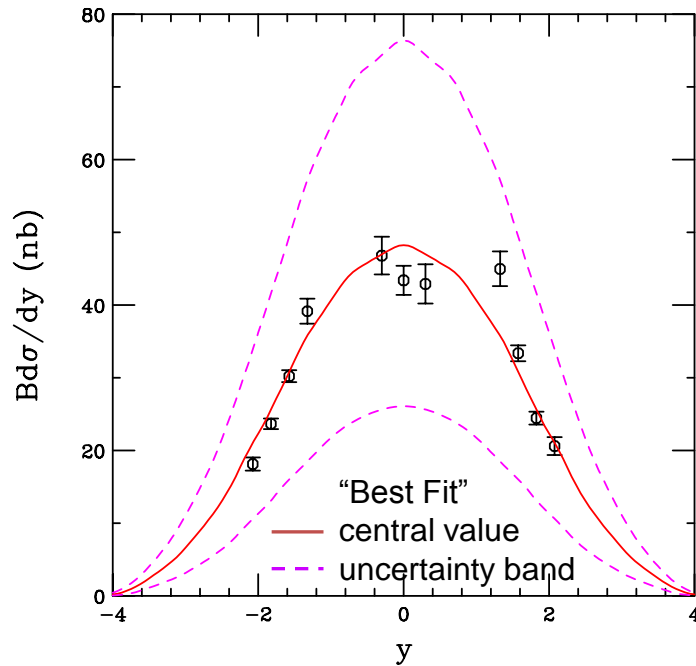
$J/\psi$  total cross section calculated using the Color Evaporation Model (CEM). In the CEM, quarkonium production is treated as  $c\bar{c}$  below the  $D\bar{D}$  threshold:

$$\sigma_Q^{\text{CEM}} = F_Q \sum_{ij} \int_{4m_Q^2}^{4m_H^2} d\hat{s} \times \int dx_1 dx_2 f_{i/p}(x_1, \mu^2) f_{j/p}(x_2, \mu^2) \hat{\sigma}_{ij}(\hat{s}) \delta(\hat{s} - x_1 x_2 s)$$

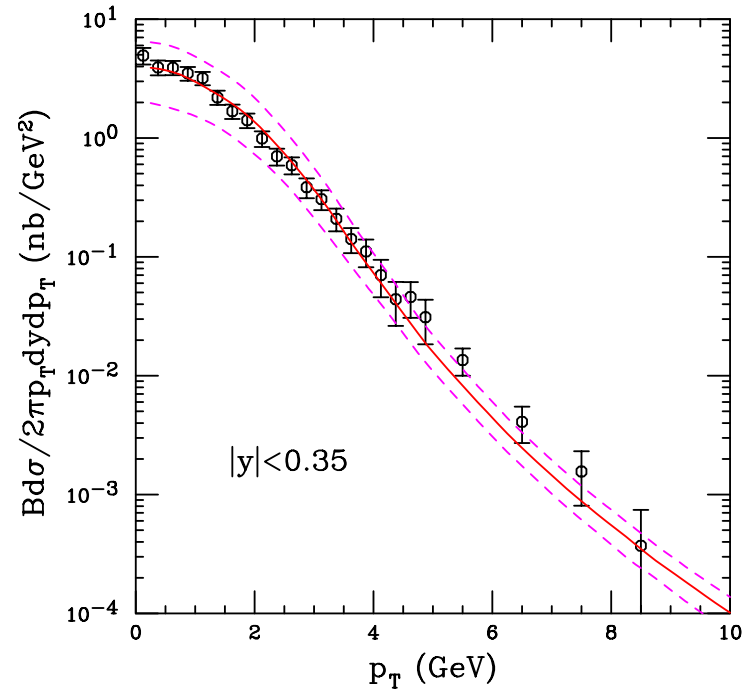
The different charmonium states are assumed to have the same  $\sqrt{s}$ ,  $p_T$ , and  $x_F$  dependence. The normalization,  $F_Q$ , is fit to the central value of the inclusive  $J/\psi$  production cross section as a function of  $\sqrt{s}$ . Other sets are scaled by central set  $F_Q$ .

The lower uncertainty band does not fit well for fiducial values -- undefined below  $\sqrt{s} < 60$  GeV. The "Best fit" charm parameters give good agreement and tighter constraints on the  $J/\psi$  cross section.

# $J/\psi$ Kinematic Distributions -- RHIC @ 200 GeV



Rapidity distribution for  $J/\psi$  at  $\sqrt{s} = 200$  GeV using the “Best Fit” parameters, compared to PHENIX data



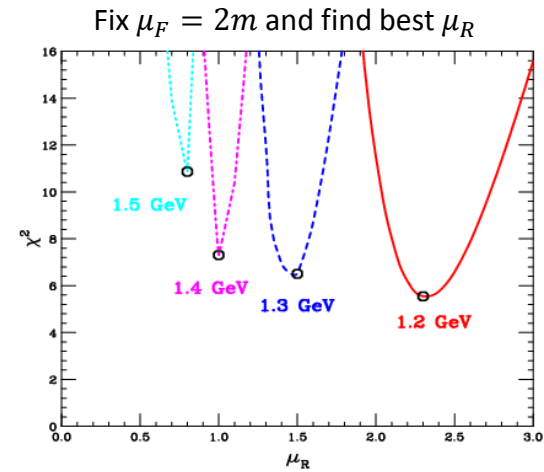
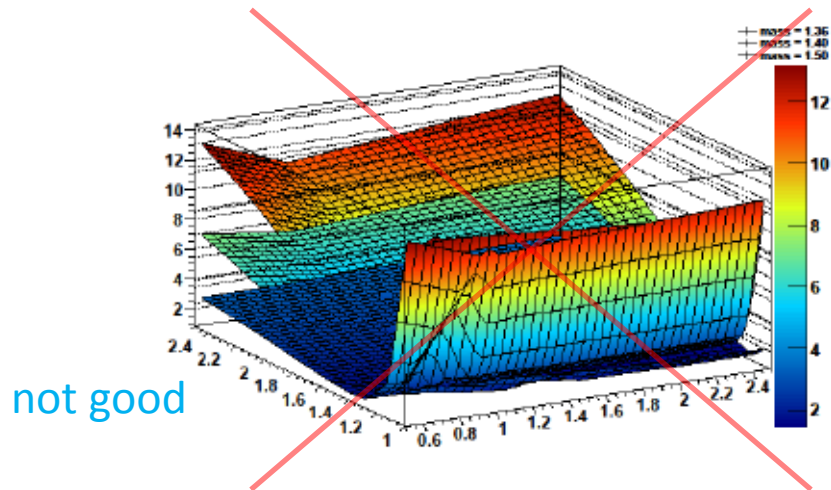
$p_T$  distribution (mid-rapidity) for  $J/\psi$  at  $\sqrt{s} = 200$  GeV using the “Best Fit” parameters, compared to PHENIX data

## Summary/Conclusions

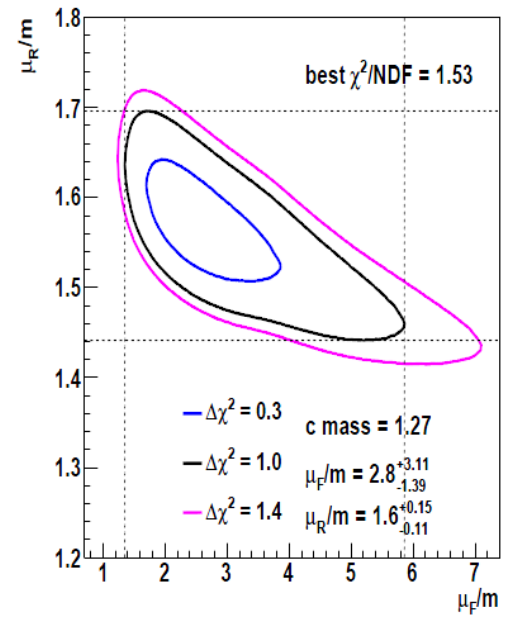
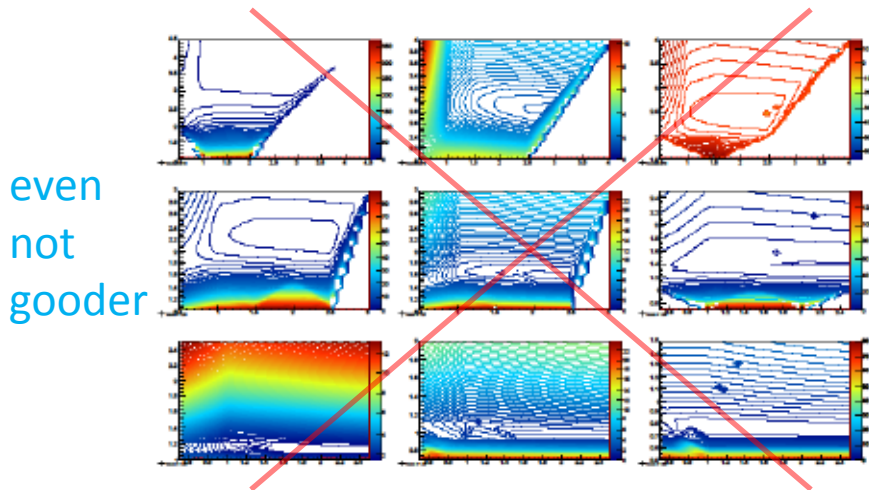
- Total charm cross sections uncertainties can be tamed by suitable choices of charm quark mass and scales
- Fiducial mass/scales provide a reasonable first guess
- Best fit mass/scales place central values and uncertainties on a more solid basis
- Best fit values lead to much tighter uncertainty bands
- $J/\psi$  predictions compare favorably to experimental results, even with the tighter uncertainty bands



# Learning How to See in Four-D



better



even better

# The “HowTo” on Uncertainty Calculations

- How it should be done:
  - Enumerate the input variables
  - Determine their statistical/systematic uncertainties
  - Central value based on means of all variables
  - Error (about the mean) =  $\sum_i \frac{\partial y}{\partial x_i} \Delta x_i$
- Why that won't work:
  - Don't know anything about input variables
  - Without knowing mean of inputs, where is central value?
  - No analytical function to apply error calculation

Problems with this approach

  - input variables have unknown distributions and unknown means
  - central value is too dependent on subjective choice of inputs

Can't assign CI to uncertainty results