Rotational Kinematics and Angular Momentum Conservation

Rotational Motion

If the force is always perpendicular to the direction of the velocity, then only the direction changes.

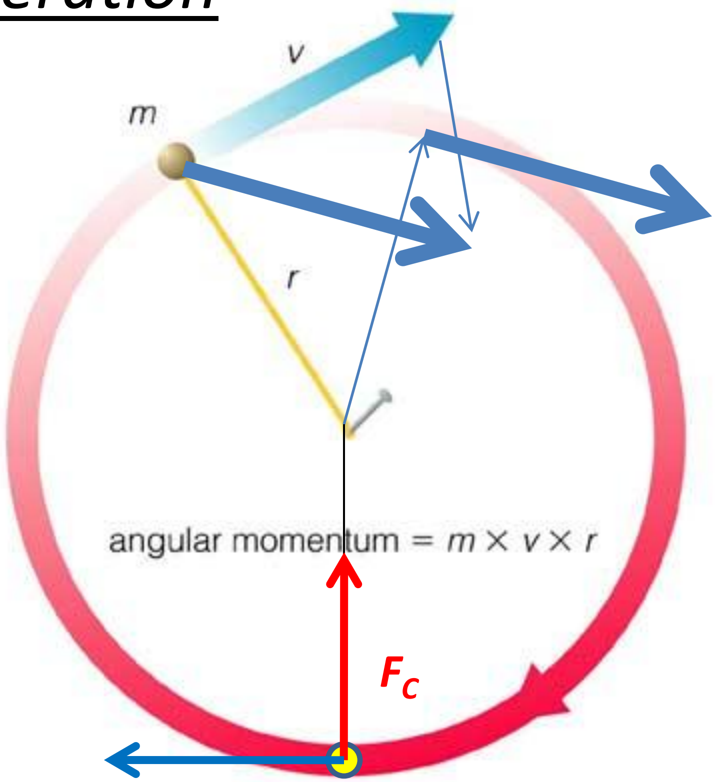
<u>*Centripetal Acceleration*</u>

<u>*Centripetal Force*</u>

$v = 2\pi r/T$

$$\frac{\Delta v}{v} = \frac{v\Delta t}{r}$$

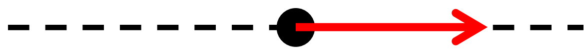
$$a_c = \frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

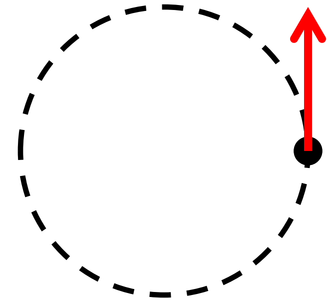


$F_c = ma_c = mv^2/r$

Angular Quantities

- There is an analogy between objects moving along a straight path and objects moving along a circular path.

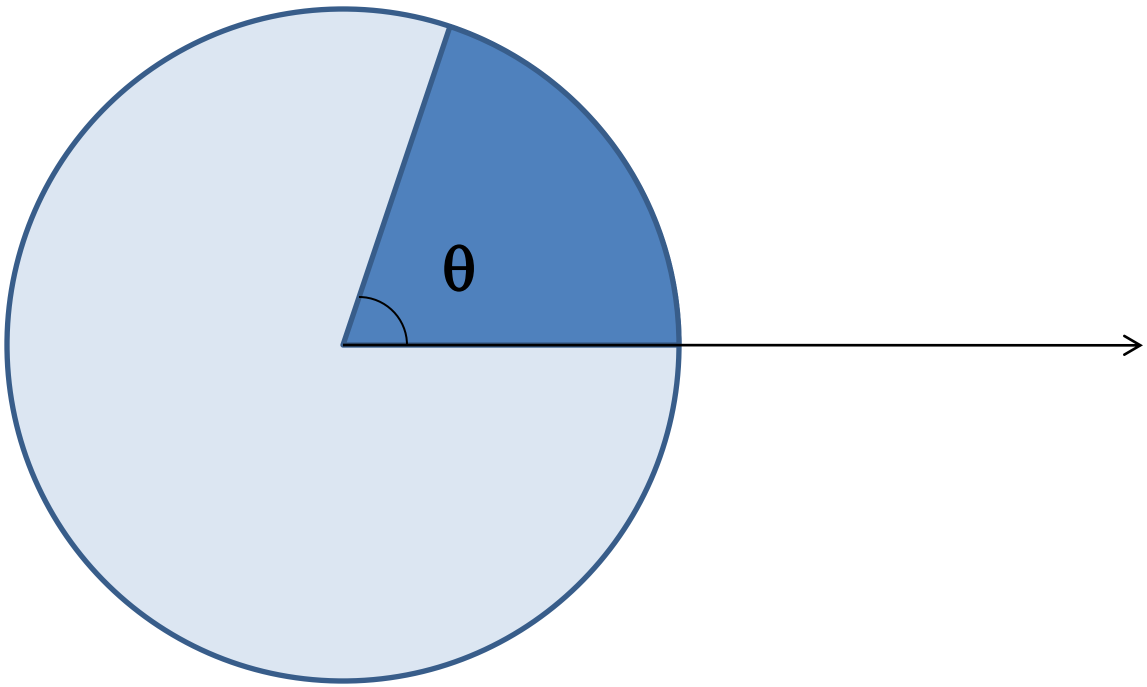




Linear		Rotational
Position (*x*)	↔	Angle (θ)
Velocity (***v***)	↔	Angular Velocity (ω)
Acceleration (***a***)	↔	Angular Acceleration (α)
Momentum (***p***)	↔	Angular Momentum (***L***)
Force (***F***)	↔	Torque (τ)
Mass (m)	↔	Moment of Inertia (*I*)

Angular Displacement

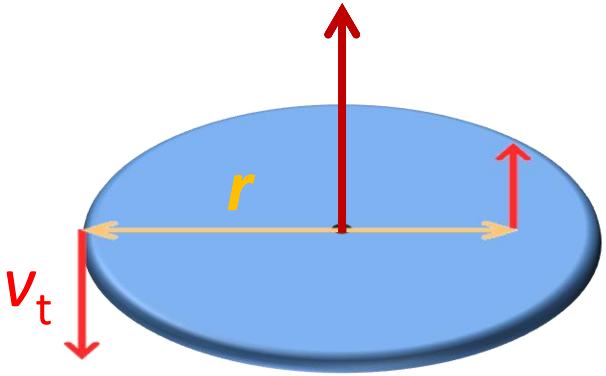
By convention, θ is measured clockwise from the *x*-axis.



Angular Velocity

- Angular velocity is a vector:
  - Right hand rule to determines direction of ω

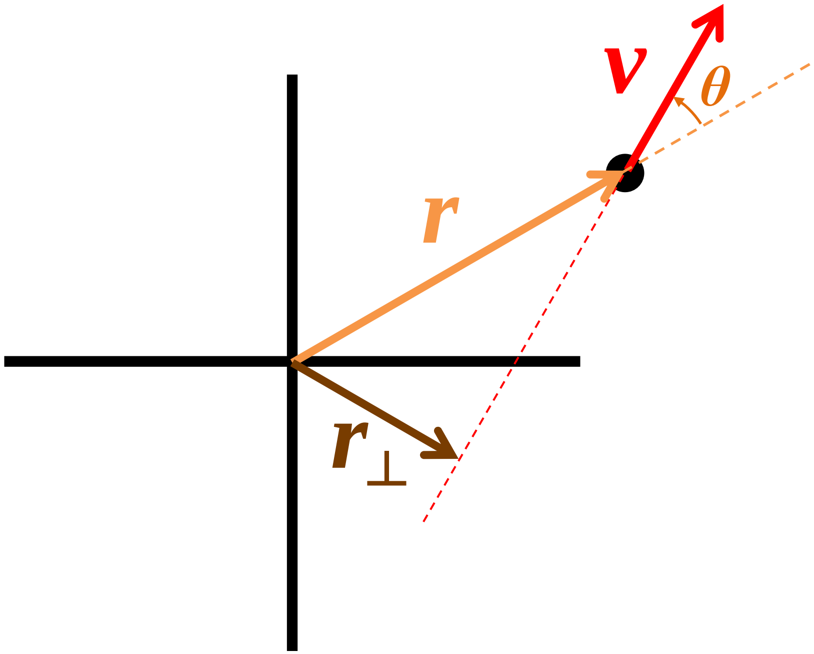
  - Velocity and radius determine magnitude of ω



$$\omega = \frac{v_t}{r}$$

Angular Velocity

- What's the angular velocity of a point particle?



$$\omega = \frac{v}{r_\perp} = \frac{v}{r\sin\theta}$$

Angular Acceleration

- Angular Acceleration: Rate of change in angular velocity

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt} = \frac{a_t}{r}$$

- A stationary disk begins to rotate. After 3 seconds, it is rotating at 60 rad/sec. What is the average angular acceleration?

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{60\,rad/s - 0\,rad/s}{3\,s} = 20\,\frac{rad}{s^2}$$

Rotational Kinematics

Rotational Motion (α = constant)	Linear Motion (a = constant)
$\omega = \omega_0 + \alpha t$	$v = v_0 + at$
$\theta = (1/2)(\omega_0 + \omega)t$	$x = (1/2)(v_0 + v)t$
$\theta = \theta_0 + \omega_0 t + (1/2)\alpha t^2$	$x = x_0 + v_0 t + (1/2)at^2$
$\omega^2 = \omega_0^2 + 2\alpha\theta$	$v^2 = v_0^2 + 2ax$

Rotational Kinematics

A commercial jet airplane is idling at the end of the run way. At idle, the turbo fans are rotating with an angular velocity of 175 rad/sec. The Tower gives the jetliner permission to take-off. The pilot winds up the jet engines with an angular acceleration of 175 rad/s^2. The engines wind up through an angular displacement of 2000 radians. What is the final angular velocity of the turbo-fan blades?

Solution:

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

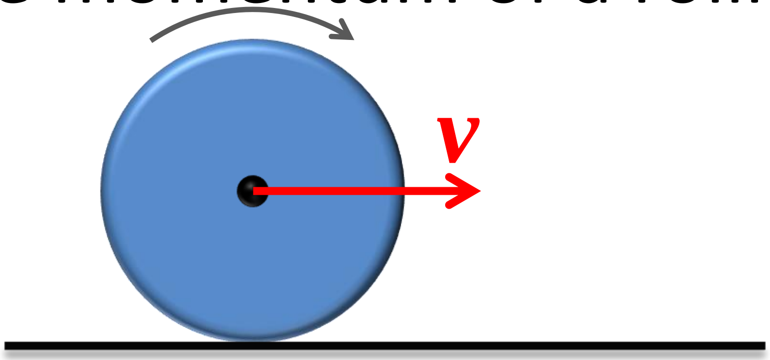
$$= (175 \text{ rad/s})^2 + 2(175 \text{ rad/s}^2)(2000 \text{ rad})$$

$$= 7.00 \times 10^5 \text{ rad}^2/\text{s}^2$$

$$\omega = 837 \text{ rad/s}$$

Angular Momentum

- What's the momentum of a rolling disk?



- Two types of motion:
  - Translation: $\vec{p} = m_{disk}\vec{v}$
  - Rotation: $\vec{L} = I_{disk}\vec{\omega}$

Angular Momentum

- Angular Momentum: Product of position vector and momentum vector

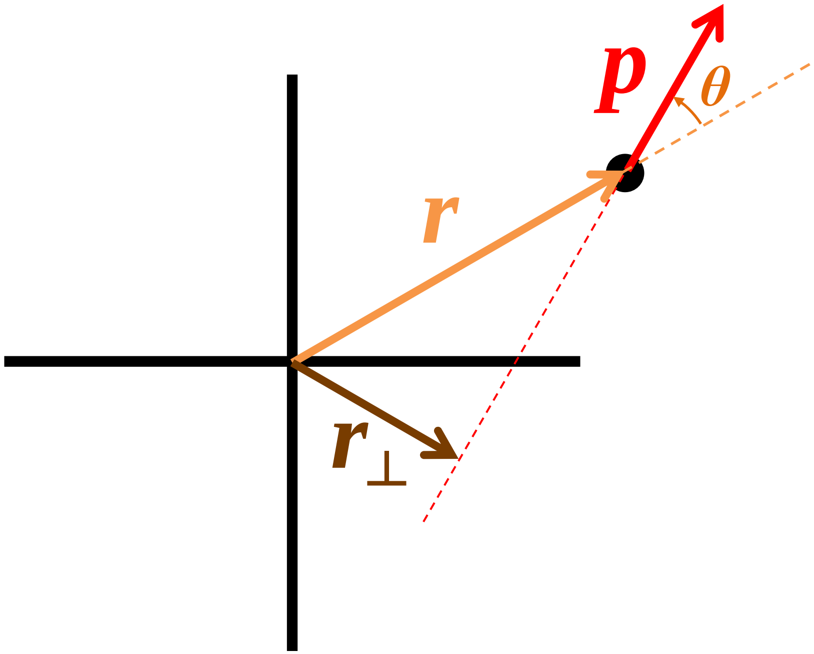
$$\vec{L} = \vec{r} \times \vec{p} = \left| rp\sin\theta \right|$$

- Why is angular momentum important? Like energy and momentum, angular momentum is conserved.
- Angular Impulse: Change in angular momentum vector

$$\Delta\vec{L} = \vec{L}_2 - \vec{L}_1$$

Angular Momentum

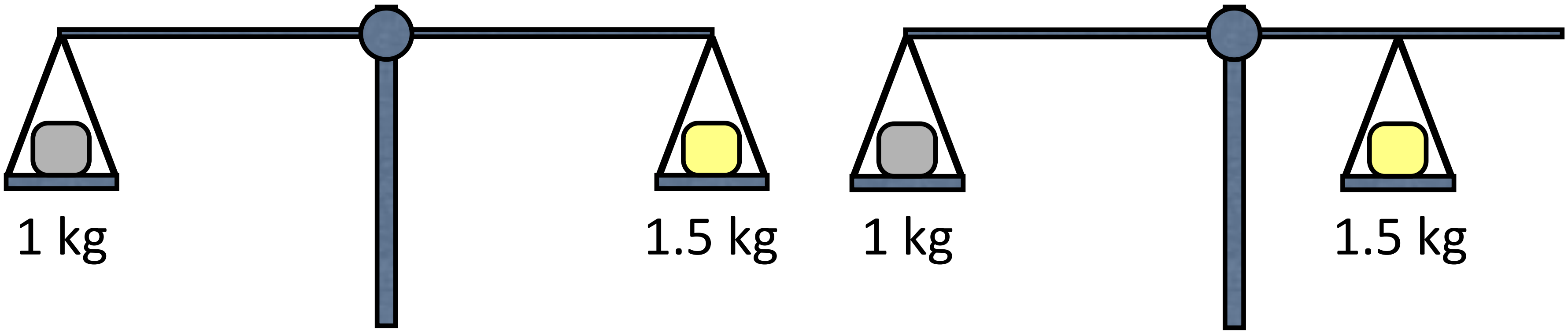
- What's the angular momentum of a point particle?



$$L = r_\perp p = r \sin\theta \; p$$

Torque

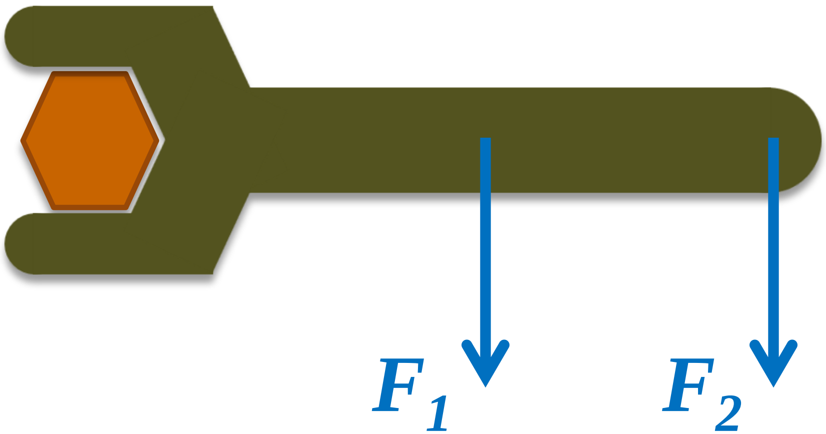
- Which way will the scale tip?



- Rotation of scale is influenced by:
  - Magnitude of forces
  - Location of forces

Torque

- Which is more effective?



- Rotation of wrench is influenced by:
  - Magnitude of forces
  - Location of forces
  - Direction of forces

DEMO: Torque Meter

Torque

- Torque: The cause or agent of angular acceleration $\vec{\tau} = \vec{r} \times \vec{F} = |rF\sin\theta|$
- The angular velocity of an object will not change unless acted upon by a torque
$$\vec{\tau}_{Net\ on\ Object} = 0 \quad \Rightarrow \quad \Delta\vec{\omega} = 0$$
- The net torque on an object is equal to the rate of change of angular momentum
$$\vec{\tau}_{Net\ on\ Object} = \frac{d\vec{L}}{dt}$$

Newton's Second Law - Rotation

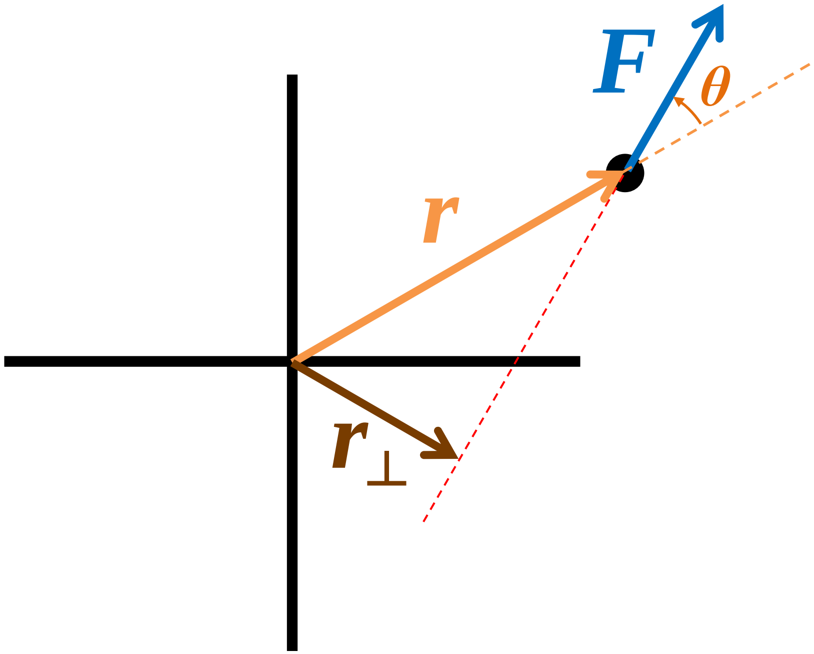
$$\vec{\tau}_{Net\ on\ Object} = \frac{d\vec{L}}{dt} = \frac{dI\omega}{dt} = I\frac{d\omega}{dt} = I\alpha$$

$$\tau = I\alpha$$

Angular impulse = $\tau\Delta t = \Delta \boldsymbol{L}$

Torque

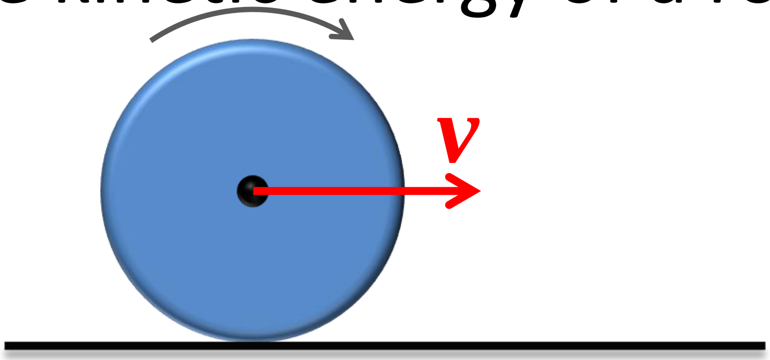
- What's the torque on a point particle?



$$\tau = r_{\perp} F = r \sin\theta \, F$$

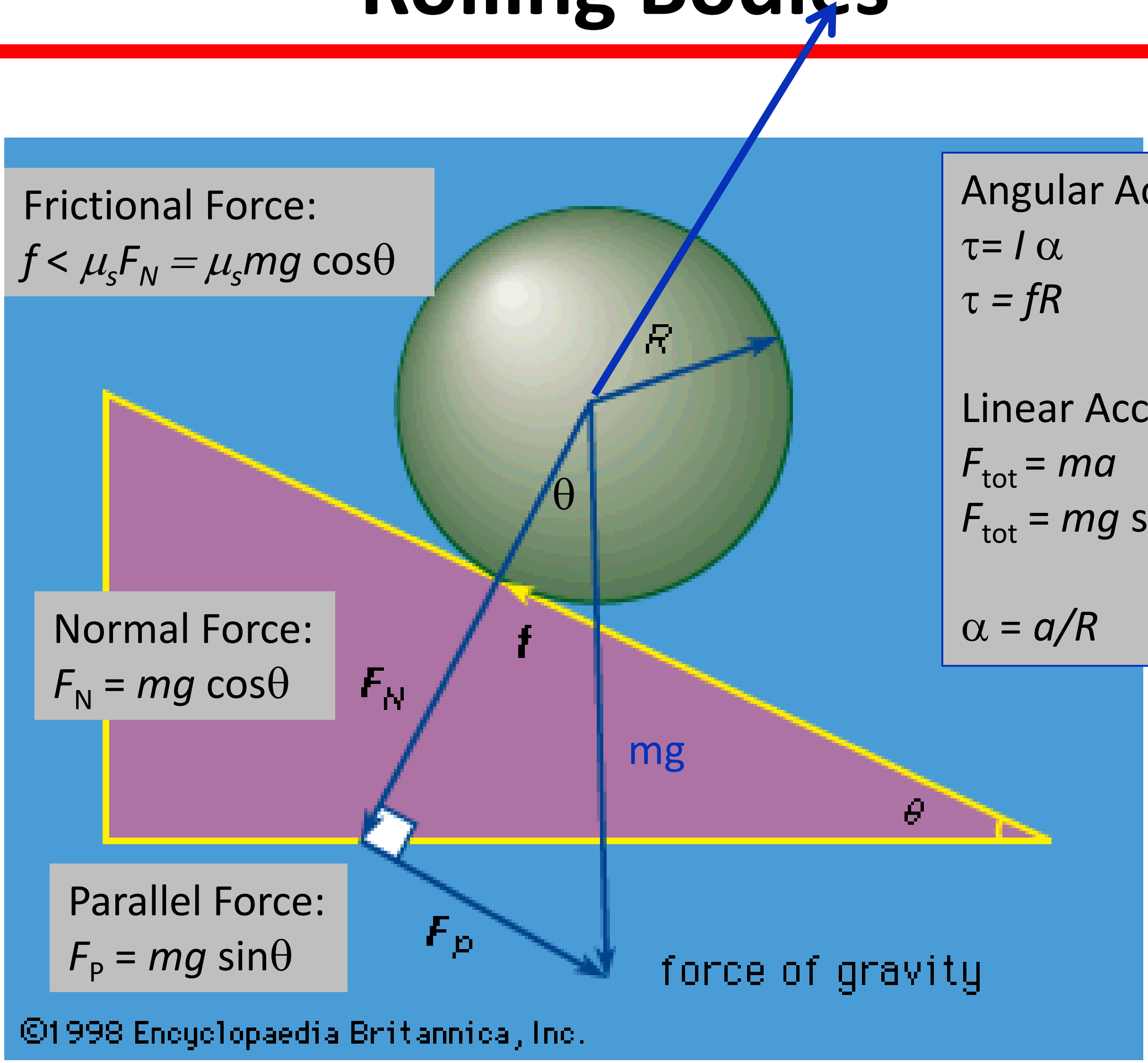
Rotational Kinetic Energy

- What's the kinetic energy of a rolling disk?



- Two types of motion:
  - Translation: $KE_{tran} = \frac{1}{2} m_{disk} v^2$
  - Rotation: $KE_{rot} = \frac{1}{2} I_{disk} \omega^2$

Rolling Bodies



Angular Acceleration

$\tau = I\,\alpha$

$\tau = fR$

Linear Acceleration

$F_{tot} = ma$

$F_{tot} = mg \sin\theta - f$

$\alpha = a/R$

Angular Acceleration

$\tau = I\alpha$

$\tau = fR$

Linear Acceleration

$F_{tot} = ma$

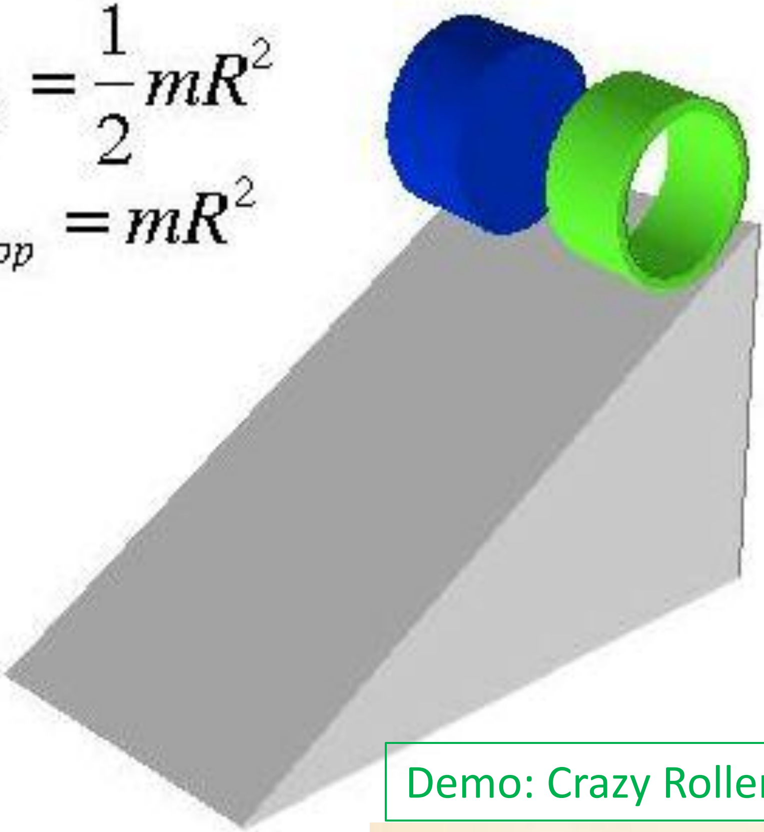
$F_{tot} = mg\sin\theta - f$

$\alpha = a/R$

Moment of Inertia

$$I_{disk} = \frac{1}{2}mR^2$$

$$I_{hoop} = mR^2$$



$$ma = mg\sin\theta - f$$

$$fR = I\alpha = \frac{Ia}{R} \Rightarrow f = \frac{Ia}{R^2}$$

$$ma = mg\sin\theta - \frac{Ia}{R^2}$$

$$ma + \frac{Ia}{R^2} = mg\sin\theta$$

$$\left(mR^2 + I\right)a = mR^2 g\sin\theta$$

$$a = \frac{mR^2 g\sin\theta}{mR^2 + I}$$

As I increases, a decreases

Demo: Crazy Rollers

Moment of Inertia

Let's use conservation of energy

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}I\,\frac{v^2}{R^2}$$

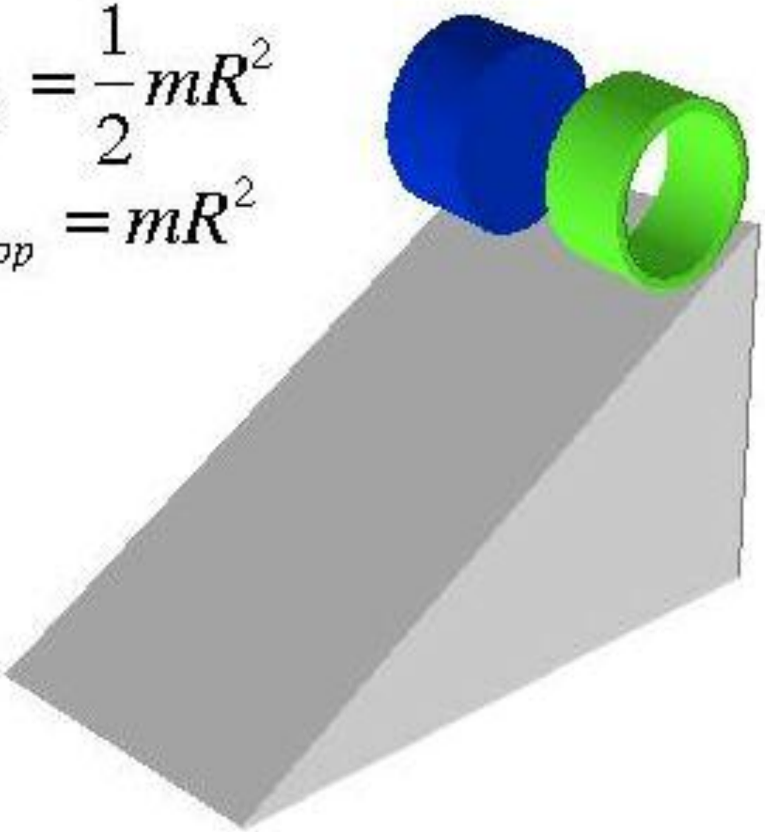
$$v^2 = \frac{2mR^2gh}{mR^2 + I}$$

$$v_{hoop} = .707\sqrt{2gh}$$

$$v_{disk} = .817\sqrt{2gh}$$

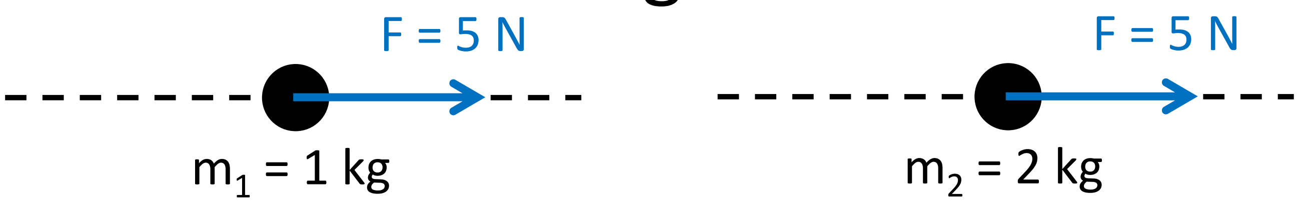
$$I_{disk} = \frac{1}{2}mR^2$$

$$I_{hoop} = mR^2$$



Moment of Inertia

- Which will have the greater acceleration?



- Which will have the greater angular acceleration?

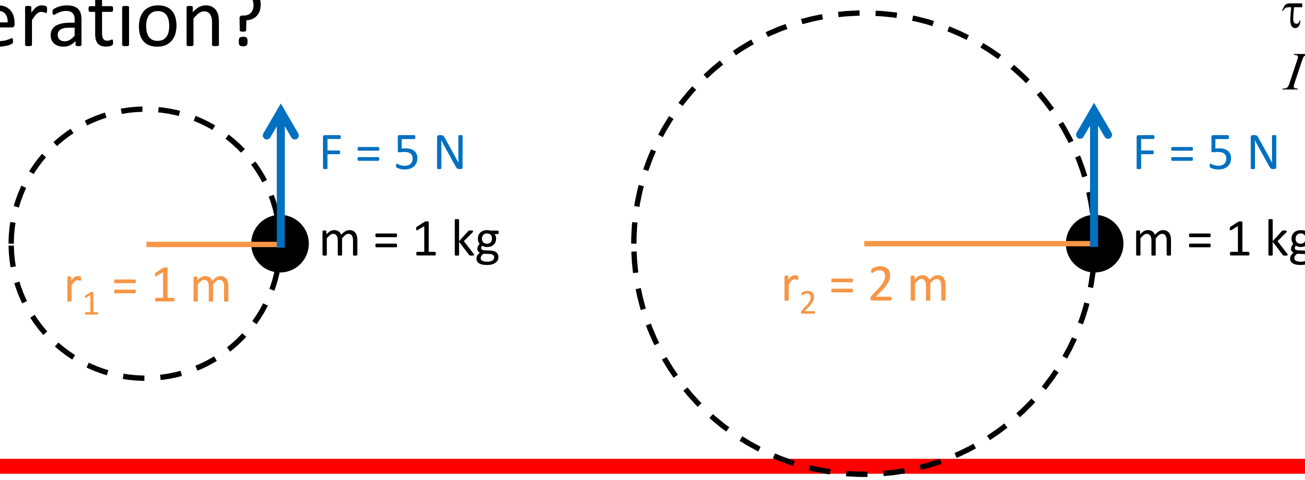
$F = ma$

$Fr = mra$

$\tau = mr^2\alpha$

$\tau = I\alpha$

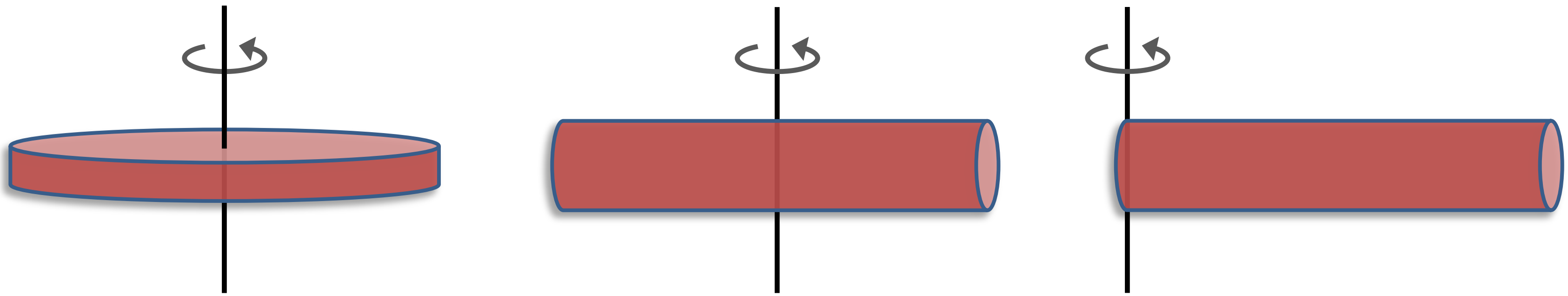
$I = mR^2$



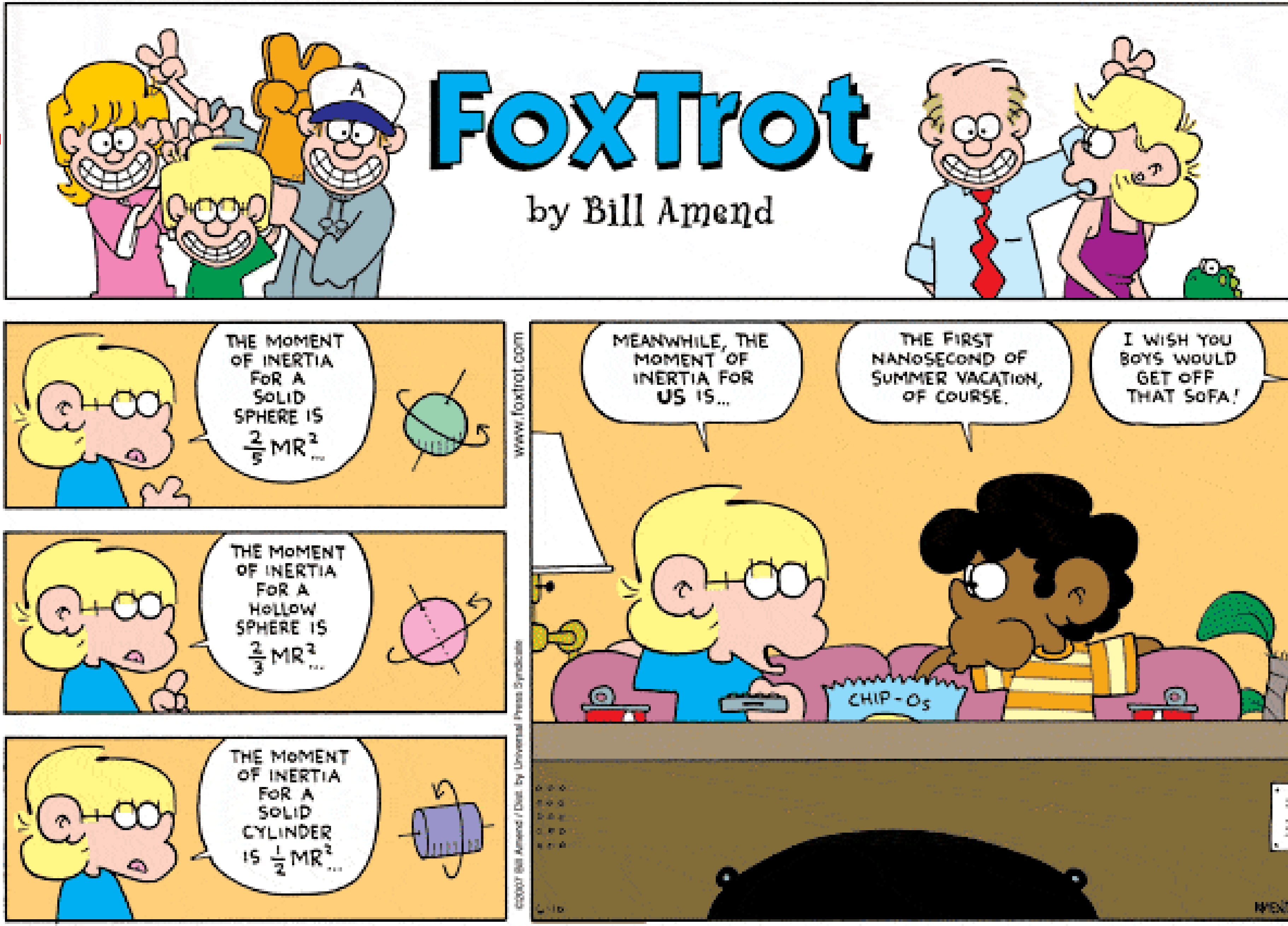
Moment of Inertia

- What's the moment of inertia of an extended object?

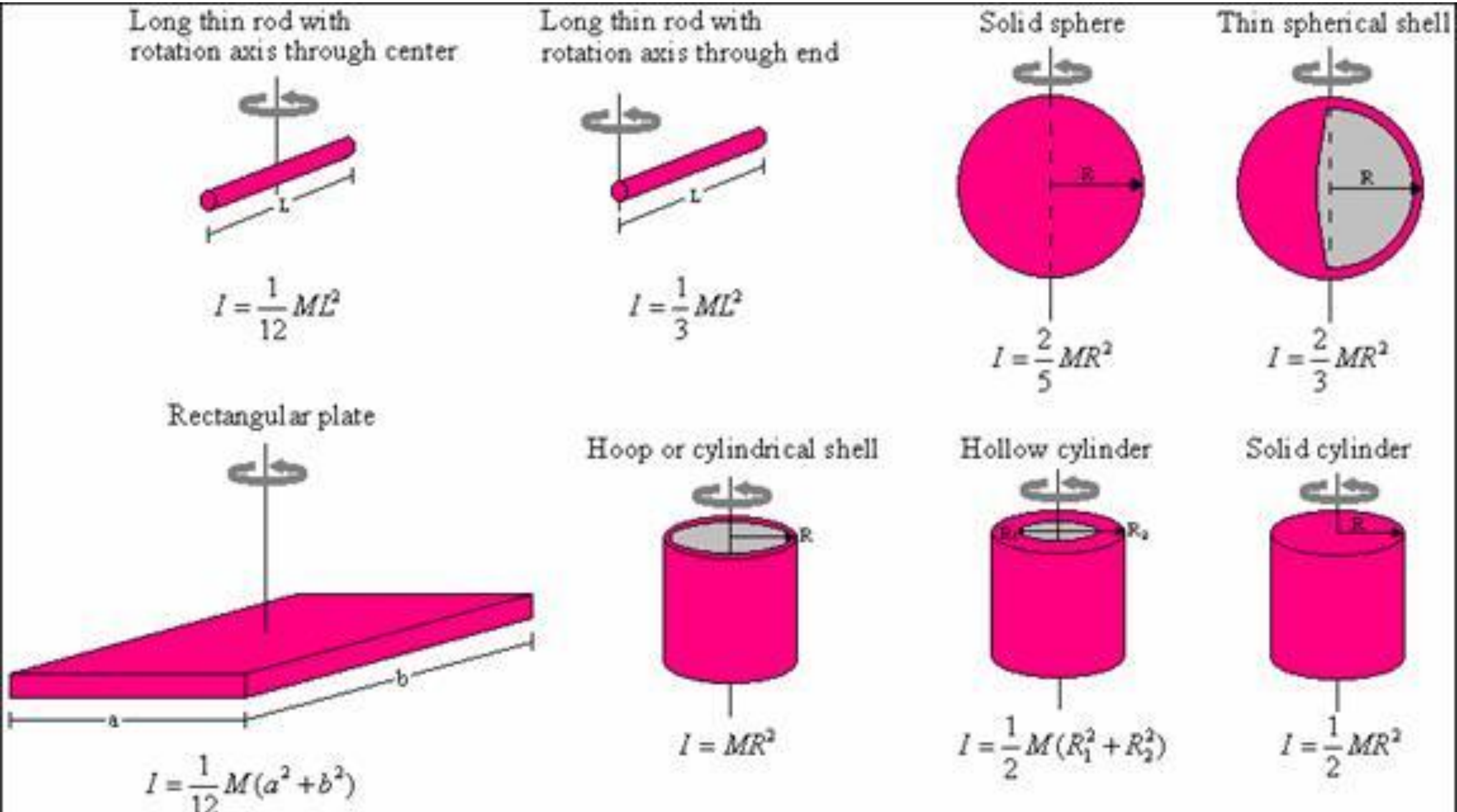
$$I = I_1 + I_2 + I_3 + \cdots = m_1 r_1^{\;2} + m_2 r_2^{\;2} + m_3 r_3^{\;2} + \cdots = \sum m_i r_i^{\;2} = \int r^2 dm$$



$$I = \frac{1}{2} MR^2 \qquad I = \frac{1}{12} ML^2 \qquad I = \frac{1}{3} ML^2$$


FoxTrot
by Bill Amend
THE MOMENT OF INERTIA FOR A SOLID SPHERE IS $\frac{2}{5}MR^2$...
THE MOMENT OF INERTIA FOR A HOLLOW SPHERE IS $\frac{2}{3}MR^2$...
THE MOMENT OF INERTIA FOR A SOLID CYLINDER IS $\frac{1}{2}MR^2$...
MEANWHILE, THE MOMENT OF INERTIA FOR US IS...
THE FIRST NANOSECOND OF SUMMER VACATION, OF COURSE.
I WISH YOU BOYS WOULD GET OFF THAT SOFA!
CHIP-Os
www.foxtrot.com

Moment of Inertia



Conservation of Angular Momentum

(Orban/Corbis/Sygma)

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Conservation of L:

$$L = I\omega$$

If moment of inertia goes own, then angular velocity must go up.

Demo: Rotation with Weights

Conservation of Angular Momentum

Demo: Wheel on a string

Demo: Gyro in a can

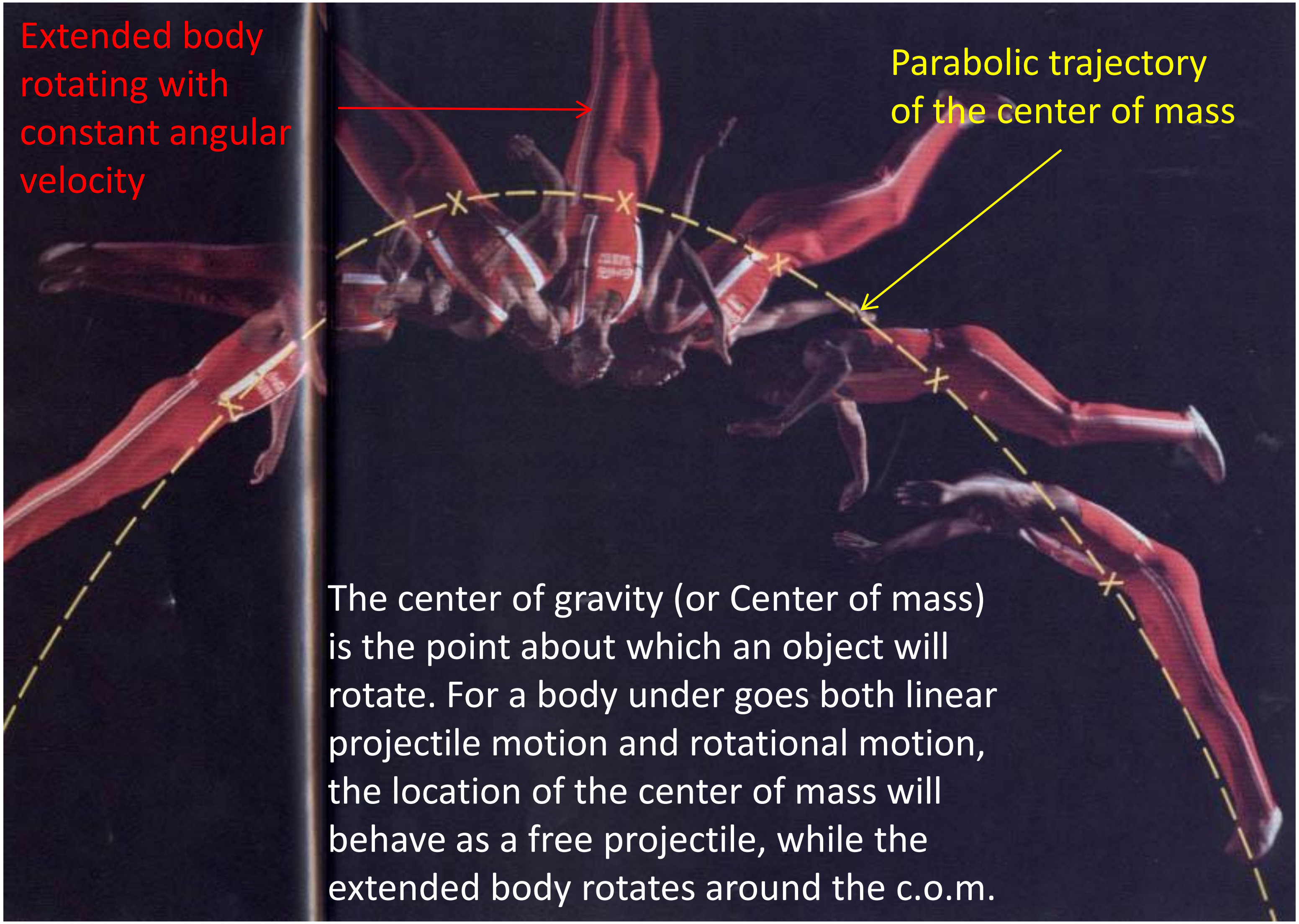
Demo: Gyroscope

Conservation of Angular Momentum

The total angular momentum vector must be conserved. If you flip the spinning wheel, then something else must create an angular momentum vector.

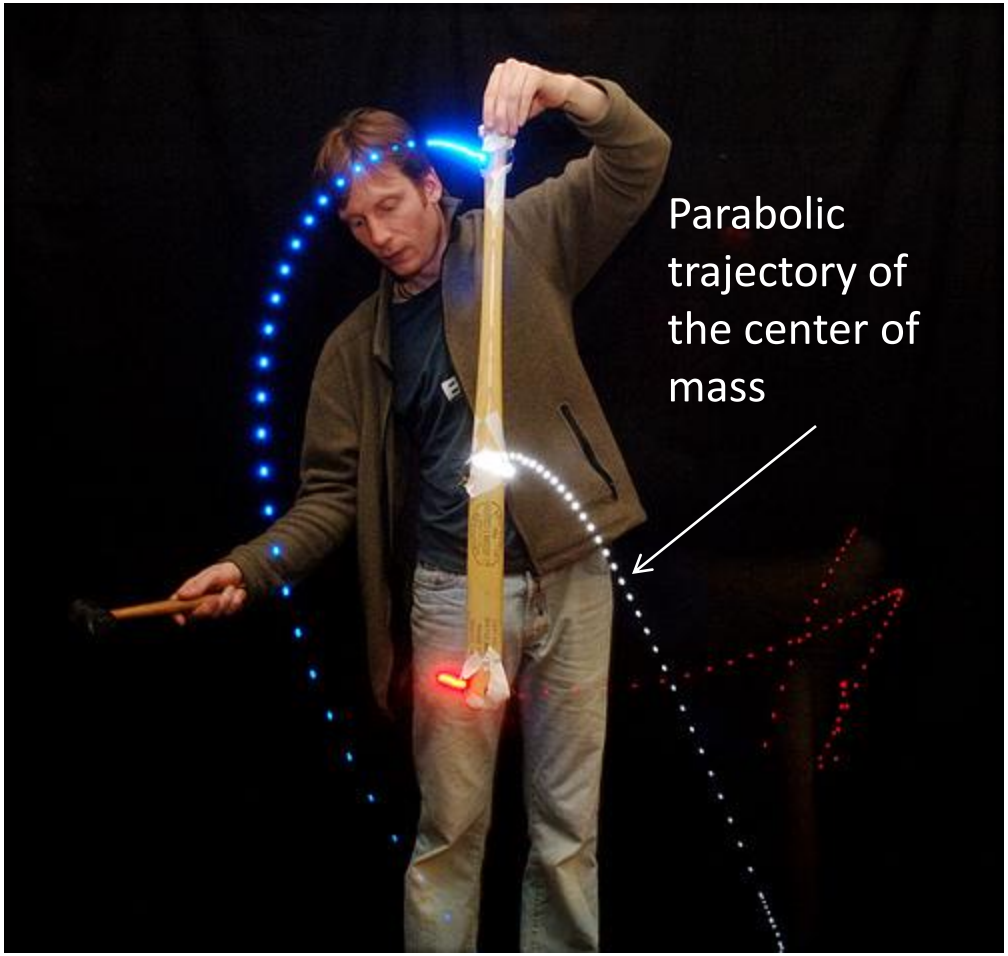
DEMO: Spinning Wheel and rotation chair

Center of Gravity



The center of gravity (or Center of mass) is the point about which an object will rotate. For a body under goes both linear projectile motion and rotational motion, the location of the center of mass will behave as a free projectile, while the extended body rotates around the c.o.m.

Center of Gravity



Angular impulse

Summary of Angular Equations

Magnitude of ω

$$\omega = \frac{v}{r_\perp}$$

Direction of ω

Same direction as Δω

$$\vec{\alpha} = \frac{\Delta \vec{\omega}}{\Delta t}$$

For point particle

$$L = r_\perp P$$

For extended body

$$\vec{L} = I\vec{\omega}$$

For extended body

$$KE_{rot} = \frac{1}{2} I\omega^2$$

Point particle or extended body

$$\tau = r_\perp F$$

Same direction as ΔL

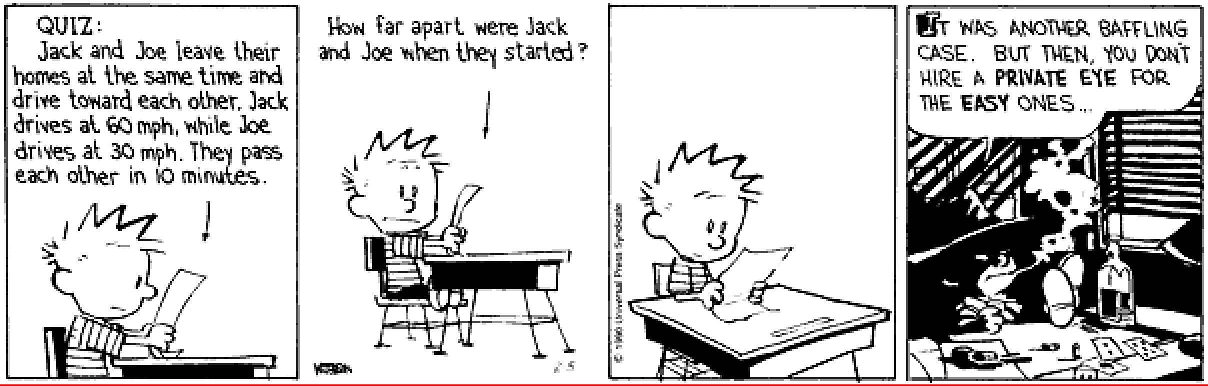
$$\vec{\tau}_{Net\ on\ Object} = \frac{\Delta \vec{L}}{\Delta t}$$

Announcements

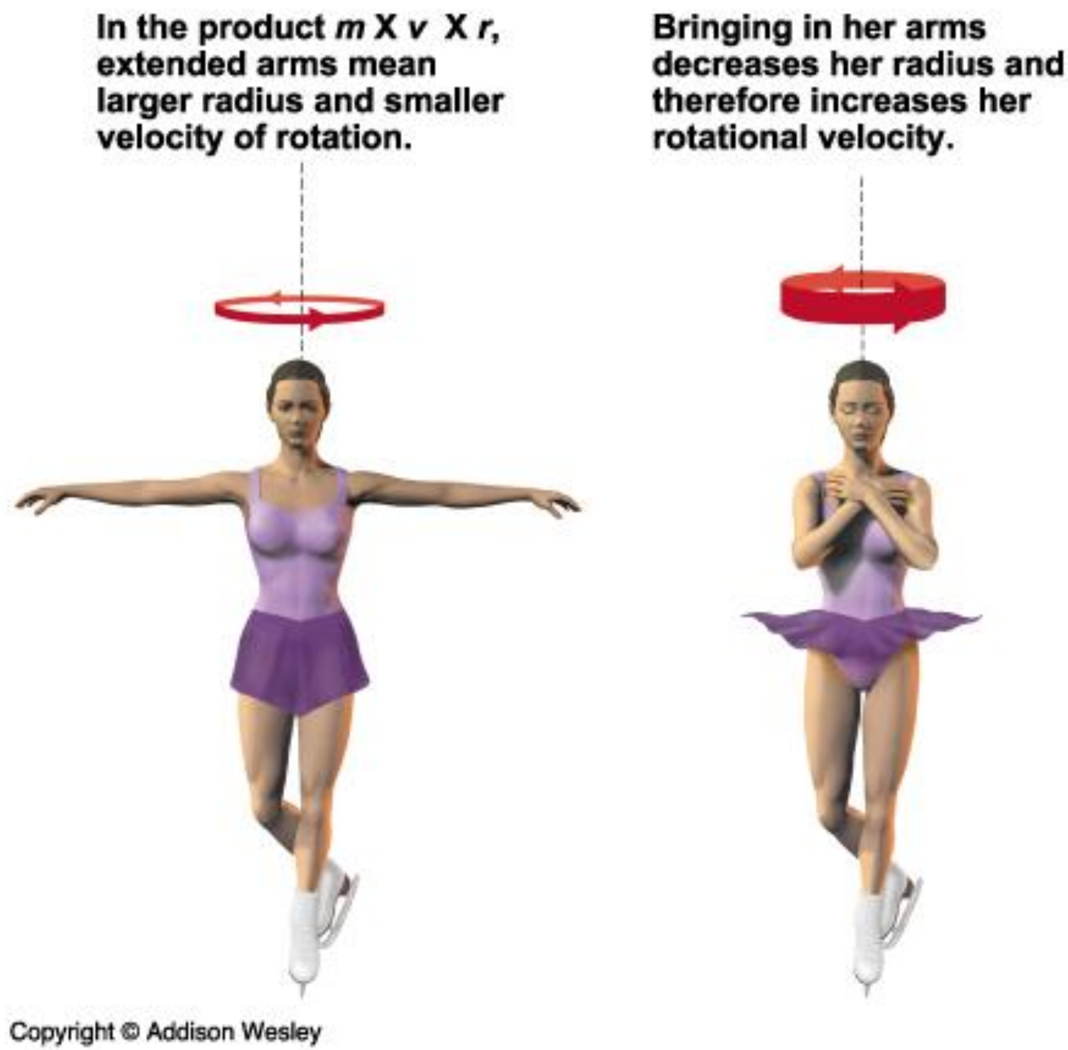
DL Sections

Winter 2010 7B-1 (A/B) D/L Assignments & Job Responsibilities

1	WF	10:30-12:50	2317 EPS	Marcus Afshar
2	MW	2:10-4:30	2317 EPS	Aaron Hernley
3	MW	4:40-7:00	2317 EPS	Rylan Conway
4	MW	7:10-9:30	2317 EPS	Rylan Conway
5	MR	8:00-10:20	2317 EPS	Robert Lynch
6	TR	10:30-12:50	2317 EPS	Aaron Hernley
7	R	2:10-4:30	2317 EPS	Justin Dhooghe
7	M	10:30-12:50	2317 EPS	Justin Dhooghe
8	TR	4:40-7:00	2317 EPS	Britney Rutherford
9	TR	7:10-9:30	2317 EPS	Britney Rutherford
10	TF	8:00-10:20	2317 EPS	Emily Ricks
11	TF	2:10-4:30	2317 EPS	Justin Dhooghe



Conservation of Angular Momentum



Torque