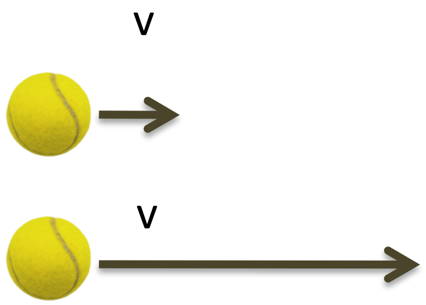
Review of Linear Momentum And Rotational Motion

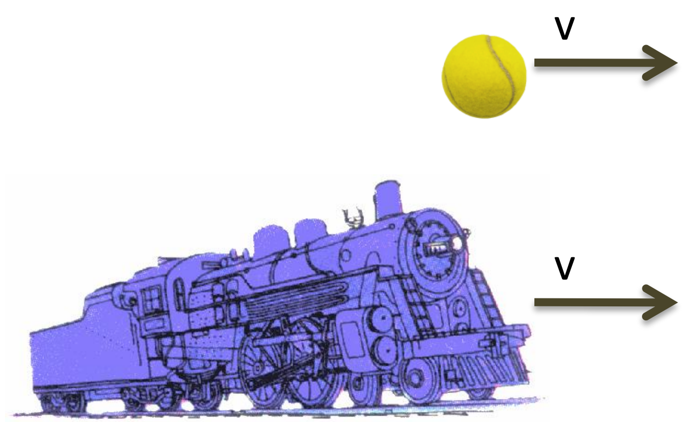
Slides 3-19 were discussed in the 7:30 Lecture

Slides 6-27 were discussed in the 9:00 Lecture

Momentum

- Choose: Which would you rather be hit by? A tennis ball going 1m/s or a tennis ball going 10m/s. Which would transfer more momentum to you?
- Choose: Which would you rather be hit by? A train going 1 m/s or a tennis ball going 1 m/s. Which would transfer more momentum to you?
- Momentum is proportional to the mass and the velocity of an object
- Momentum has direction and therefore is a vector!





$$p = mv$$

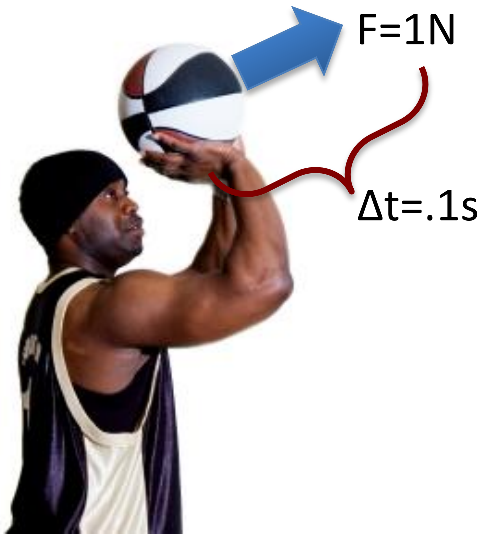
Momentum and Impulse

- Q: How would you give an object momentum?
- We must apply a force for a given amount of time. The greater the force, the greater the change in momentum. The longer we apply the force, the greater the change in momentum.
- A change in momentum is called an impulse J
- Momentum has direction and therefore is a vector! Impulse is also a vector.

$$\Delta p = F\Delta t$$

$$F = \frac{\Delta p}{\Delta t} = \frac{dp}{dt}$$

$$J = F\Delta t$$



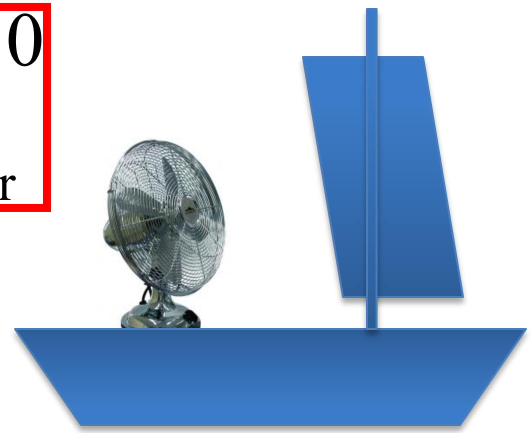
Q: Find the impulse the player gives to the ball. If the mass of the basketball is 0.5 kg, what is the speed the ball leaves the player's hand with?

Conservation of Momentum

- The other big law after conservation of Energy.
- Q: If there is a system with no external force acting upon it, what is the change in momentum of the system?

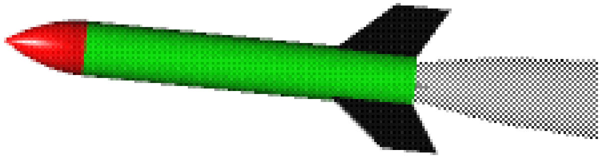
$$F = 0 \rightarrow \Delta p = 0$$

$$p_{\text{before}} = p_{\text{after}}$$



Q? Will the boat move?

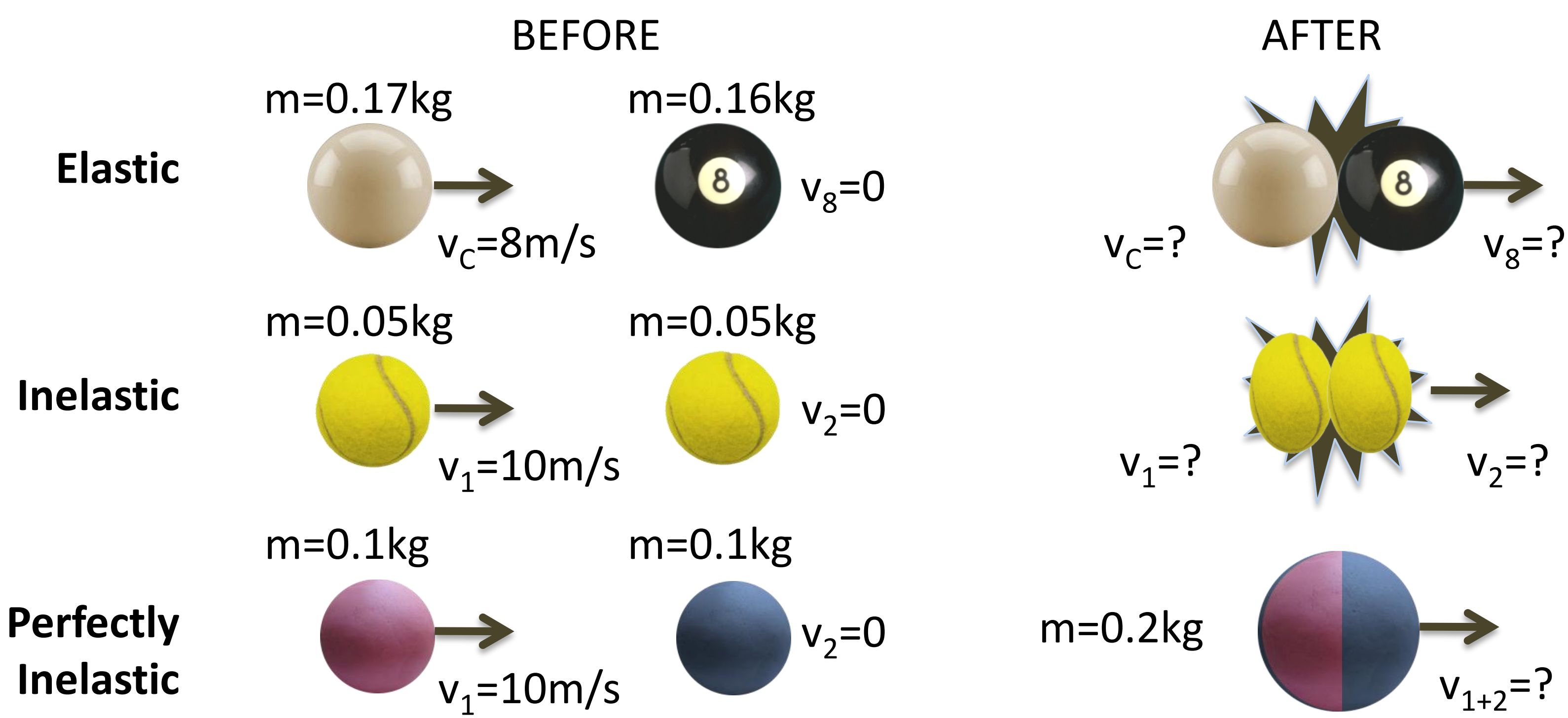
Q: Why does the tank feel a recoil?



Q: Take a compressed fuel rocket. Which fuel would be better for this rocket, water or air?

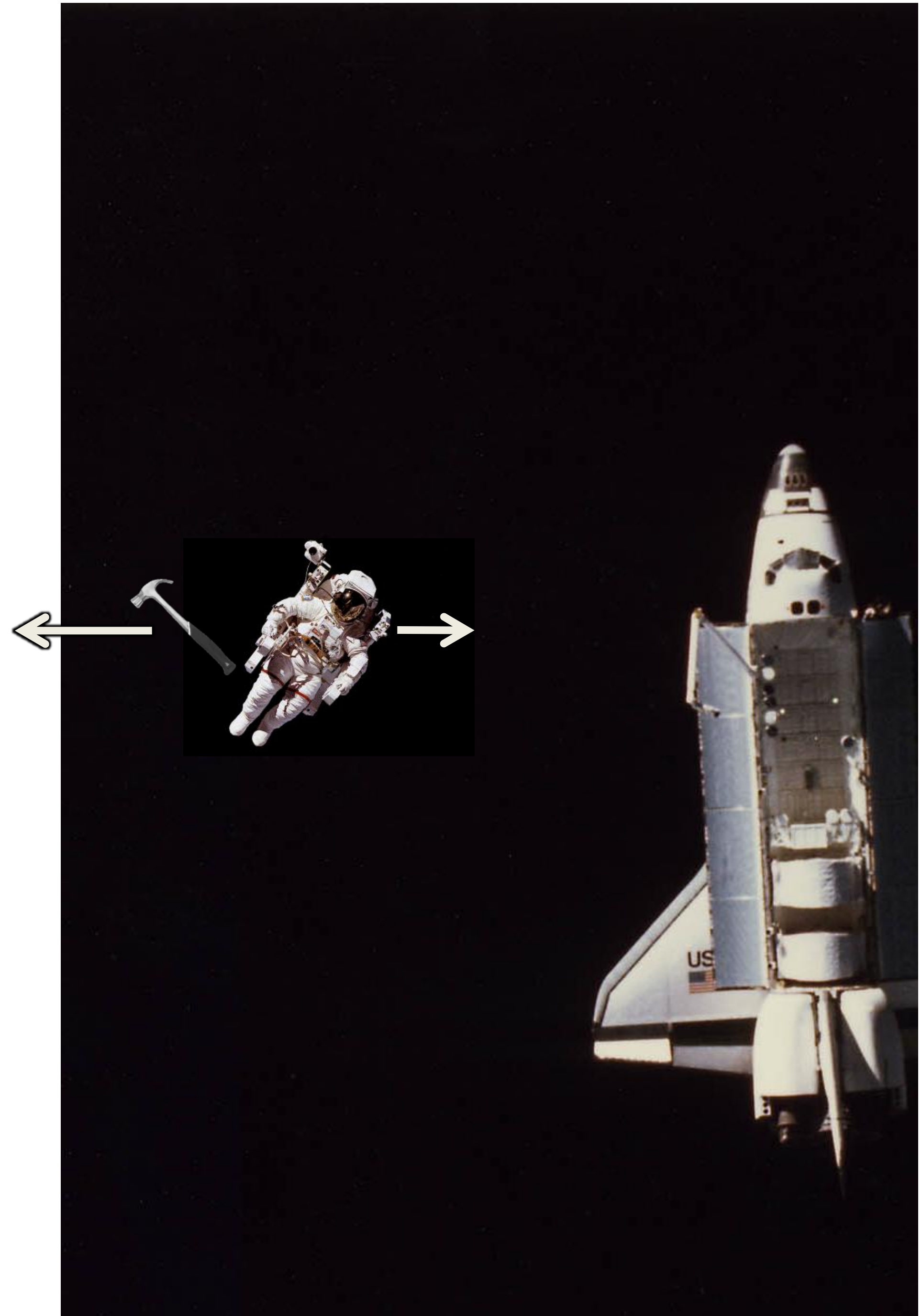
Collisions:

Type of Collision	Description	Example	Momentum (p_{tot})	Kinetic Energy (Ke_{tot})
Elastic	Perfectly bounce off each other – no deformation	Billiard Balls	Conserved	Conserved
Inelastic	Bounce with some Deformation	Tennis Balls	Conserved	Not Consv'd
Perfectly Inelastic	Stick to each other	Play-doh Balls	Conserved	Not Consv'd



- Q: An astronaut is performing a spacewalk when his tether breaks. He happens to be holding a 1kg tool. Because the astronaut was paying attention in his physics class, he knows that he can use **conservation of momentum** to his advantage. How?

- Q: An astronaut is performing a spacewalk when his tether breaks. He happens to be holding a 1kg tool. Because the astronaut was paying attention in his physics class, he knows that he can use **conservation of momentum** to his advantage. How?
- He can manage to throw the tool at 10m/s
- If the astronaut is 10m away from the space shuttle and his mass is 100kg, find the time it takes for him to get back to the Space Shuttle.



The Definition of Impulse

Recall that *work* was resulted in a change in *energy, where work is*

$$W = \int_{x_i}^{x_f} \vec{F}(x) \bullet d\vec{x} = E_f - E_i = \Delta E$$

We can similarly define a quantity called the *impulse*, which is the time integral o f the applied force

$$\vec{J} = \int_{t_i}^{t_f} \vec{F}(t) dt = \vec{p}_f - \vec{p}_i = \Delta\vec{p}$$

J is a vector quantity, which is applied to the vector ***p***.

Conservation of Momentum

$$Net\vec{J}_{ext} = \Sigma\vec{J}_{ext} = \int \Sigma\vec{F}_{ext}(t)dt = \vec{p}_f - \vec{p}_i = \Delta\vec{p}_{system}$$

If the net external impulse in a given direction acting on the system is zero, then there is no change in the linear momentum of the system in that direction; Otherwise there is a change in the momentum equal to the net external impulse.

Conservation of Momentum!

Compare to conservation of Energy

Rockets

Consider a model rocket. How high will a 0.2 kg model rocket fly if your install a class “C” Estes rocket engine? An Estes “C” engine provide 8.0 Newton-seconds of impulse.

$$\vec{J} = \Delta\vec{p} = m\Delta\vec{v} = (0.2kg)(v)$$

$$v = (8Ns)/(0.2kg) = 40m/s$$

$$\Delta PE = \Delta KE$$

$$mgh = (1/2)mv^2$$

$$h = (1/2)v^2/g = (0.5)(1600m^2/s^2)/(10m/s^2) = 80m$$

Collisions

A ***collision*** occurs when two free bodies make contact. The contact results in a repulsive normal force. The magnitude of the force and the duration of the contact will determine the resulting impulse. Newton's 3rd law indicates that each of the two participating bodies will experience an equal but opposite impulse.

All collisions conserve both energy and momentum, however we classify collisions as being either ***elastic*** on ***inelastic*** based upon whether kinetic energy is conserved.

Elastic collisions conserve both momentum and kinetic energy.

In an ***inelastic*** collision some of the energy of the collisions is dissipated as internal energy within the bodies.

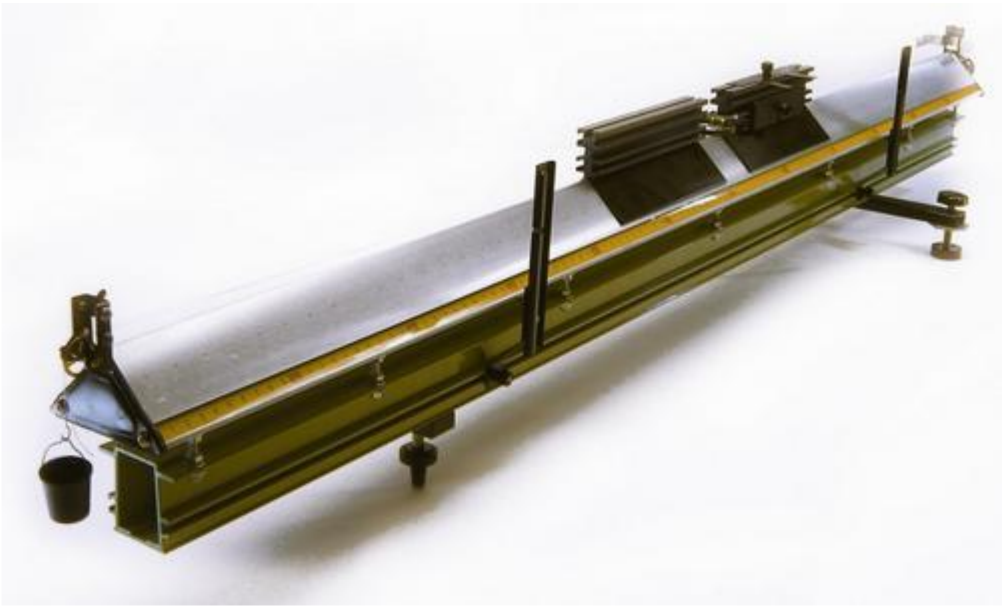
In a ***completely inelastic*** collision, the two bodies stick together and there is only center of mass motion remaining after the collision

1D Elastic Collisions

Elastic Collisions:

$$KE_i = KE_f$$

$$\vec{p}_{tot,i} = \vec{p}_{tot,f}$$



Try cases of:

- m1=m2
- M1=2m2
- m1=(1/2)M2
- v1>v2
- v1<v2

Demo: collision table

1D Inelastic Collisions

Inelastic Collisions:

$$KE_i \neq KE_f$$

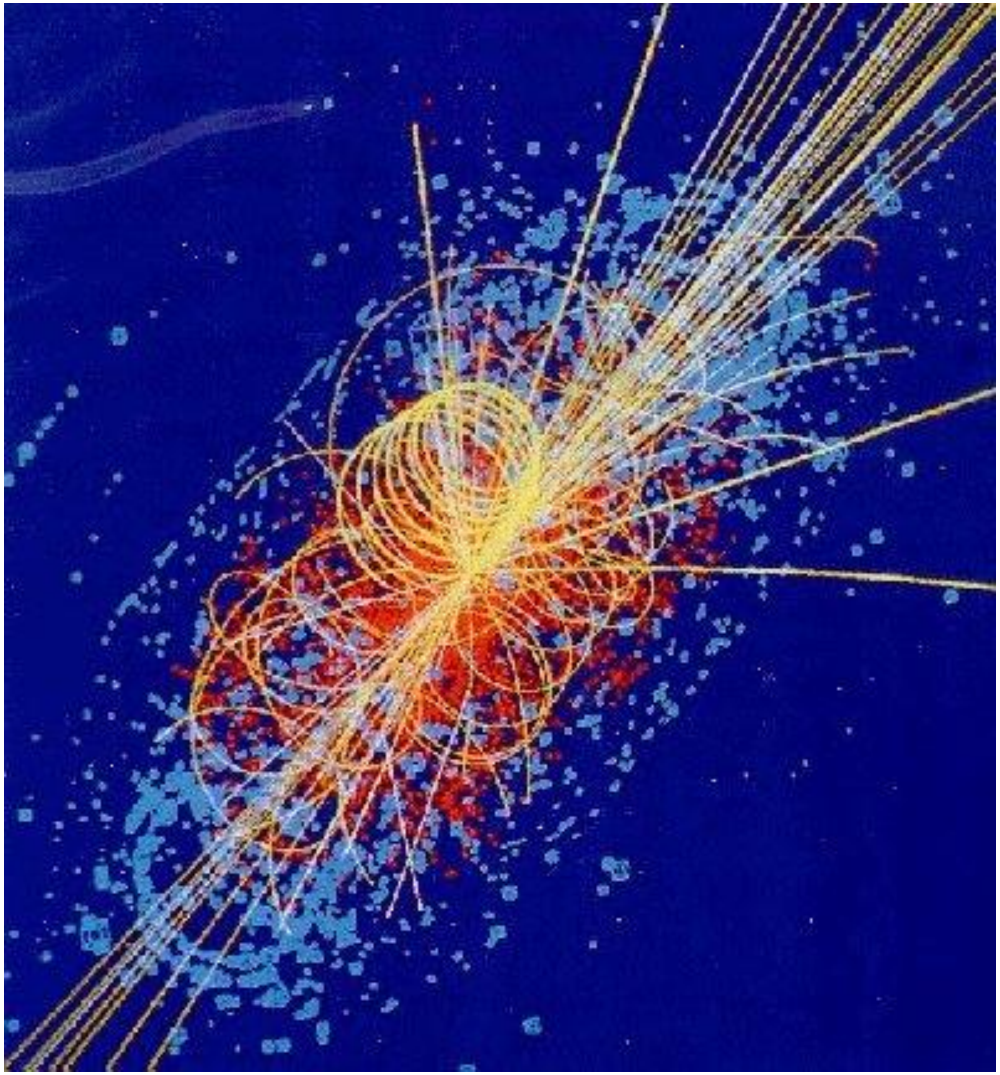
$$\vec{p}_{tot,i} = \vec{p}_{tot,f}$$

Try cases of:
Equal masses
- m1=m2
- M1=2m2
- m1=(1/2)M2
- v1>v2
- v1<v2

Demo: collision table

A Particle Physics Collision

Is this an elastic on inelastic collision?



Inelastic Collisions

Why do cars have crumple zones?

Consider an older model car with a 5 cm bumper and a rigid frame.

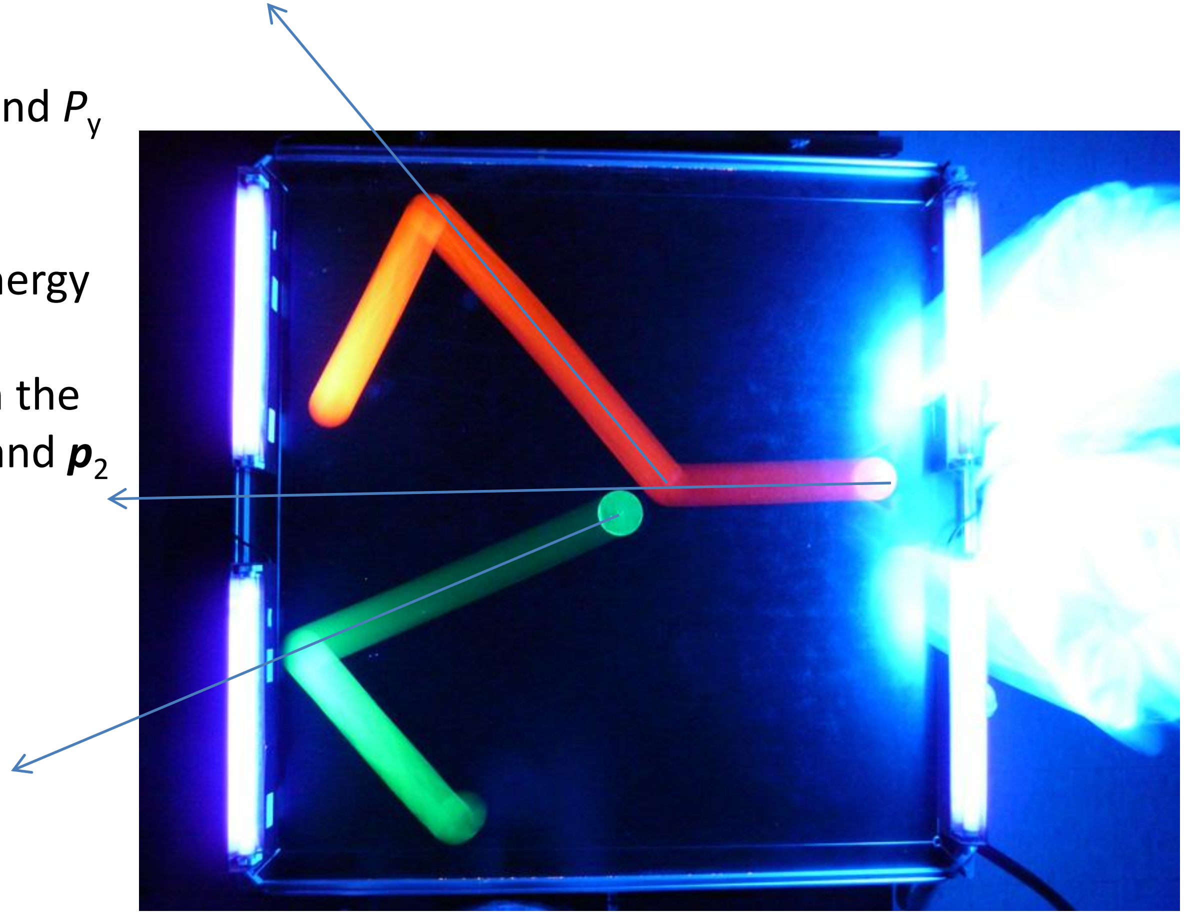
Compare to a more model car with a 25 cm crumple zone.

2D Collisions

Must conserve P_x and P_y

Elastic collisions conserve kinetic energy

When $m_1=m_2$, then the angle between $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$ will be 90 degree.

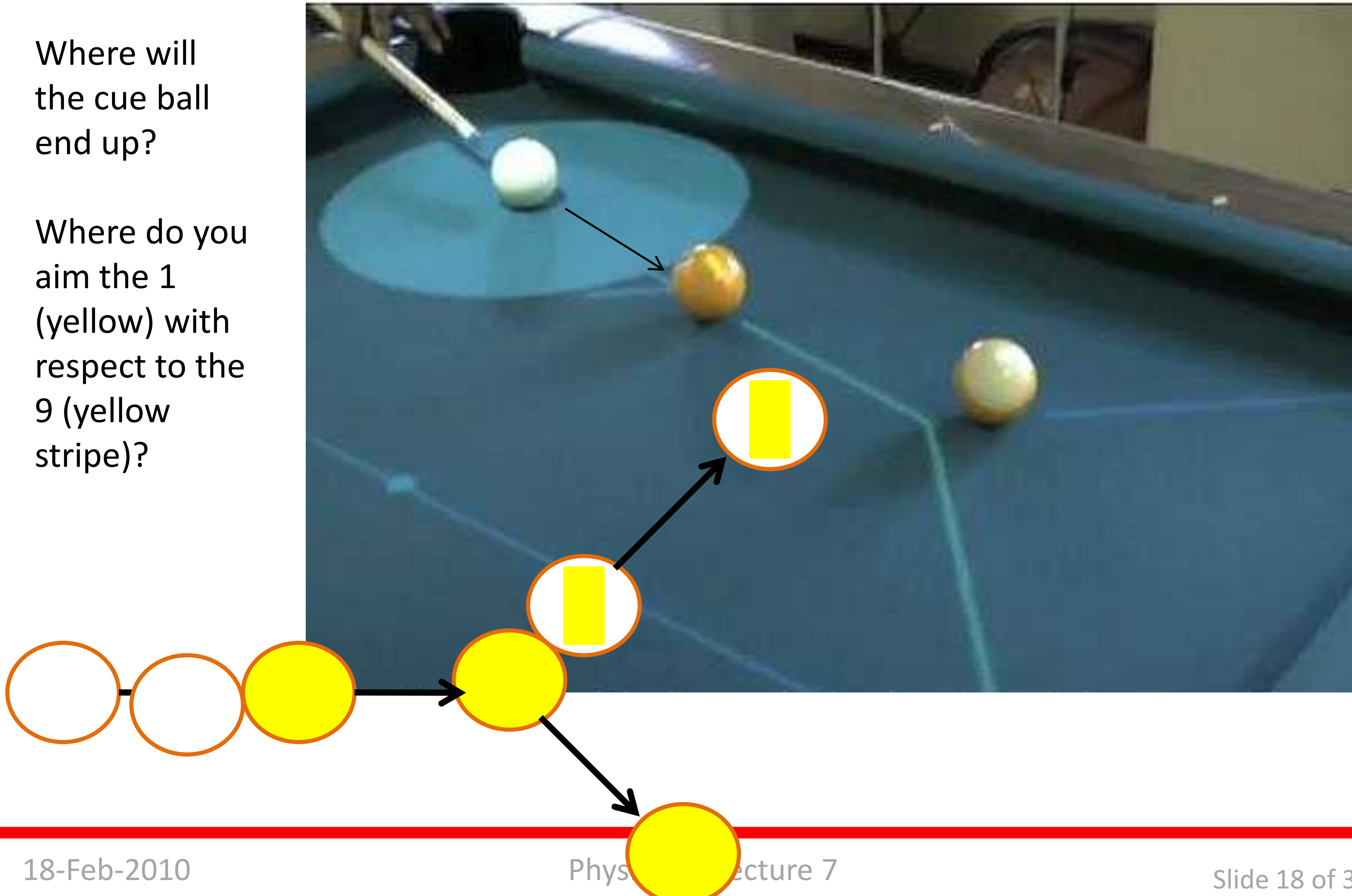


Demo: Air puck

2D Collisions

Where will the cue ball end up?

Where do you aim the 1 (yellow) with respect to the 9 (yellow stripe)?



2D Collisions

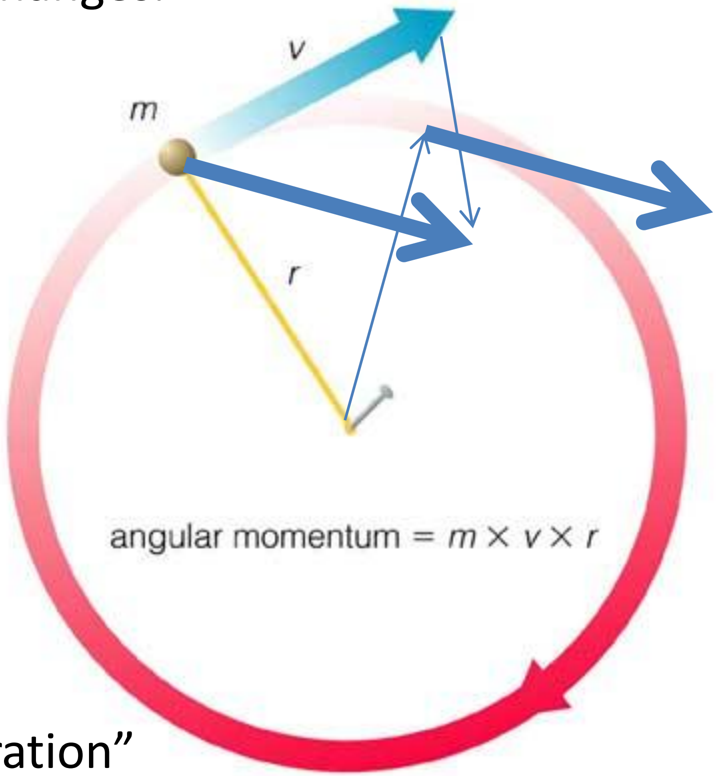
Can the American Curling team knock out both the yellow stones?

Where should they aim their stone?

Rotational Motion

Can an object be continually accelerating without gaining any speed?

If the force is always perpendicular to the direction of the velocity, then only the direction changes.



$v = 2\pi r/T$

$$\frac{\Delta v}{v} = \frac{v\Delta t}{r}$$

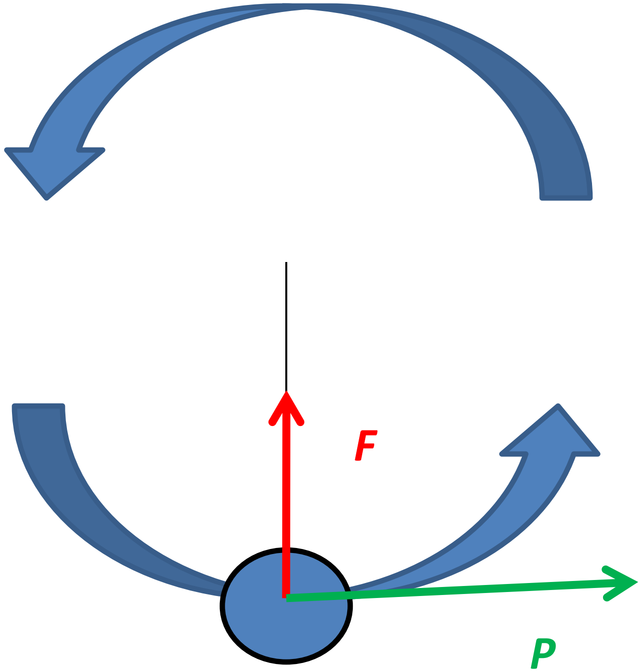
$$a_c = \frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

"centripetal acceleration"

Rotational Motion

Centripetal Force

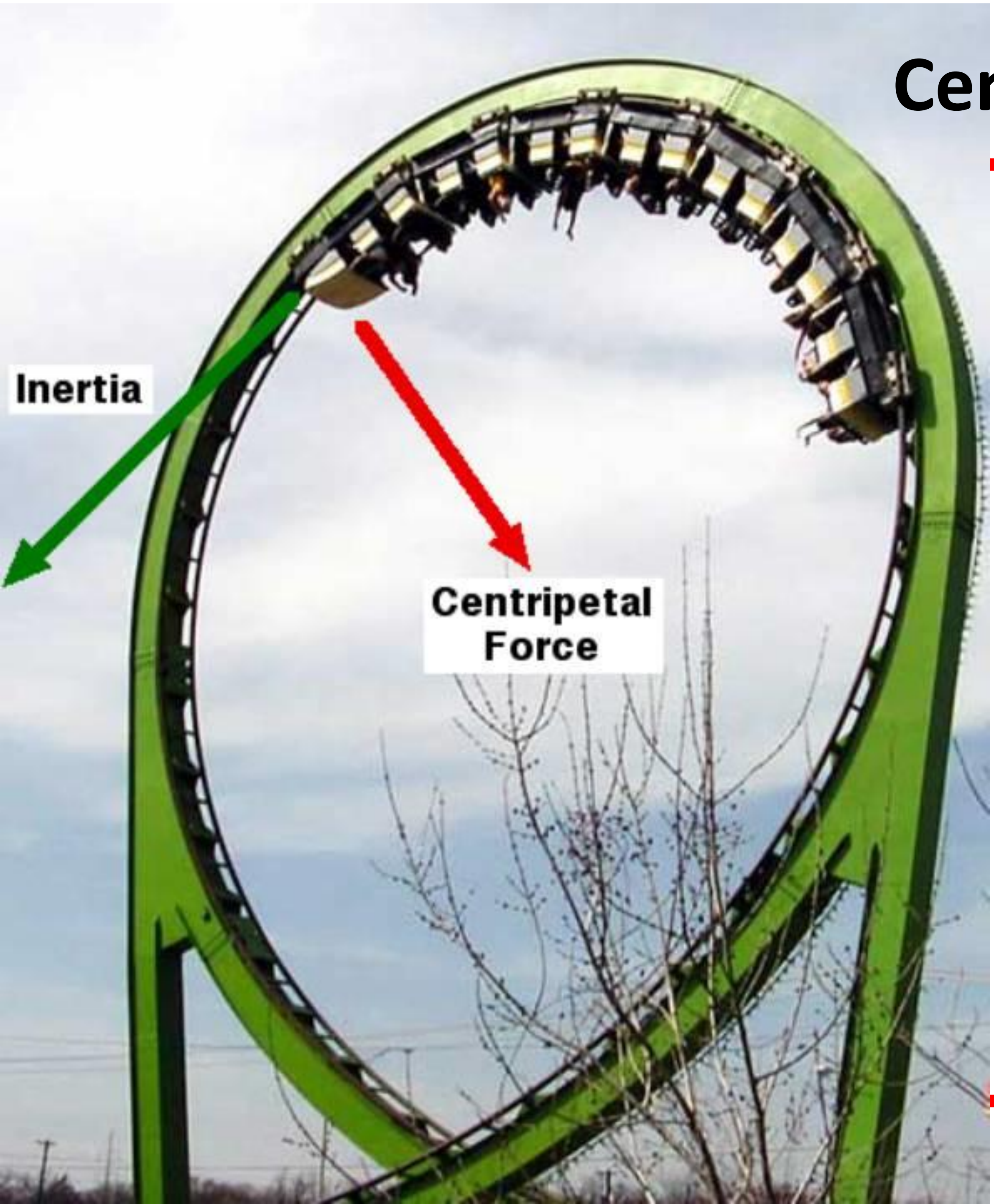
$F_c = ma_c = mv^2/r$



Spin the ball, then let it go. Due to conservation of momentum, it must go in the straight line

Demo: Ball on a string

Centripetal Force



How fast must the roller coaster be going so that the riders still feel a 0.5 g force even at the top of the 10 meter radius loop-the-loop?

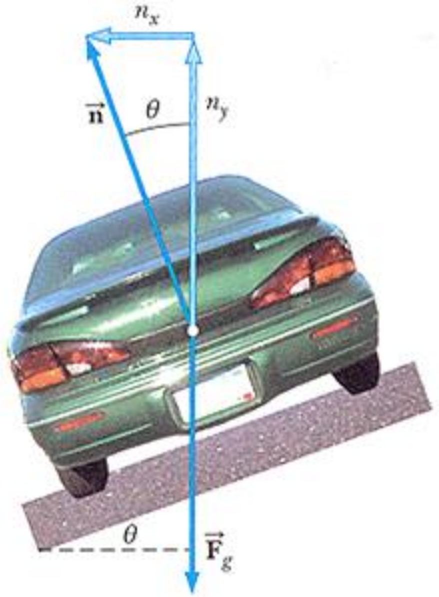
F_c = (3/2)mg = mv^2/r

v^2 = (3/2)rg = (3/2)(10)(10)

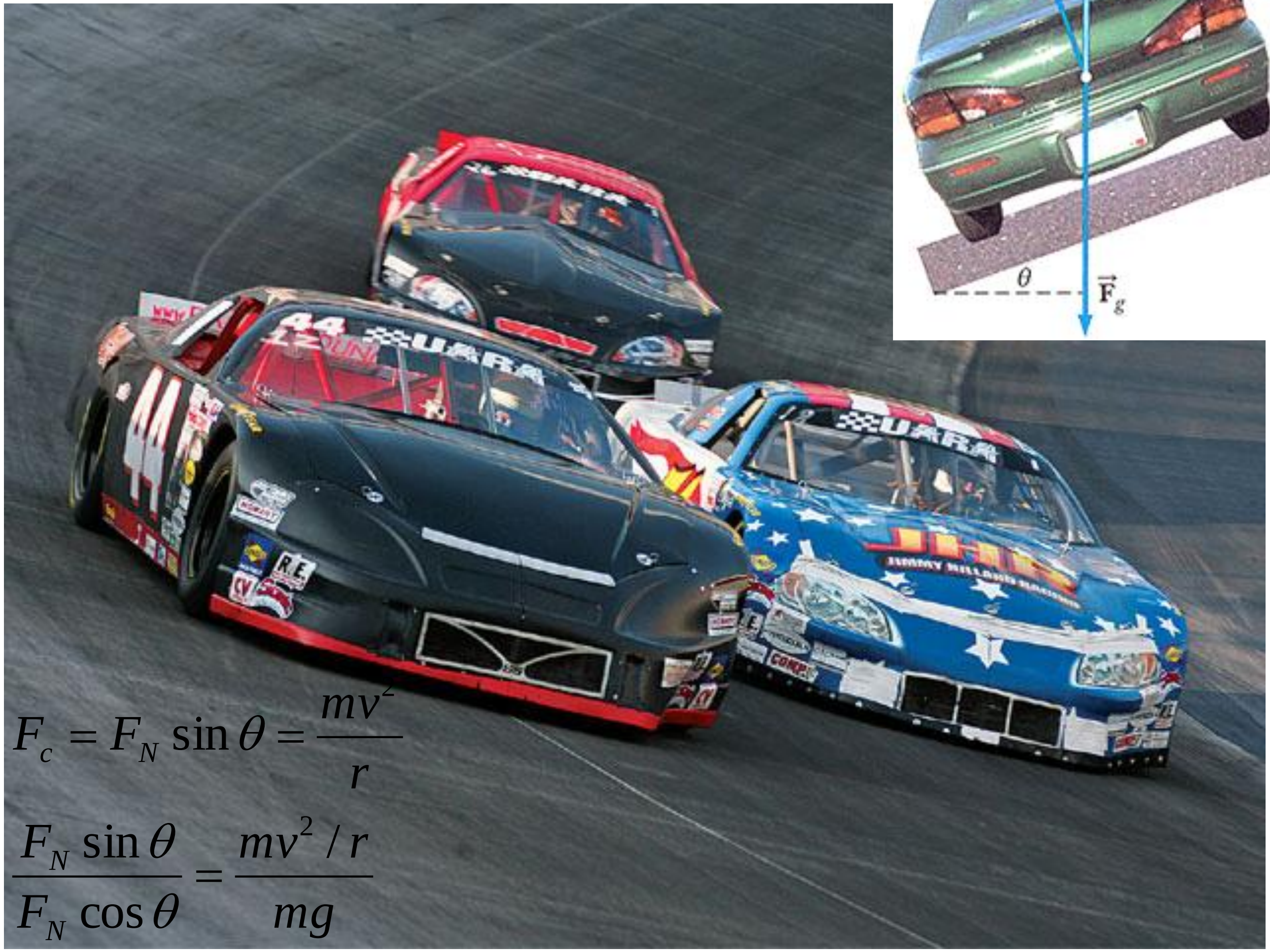
v = 12.2 m/s = 27.5 mph

What provides the centripetal force?

Centripetal Force



What is the centripetal force needed to keep the NASCAR 1000 kg cars on the track when taking an R = 200 m curve at 200 mph (320 km/hour)?



$$F_c = F_N \sin\theta = \frac{mv^2}{r}$$

$$\frac{F_N \sin\theta}{F_N \cos\theta} = \frac{mv^2/r}{mg}$$

$$\tan\theta = v^2/rg$$

$F = mv^2/r$

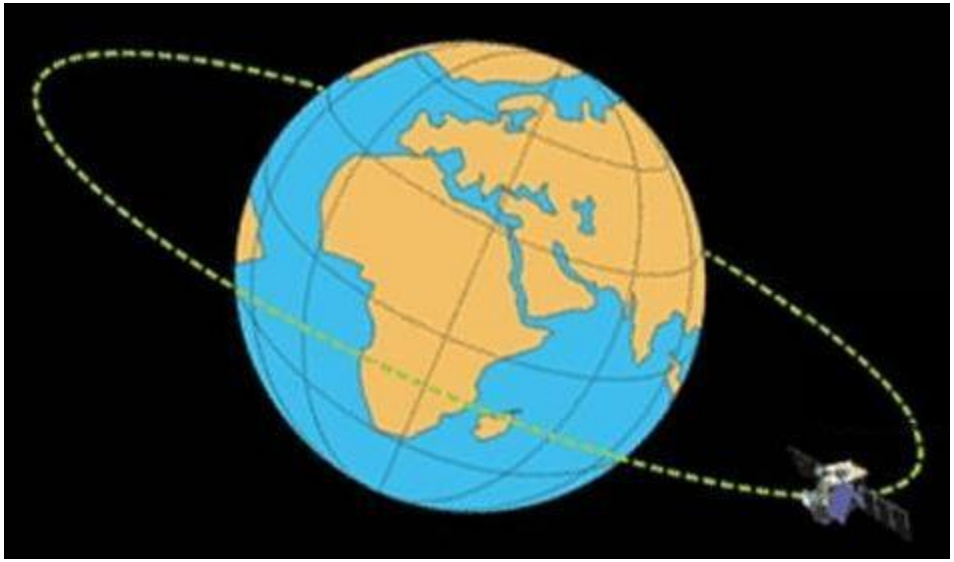
$= (1000)(88.9)^2/200$

$= 3.95\text{x}10^4$ N

What provides this centripetal force?

Centripetal Force

How far above the surface of the earth must one launch a communications satellite to achieve an geosynchronous orbit?



r = 42,164 km

Note R_E = 6378 km

=> Launch altitude above 36000 km

Note also, low earth orbit is anything less than 200 km above the surface of the earth

What provides the centripetal force?

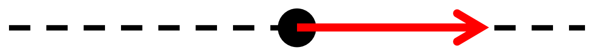
$$F_c = G\frac{mM}{r^2} = mv^2 / r$$

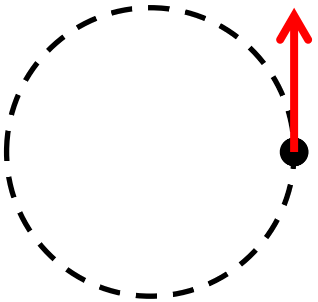
$$v = \sqrt{\frac{GM}{r}} = \frac{2\pi r}{T}$$

$$T = \frac{2\pi r^{3/2}}{\sqrt{GM}}$$

Angular Quantities

- There is an analogy between objects moving along a straight path and objects moving along a circular path.

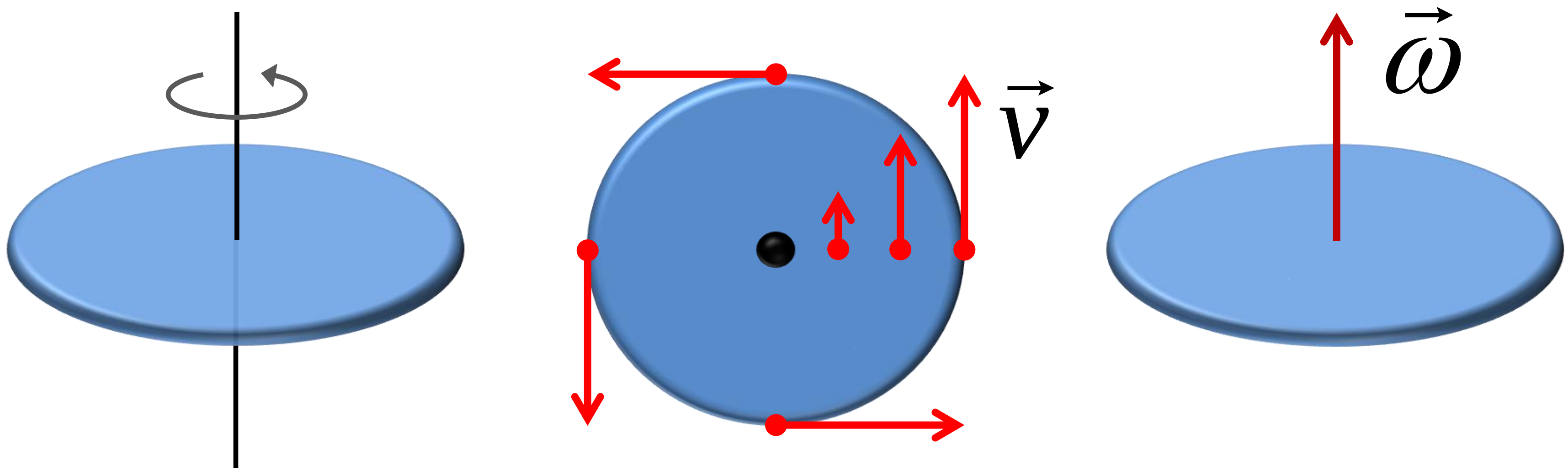




Position	$\leftrightarrow$	Angle
Velocity	$\leftrightarrow$	Angular Velocity
Acceleration	$\leftrightarrow$	Angular Acceleration
Momentum	$\leftrightarrow$	Angular Momentum
Force	$\leftrightarrow$	Torque

Angular Velocity

- How do we represent the motion of a rotating disk?

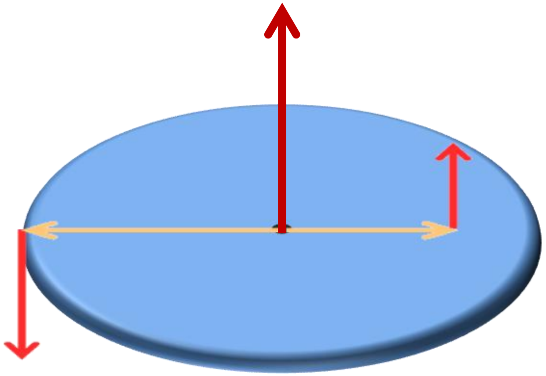


Demo: Accelerometer

Angular Velocity

- Angular velocity is a vector:
  - Right hand rule to determines direction of ω

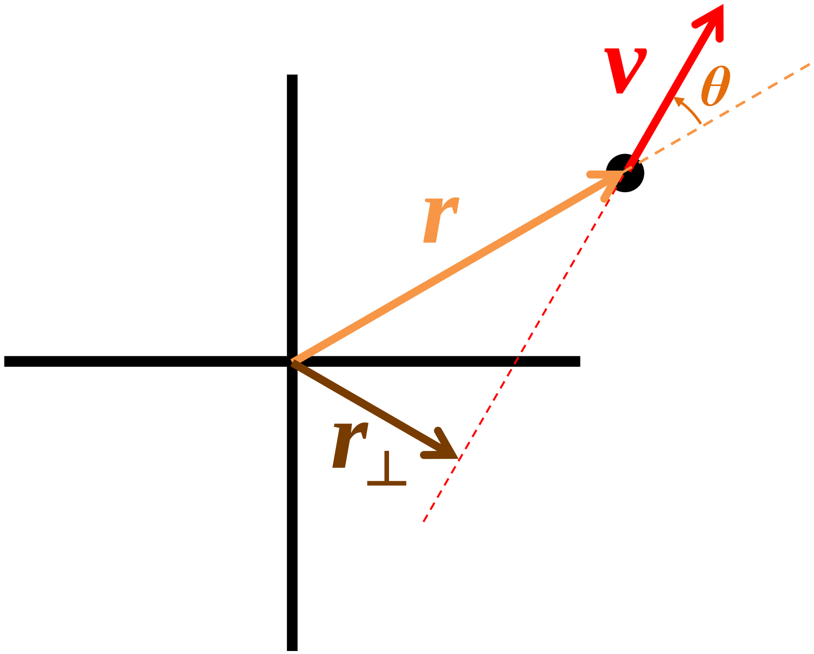
  - Velocity and radius determine magnitude of ω



$$\omega = \frac{v}{r}$$

Angular Velocity

- What's the angular velocity of a point particle?



$$\omega = \frac{v}{r_\perp} = \frac{v}{r\sin\theta}$$

Angular Acceleration

- Angular Acceleration: Rate of change in angular velocity

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt}$$

Demo: Atwood Machine

- A stationary disk begins to rotate. After 3 seconds, it is rotating at 60 rad/sec. What is the average angular acceleration?

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{60\,rad/s - 0\,rad/s}{3\,s} = 20\,\frac{rad}{s^2}$$

Angular Momentum

- Angular Momentum: Product of position vector and momentum vector

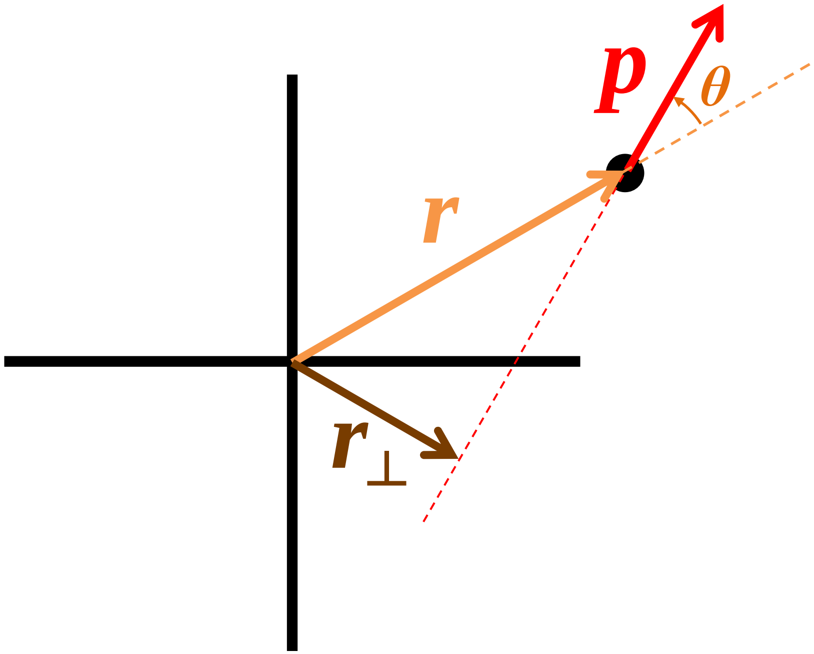
$$\vec{L} = \vec{r} \times \vec{p}$$

- Why is angular momentum important? Like energy and momentum, angular momentum is conserved.
- Angular Impulse: Change in angular momentum vector

$$\Delta\vec{L} = \vec{L}_2 - \vec{L}_1$$

Angular Momentum

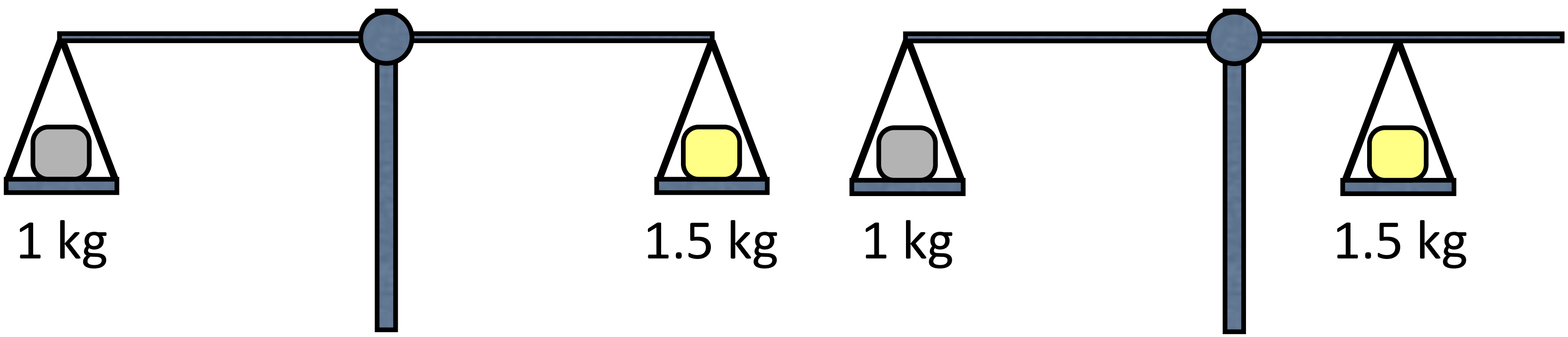
- What’s the angular momentum of a point particle?



$$L = r_{\perp} p = r \sin\theta \; p$$

Torque

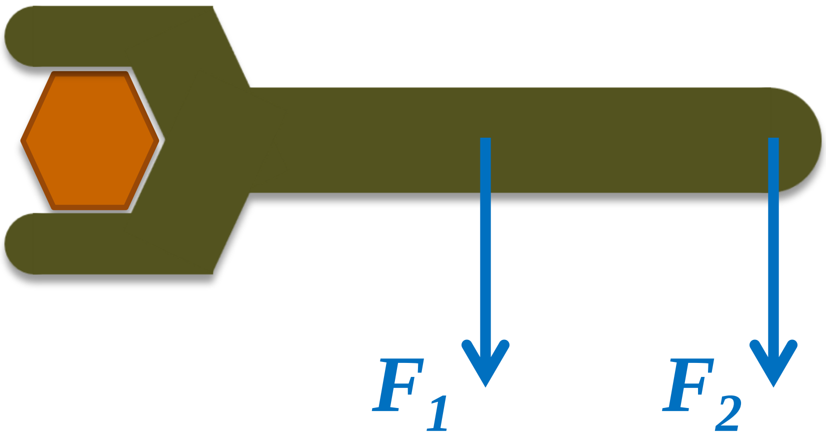
- Which way will the scale tip?



- Rotation of scale is influenced by:
  - Magnitude of forces
  - Location of forces

Torque

- Which is more effective?



- Rotation of wrench is influenced by:
  - Magnitude of forces
  - Location of forces
  - Direction of forces

Torque

- Torque: The cause or agent of angular acceleration

$$\vec{\tau} = \vec{r} \times \vec{F}$$

- The angular velocity of an object will not change unless acted upon by a torque

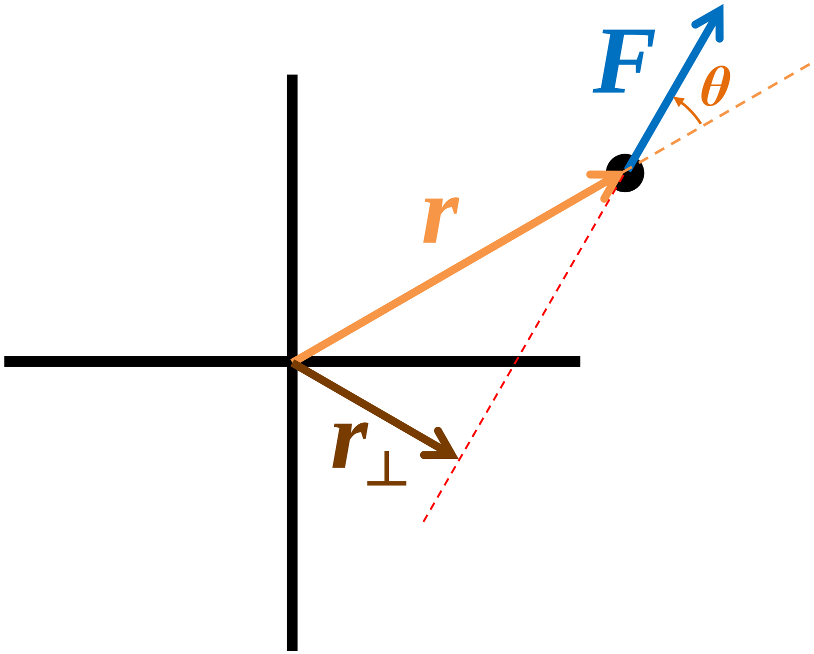
$$\vec{\tau}_{Net\ on\ Object} = 0 \quad \Rightarrow \quad \Delta\vec{\omega} = 0$$

- The net torque on an object is equal to the rate of change of angular momentum

$$\boxed{\vec{\tau}_{Net\ on\ Object} = \frac{d\vec{L}}{dt}}$$

Torque

- What's the torque on a point particle?



$$\tau = r_\perp F = r \sin\theta \, F$$

DL Sections

Winter 2010 7B-2 (C/D) D/L Assignments & Job Responsibilities

1	MR	10:30-12:50	2319 EPS	John McRaven
2	TR	2:10-4:30	2319 EPS	Miles Frampton
3	TR	4:40-7:00	2319 EPS	Ben Richard
4	TR	7:10-9:30	2319 EPS	Ben Richard
5	TF	10:30-12:50	2319 EPS	Robert Lynch
6	MW	8:00-10:30	2319 EPS	Miles Frampton
7	MW	2:10-4:30	2319 EPS	Emily Ricks
8	MW	4:40-7:00	2319 EPS	McCullen Sandora
9	MW	7:10-9:30	2319 EPS	McCullen Sandora