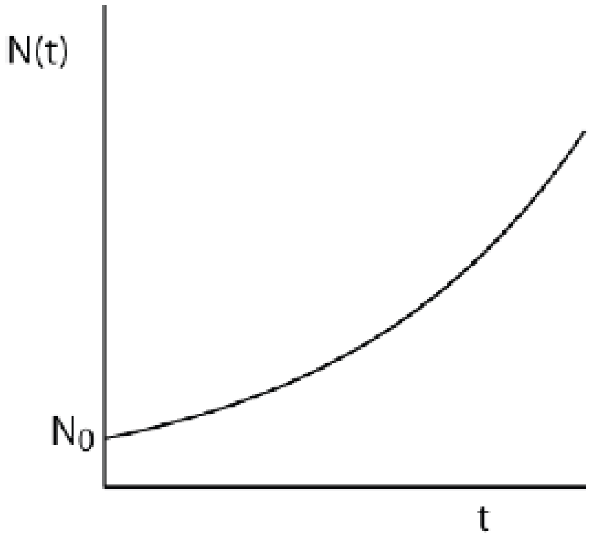
Review of Exponential Change Model and Forces

Exponential Growth

- the behavior of N(t) is a rapidly increasing function of time:



$$N(t) = N_0 e^{kt}$$

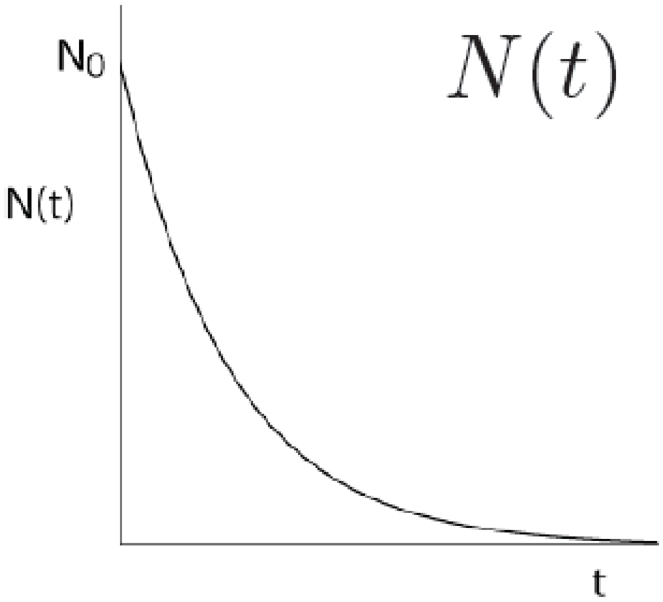
- N_o is the value of N(t=0)

Exponential decay

- equation is similar to growth case:

$$\frac{d}{dt}N(t) = -kN(t)$$

$$N(t) = N_o e^{-kt}$$



k > 0, but slope of N(t) is <u>always</u> negative!

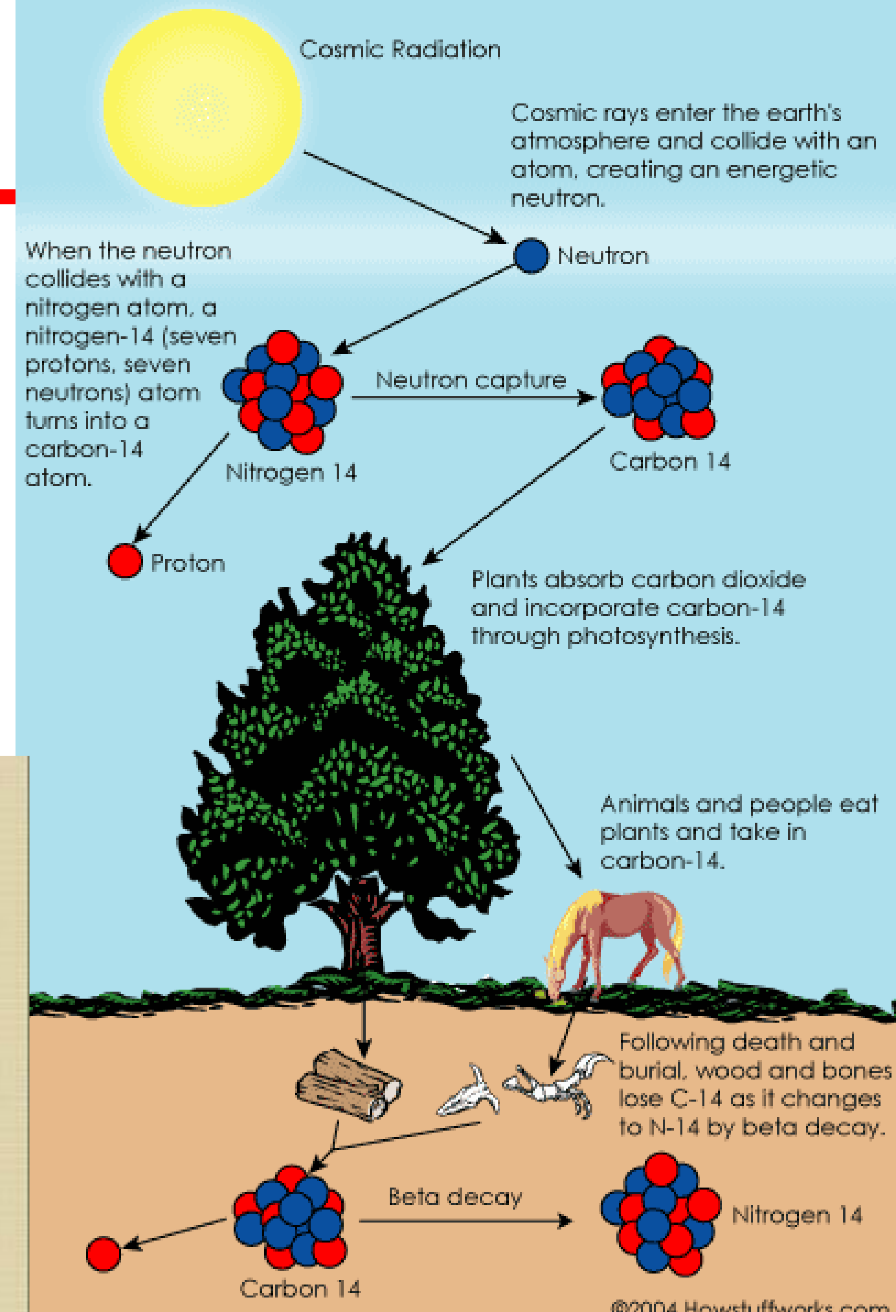
N never gets to zero!

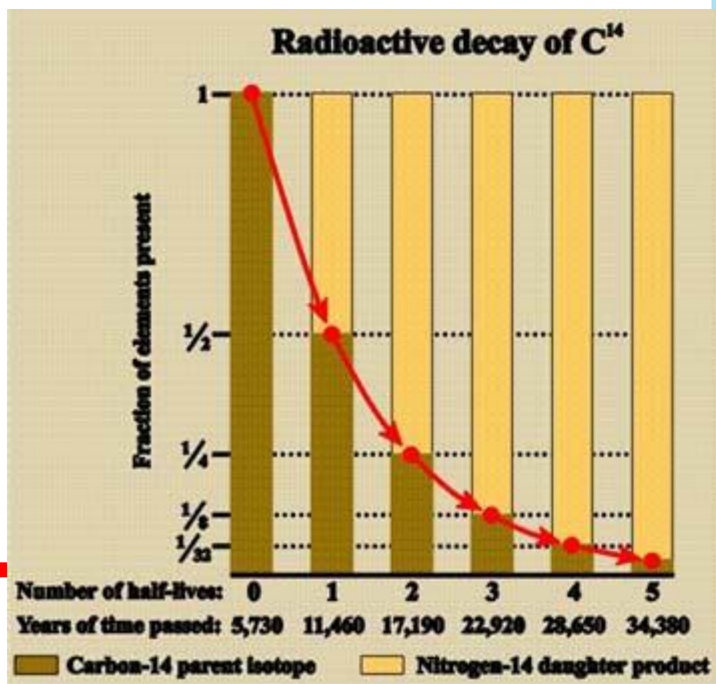
The half lie of carbon-14 is 5730 years. How old a sample that has only 10% of its original carbon-14?

$$t_{1/2} = \frac{\ln(2)}{\lambda} = \tau \ln(2) \Rightarrow \tau = 5730 / \ln(2) = 8267\,years$$

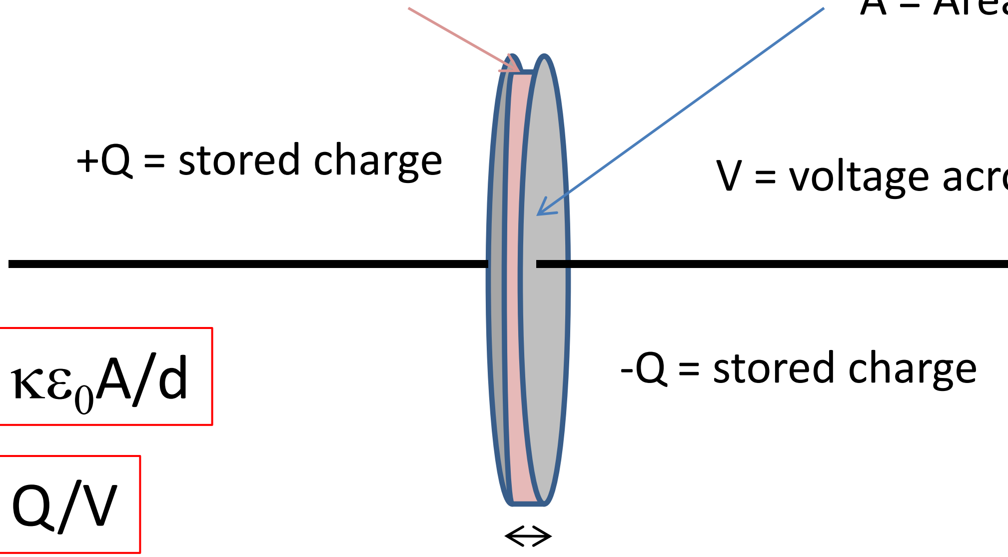
$$N(t) = N_0 e^{-\lambda t} = N_0 e^{-t/\tau} \Rightarrow \frac{-t}{\tau} = \ln[N(t)/N_0]$$

$$t = -\tau \ln(0.1) = -8256 \times 2.303 = 19{,}010\,years$$





The Parallel Plate Capacitor



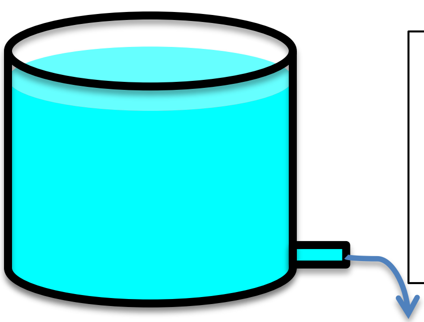
$C = \kappa\varepsilon_0 A/d$

$C = Q/V$

$$C = \kappa\varepsilon_0 \frac{A}{d} = [\mathrm{F}],\ \text{Farads}$$

$$\varepsilon_0 = 8.85 \times 10^{-12}\left[\frac{\mathrm{As}}{\mathrm{Vm}}\right]$$

$$\kappa = 1 \text{ for air or vaccum}$$



Electrical *Capacitance* is similar to the cross sectional area of a fluid reservoir or standpipe. Electrical charge corresponds to amount (volume) of the stored fluid. And voltage corresponds to the height of the fluid column.

Exponential Change in Circuits: Capacitors - Charging

- we can write an equation, then, for the current:

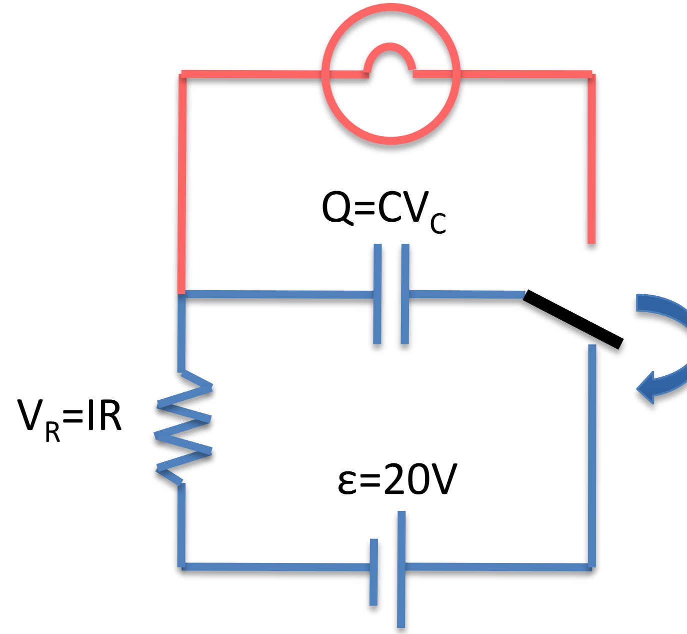
$$\mathcal{E} - R\frac{dQ}{dt} = \frac{Q}{C}$$

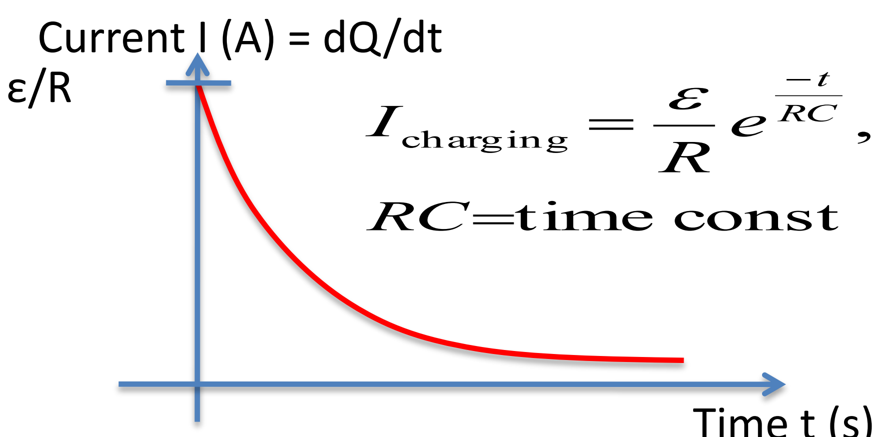
$$\frac{\mathcal{E}}{R} - \frac{Q}{RC} = \frac{dQ}{dt}$$

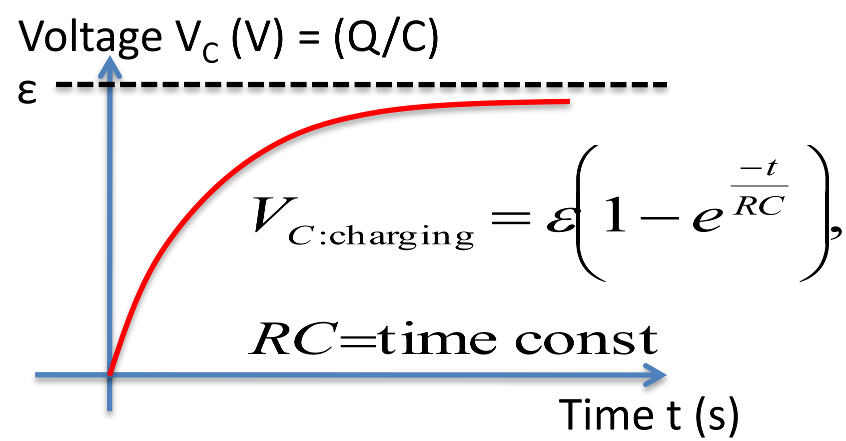
- solution: $Q(t) = \mathcal{E}C(1 - e^{-t/RC})$

V = Q/C

I = dQ/dt







Exponential Change in Circuits: Capacitors - Discharging

V = Q/C

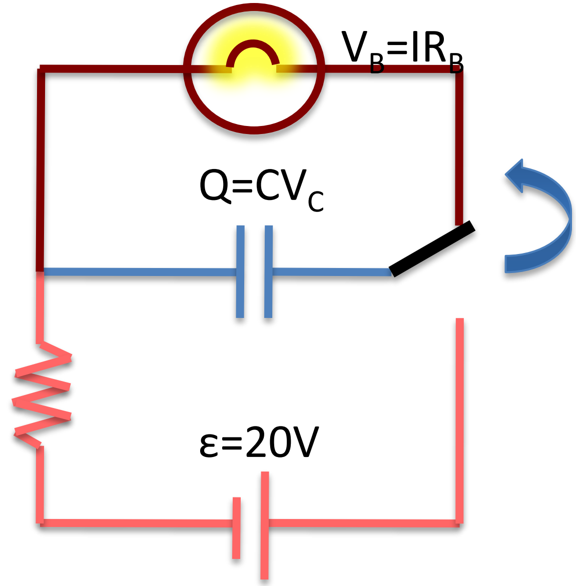
I = dQ/dt

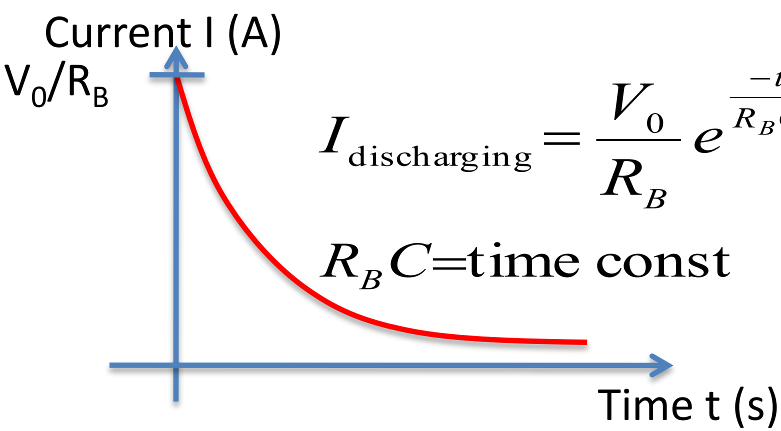
$$V_C = -IR$$

$$Q/C = -\frac{dQ}{dt}R$$

$$\frac{dQ}{dt} = -\frac{1}{RC}Q(t)$$

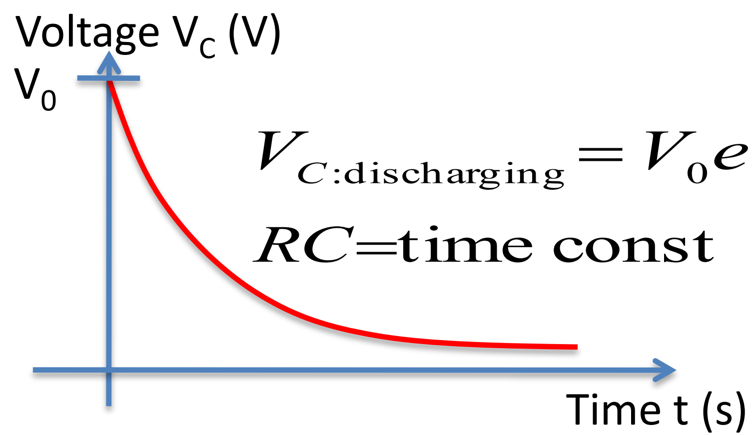
$$Q(t) = Q_0 e^{-t/RC}$$





$$I_{\text{discharging}} = \frac{V_0}{R_B} e^{\frac{-t}{R_B C}},$$

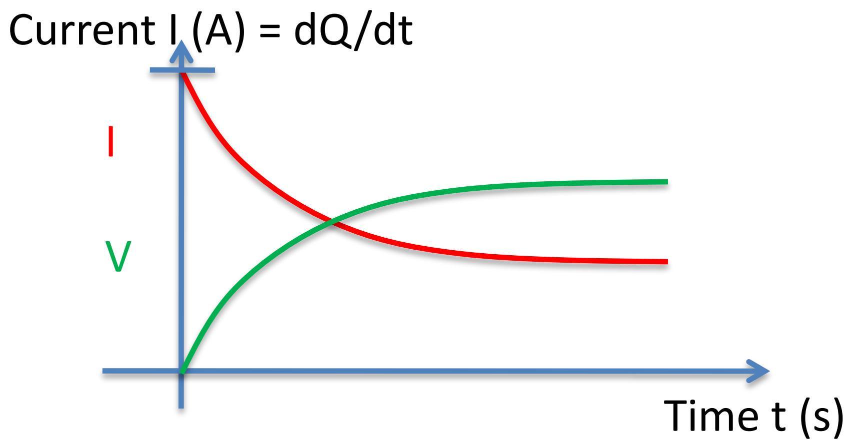
$R_B C$=time const

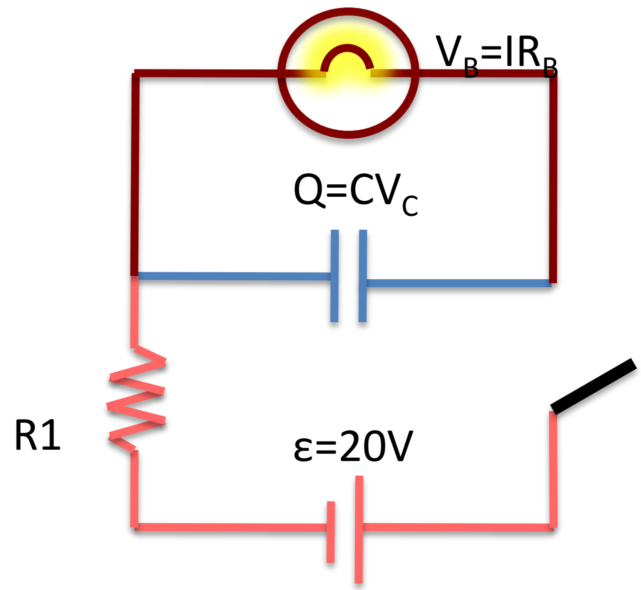


$$V_{C:\text{discharging}} = V_0 e^{\frac{-t}{RC}},$$

RC=time const

The switch is closed at time t=0. Make a plot of current as a function of time in R1. Then make a plot of the voltage across the capacitor as a function of time.

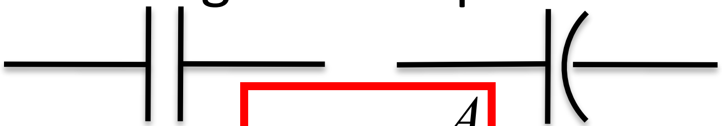




$I(t=0)=\mathcal{E}/R1$ $I(t=\text{infinity})=\mathcal{E}/(R1+RB)$ $VC(t=\text{infinity})=\mathcal{E}-\mathcal{E}R1/(R1+RB)$

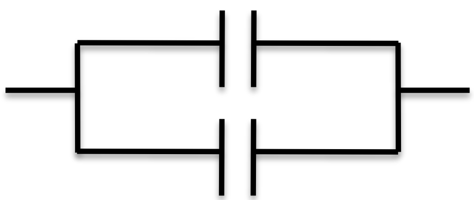
Capacitors in Series and Parallel

- Circuit Diagrams: Capacitors



$$C = \kappa \varepsilon_0 \frac{A}{d}$$

- Capacitors in parallel (~2xA)



$$C_{parallel} = C_1 + C_2 + \ldots$$

- Capacitors in series (~2xd)

$$\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$$

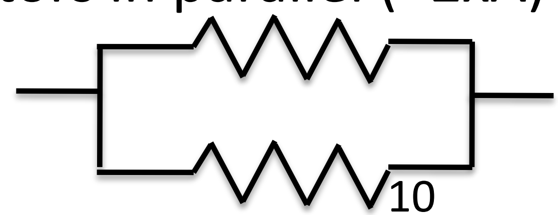
- Circuit Diagrams: Resistors

$$R = \rho \frac{L}{A}$$

- Resistors in Series (~2xL)

$$R_{series} = R_1 + R_2 + \ldots$$

- Resistors in parallel (~2xA)



$$\frac{1}{R_{parallel}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$$

Capacitors: Energy Stored in a capacitor

- Because resistors dissipate power, we wrote a an equation for the power dissipated in a Resistor:

$$P = IV, \text{ using } V = IR:$$
$$P = I^2 R \quad \text{or} \quad P = \frac{V^2}{R}$$

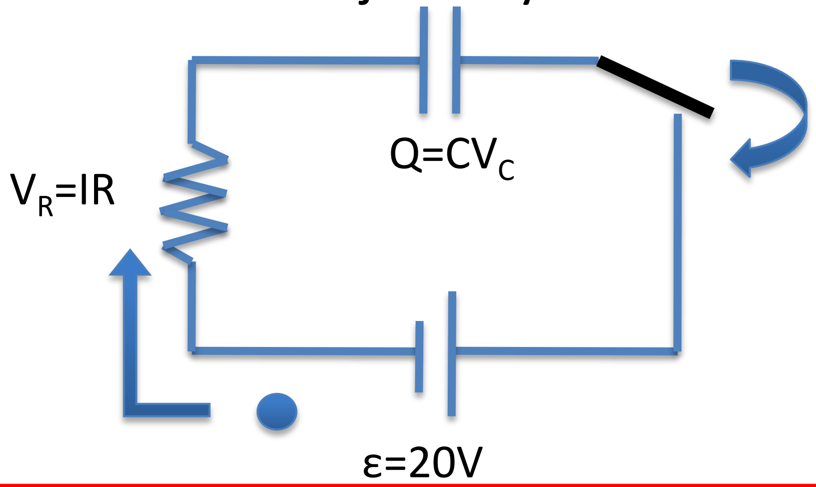
Note: Since *I* is same for resistors in series, identical resistors in series will have the same power loss. Since *V* is the same for resistors in parallel, identical resistors in parallel will have the same power loss

- Because capacitors are used to store charge and energy, we concentrate on the energy stored in a capacitor.
- We imagine the first and the last electrons to make the journey to the capacitor. What are their ΔPE's?

$\Delta PE_{first} = q\Delta V$, $\Delta V=20$ $\Delta PE_{last} = q\Delta V$, $\Delta V=0$

Thus on average for the whole charge:

$$PE = \frac{1}{2}QV, \text{ using } Q = CV$$
$$PE = \frac{1}{2}CV^2$$



Outline

Galilean Space-time; Galilean Relativity

Vectors – Vector Addition and Subtraction – r, v, a

Forces

- Newton's 3rd law – Equal and opposite forces
- Net Force – SF
- Force Diagrams – center of mass
- Long range vs. contact forces
- Balanced forces – Inertia, Newton's 1st Law

Two Fundamental Forces Fe and Fg

Fe=kq1q2/r2	Fg=Gm1m2/r2
ke=9x109 Nm2/C2	G=6.672x10-11 Nm2/kg2
qe=1.602x10-19 C	

Forces we feel – gravity of the earth

Contact forces – electromagnetic

Little g → g=GME/rE2

Orbits – Big Gravity

Springs – Hooke's Law F=-kx

Normal Force – Perpendicular contact force – Parallel contact force, friction

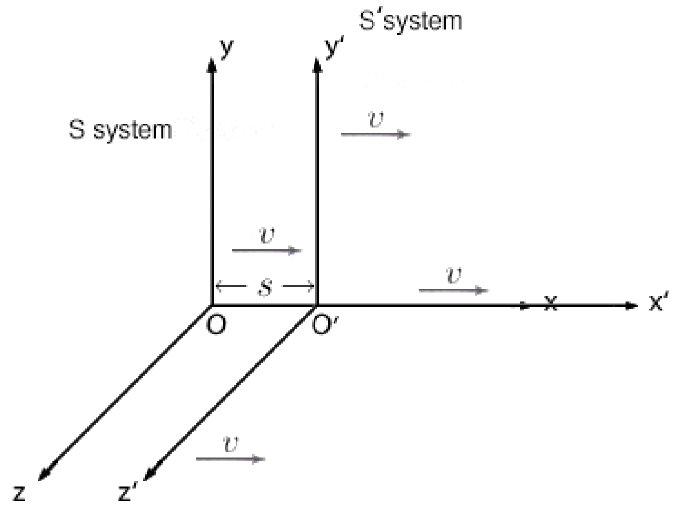
Drag forces

Galilean Space-time and Galilean Relativity

The motion of uniformly accelerated objects was studied by Galileo as the subject of **kinematics**.

Galileo's Principle of Inertia stated: "*A body moving on a level surface will continue in the same direction at constant speed unless disturbed*." This principle was incorporated into Newton's laws of motion (first law).

Galileo's concept of inertia refuted the generally accepted Aristotelian hypothesis that objects generally slow down.

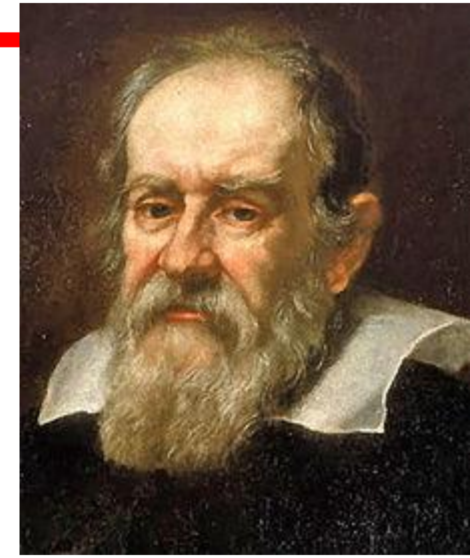


$$\left.\begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned}\right\}$$

Galilean Transformation Equations

$$\left.\begin{aligned} x &= x' + vt' \\ y &= y' \\ z &= z' \\ t &= t' \end{aligned}\right\}$$

Galilean Inverse Transformation Equations



Galileo 1564-1642

Galileo introduced space-time and the concept of an inertial reference frame.

Vector - Definition

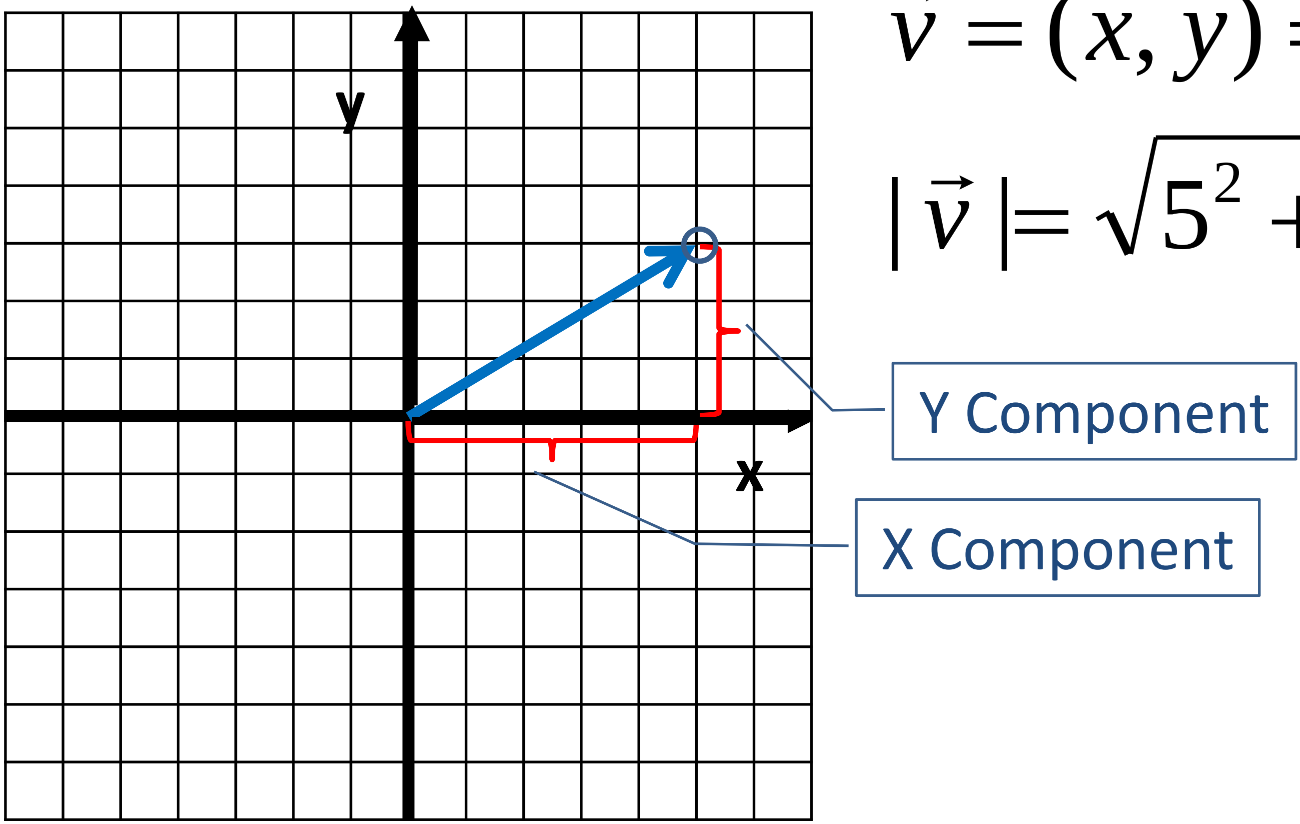
- Merriam-Webster defines vector as:
  1. **a quantity that has magnitude and direction and that is commonly represented by a directed line segment whose length represents the magnitude and whose orientation in space represents the direction**
  2. an organism (as an insect) that transmits a pathogen
  3. an agent (as a plasmid or virus) that contains or carries modified genetic material (as recombinant DNA) and can be used to introduce exogenous genes into the genome of an organism

Vectors will either be written in bold (***v***) or with an overstrike ($\vec{v}$)

Vector - Applications

- Vectors are used to represent:
  - Position of an object
  - Velocity of an object
  - Acceleration of an object
  - Force on an object
- Two representations of vectors:
  - Cartesian Coordinates
  - Polar Coordinates

Cartesian Coordinates

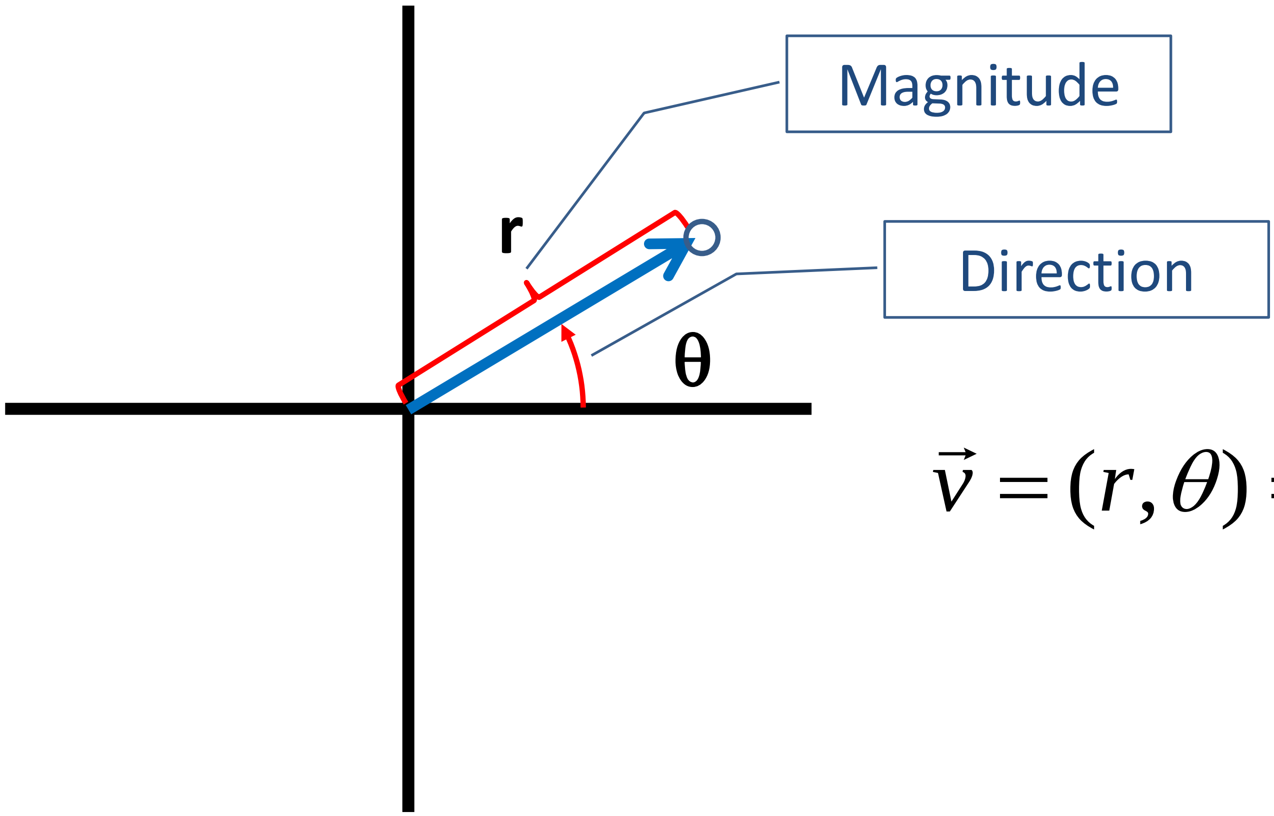


$$\vec{v} = (x, y) = (5,3)$$

$$|\vec{v}| = \sqrt{5^2 + 3^2} = 5.83$$

Rene Descartes
1596-1650

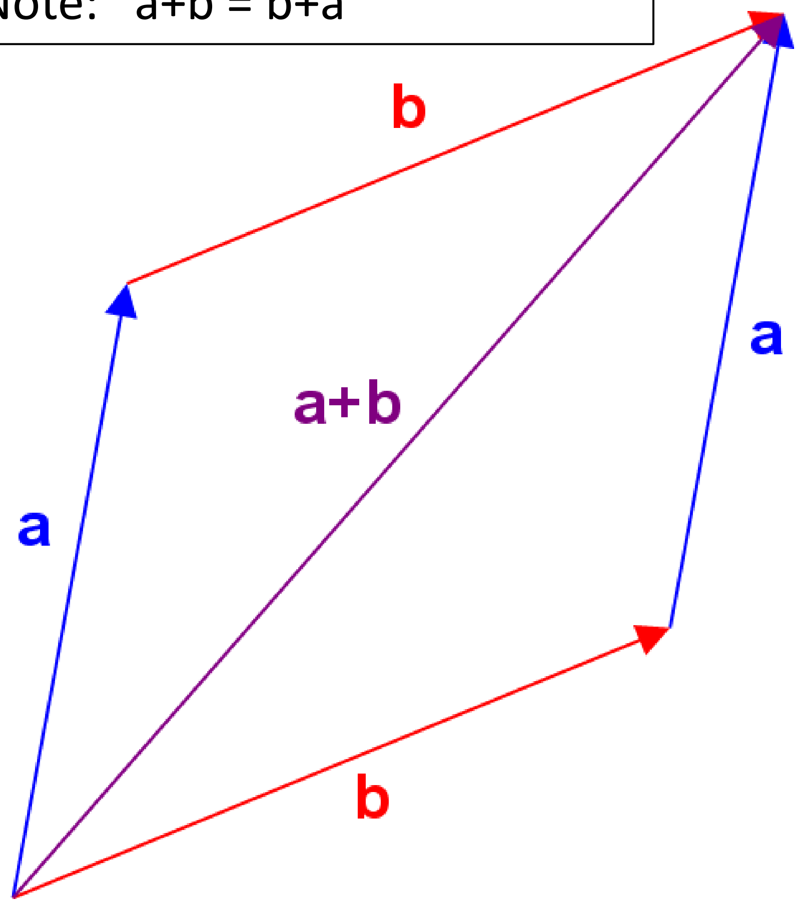
Polar Coordinates



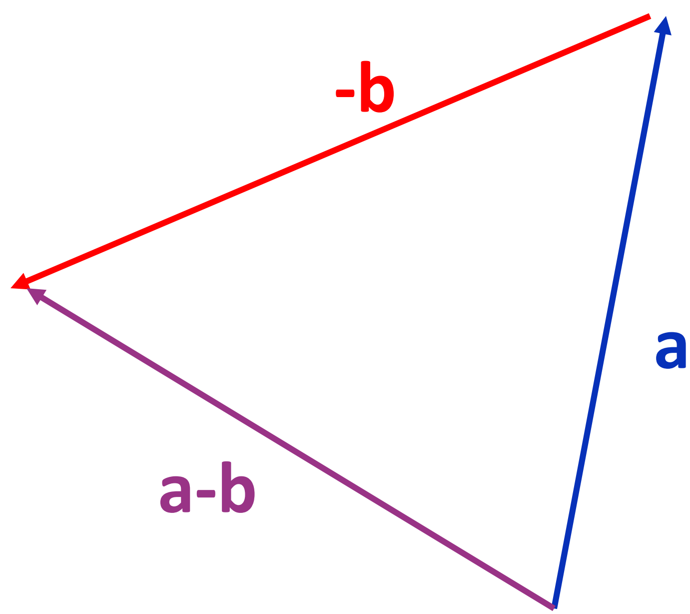
$$\vec{v} = (r, \theta) = (5.83, 31^o)$$

Vector Addition and Subtraction

Vectors are added head to tail
Note: a+b = b+a



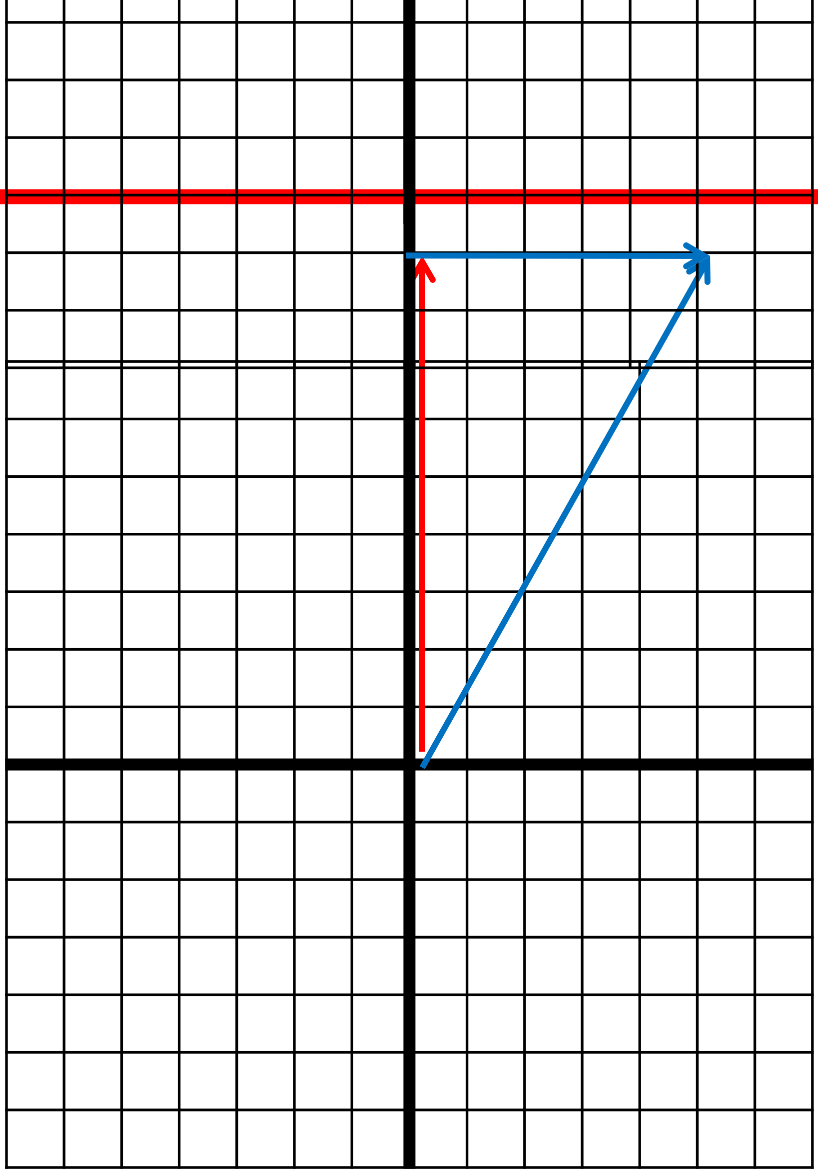
Vectors are subtracted by added –b to vector a
Note: a-b does not equal b-a
(a-b) = -(b-a)



Consider two vectors; the first vector (**A**) has a magnitude of 10 and a direction 60 degrees counter clockwise form the x-axis. The second vector (**B**) has magnitude 5 and is directed in the positive x direction.

What is the magnitude and direction of **A-B**?

Draw the resulting vectors.



Magnitude of A-B is $5\sqrt{3}$ direction is in the y direction. Magnitude is 8.66.

The Vectors of Kinematics

- Position: $\vec{r}$
- Velocity: $\vec{v}_{ins} = \frac{d\vec{r}}{dt}$
- Acceleration: $\vec{a}_{ins} = \frac{d\vec{v}}{dt}$
- Momentum: $\vec{p} = m\vec{v}$
- Force: $\vec{F}_{A\ on\ B}$

The Four Fundamental Forces

Newton's Laws of Motion

- **1st Law**: The velocity of an object will not change unless acted upon by a force
- **2nd Law**: The net force on an object is equal to the rate of change of momentum
- **3rd Law**: For every force there is an equal but opposite force

Newton's First Law of Motion

- 1st Law: The velocity of an object will not change unless acted upon by a net force

$$\vec{F}_{Net\ on\ Object} = 0 \quad \Rightarrow \quad \Delta\vec{v} = 0$$

- When $F_{Net\ on\ Object} = 0$,
  - an object at rest continues to stay at rest
  - an object in motion continues to move at constant speed along a straight path
- a.k.a. The Law of Inertia

Newton's Second Law of Motion

- 2^{nd} Law: The net force on an object is equal to the rate of change of momentum

$$\vec{F}_{Net\ on\ Object} = \frac{d\vec{p}}{dt}$$

- Force is proportional to acceleration:

$$\vec{F}_{Net\ on\ Object} = \frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a}$$

- a.k.a. The Law of Resultant Force

Newton's Third Law of Motion

- 3rd Law: For every force there is an equal but opposite force

$$\vec{F}_{A\ on\ B} = -\vec{F}_{B\ on\ A}$$

- $F_{A\,on\,B}$ and $F_{B\,on\,A}$ are referred to as an action-reaction pair.
  - They act on different objects
- a.k.a. The Law of Reciprocal Actions

Net Force – $\Sigma\boldsymbol{F}$

In the previous few slides, we have discussed $\boldsymbol{F}_{\text{net}}$ but what is the "*net force*"?

The net force is a vector sum of all the different forces that are acting upon the object of interest.

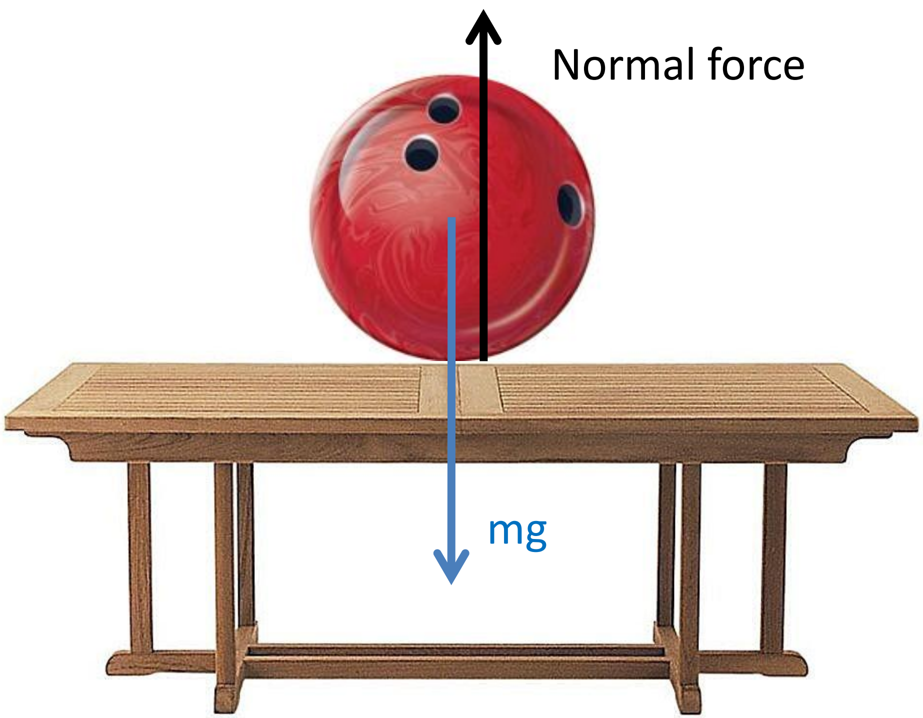
$$\vec{F}_{Net\ on\ Object} = \sum \vec{F}$$

Although each force acts independently, that final results is just the sum of the various contributions.

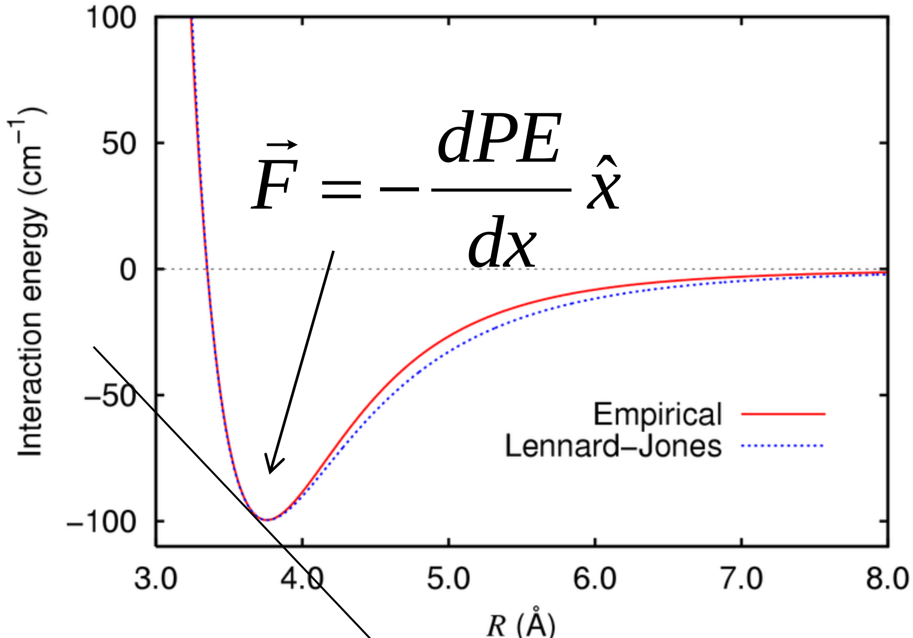
Types of Force

- Contact forces: Require physical contact between two objects (action and reaction)
  - Friction force: Acts parallel to contact surface
  - Normal force: Acts perpendicular to contact surface
- Long-range forces: Require presence of a field between two objects (action at a distance)
  - Gravitational force: Exerted by one massive object on another massive object
  - Electrostatic force: Exerted by one charged object on another charged object

Contact forces – Normal Force (electromagnetic)



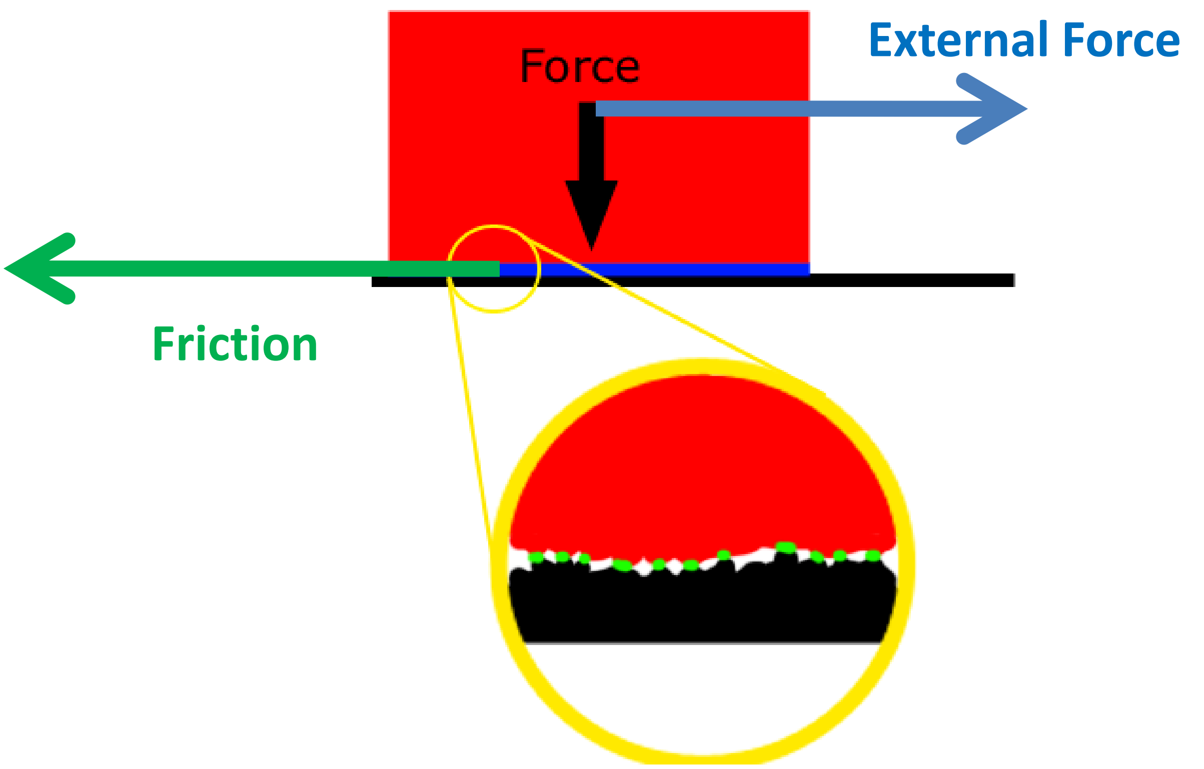
The downward gravitational force causes the bowling ball to compress the surface of the table. A very small compression will result in a very large repulsive force



$$\vec{F} = -\frac{dPE}{dx}\hat{x}$$

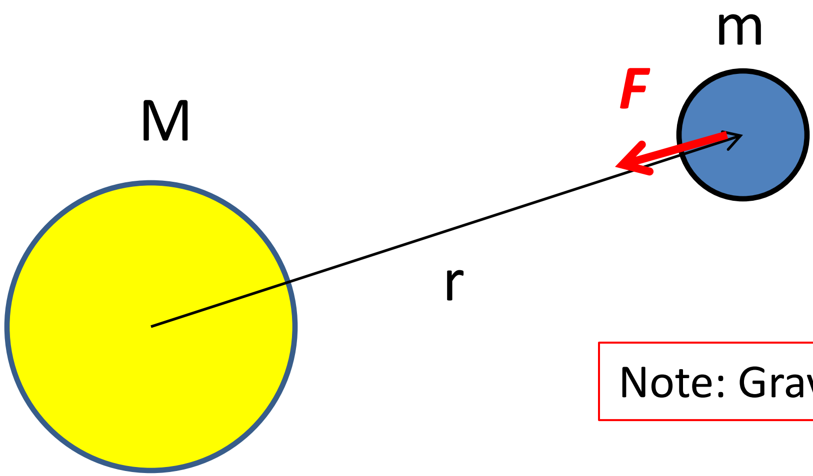
Contact forces – Friction (electromagnetic)

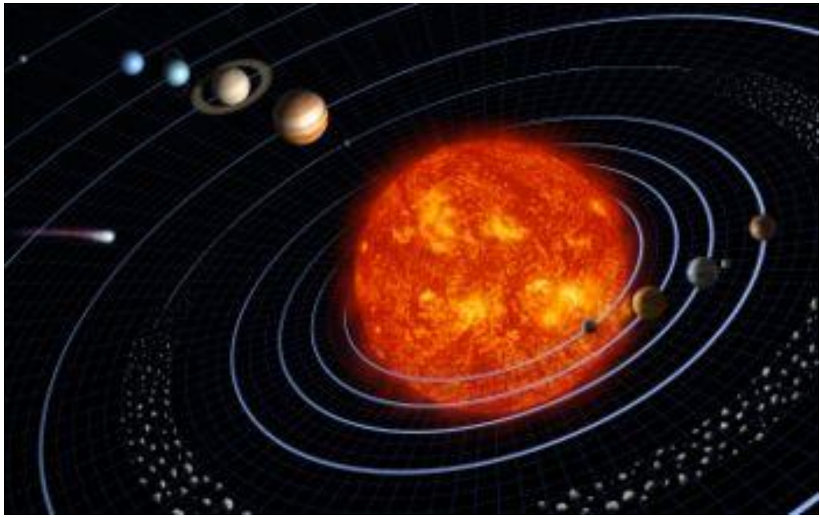
When you try to slide one object parallel to another the microscopic surface features will get compressed (Lennard-Jones) and will generate a frictional force to oppose any motion



Long Range Force -- $F_g = Gm_1m_2/r^2$ – Gravitational

$$\vec{F}_G = -G\frac{mM}{r^2}\hat{r}$$



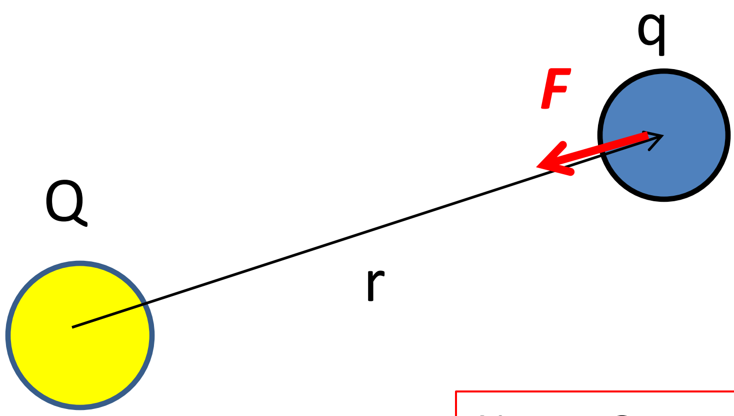


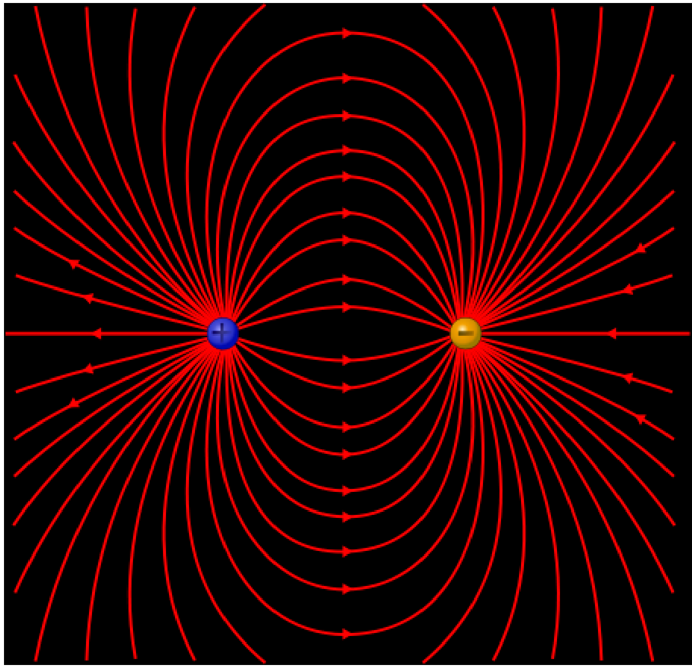
Note: Gravity is always attractive

G = 6.67 X 10^{-11} Nm^2/kg^2 "universal gravitational constant"
M = mass of the object which creates the field
m = mass of the object which experiences the field
r = distance between m and M (pointing from M to m)

Long Range Force -- $F_e = kq_1q_2/r^2$ -- Electromagnetic

$$\vec{F}_E = k\frac{qQ}{r^2}\hat{r}$$





Note: Opposites attract, like signs repel

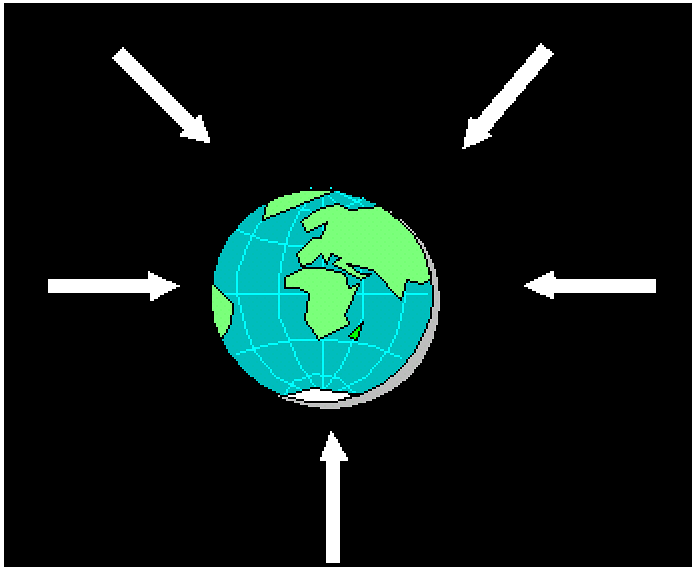
k = 9.0 x 10^9 Nm^2/C^2 "universal gravitational constant"

Q = charge of the object which creates the field

q = charge of the object which experiences the field

r = distance between q and Q

Forces we feel – gravity of the earth



At the fundamental level, the electric force is 10^{36} times stronger than the gravitational force!

In our everyday lives, we are aware of the long range force of gravity. If you try to jump up, gravity will pull you back down.

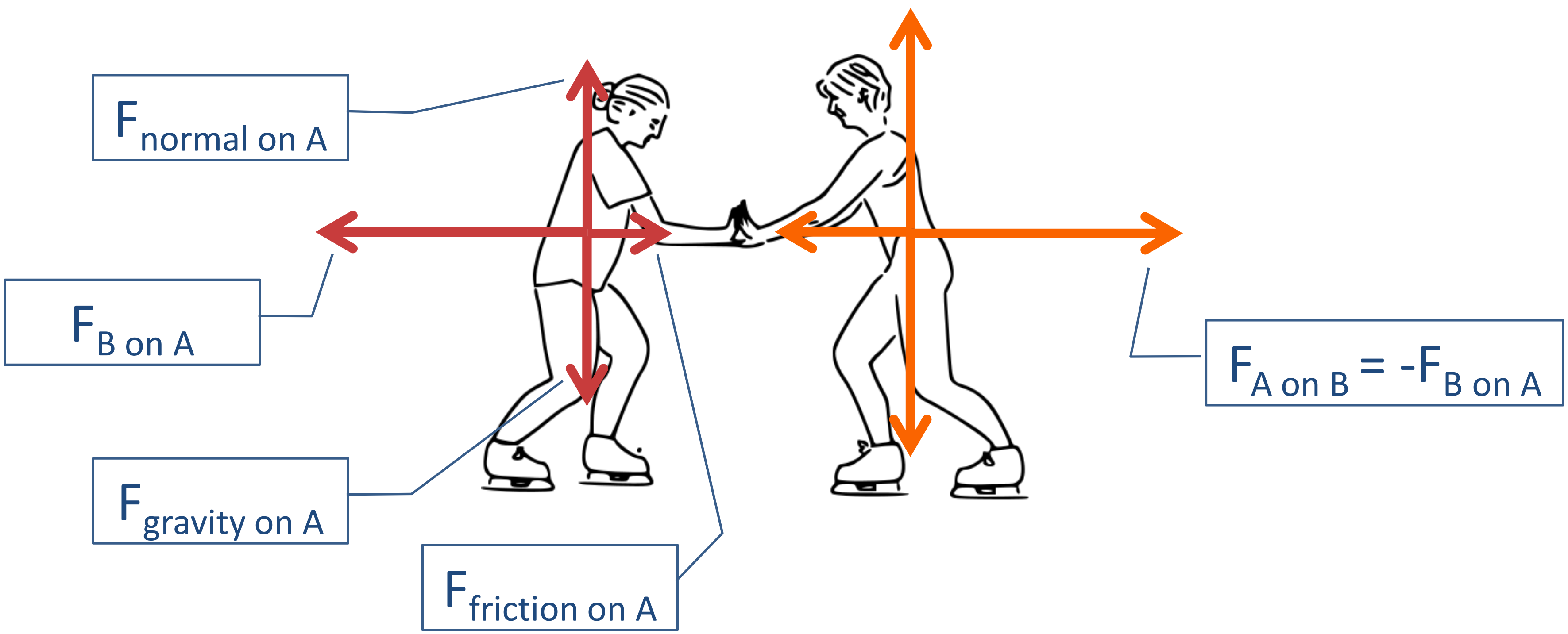
We are aware of friction, especially when you are sliding something across a floor.

We do not really appreciate the normal force; we tend to think of ourselves pushing down on the chair, rather than the chair pushing up.

And we almost never experience the long range electric force – because it is SO strong, that electric charges always want to move to neutralize any local excess.

Force Diagram

- What are the forces on Alice?
- What are the forces on Bob?



Little g → $g=GM_E/r_E^2$

We know that: $$\vec{F}_G = -G\frac{mM}{r^2}\hat{r}$$

We also know that: $$\vec{F} = m\vec{a}$$

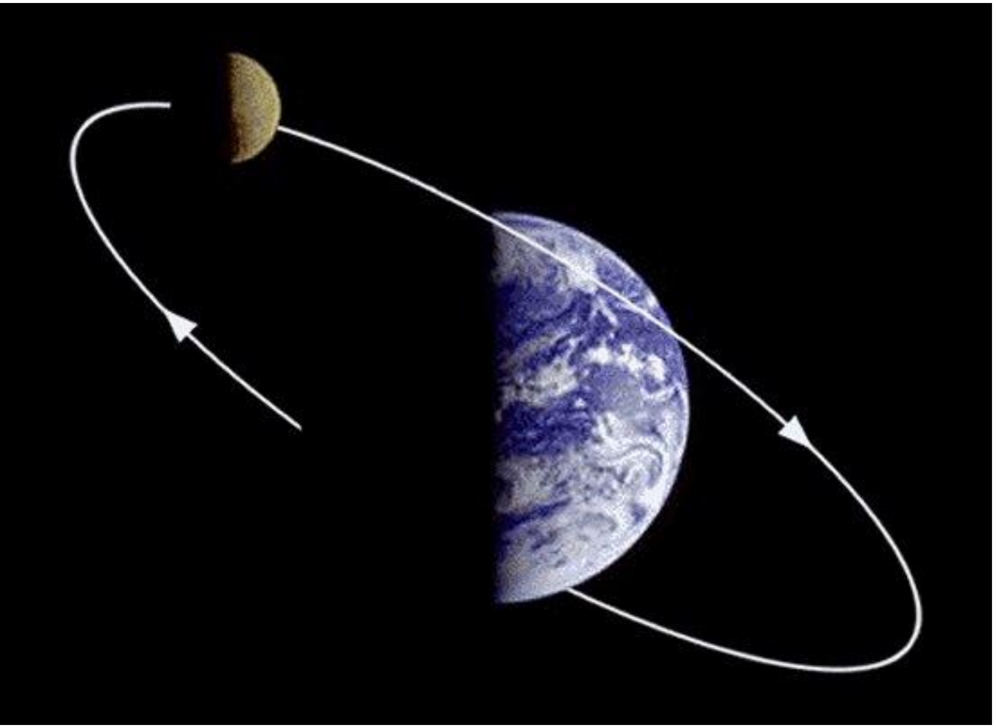
Therefore, we can define the acceleration due to gravity at the surface of the earth as *g*:

$$g = GM_\oplus / r_\oplus^2$$

$$g = (6.672\times10^{-11} Nm^2/kg^2)\frac{5.98\times10^{24} kg}{(6378km)^2}$$

$$\boxed{g = 9.808 kgm/s^2}$$

Orbits – Big Gravity



It is only on the astronomical scale that we experience the $1/r^2$ dependence of gravity (on the surface of the earth, the acceleration due to gravity is always *g*).

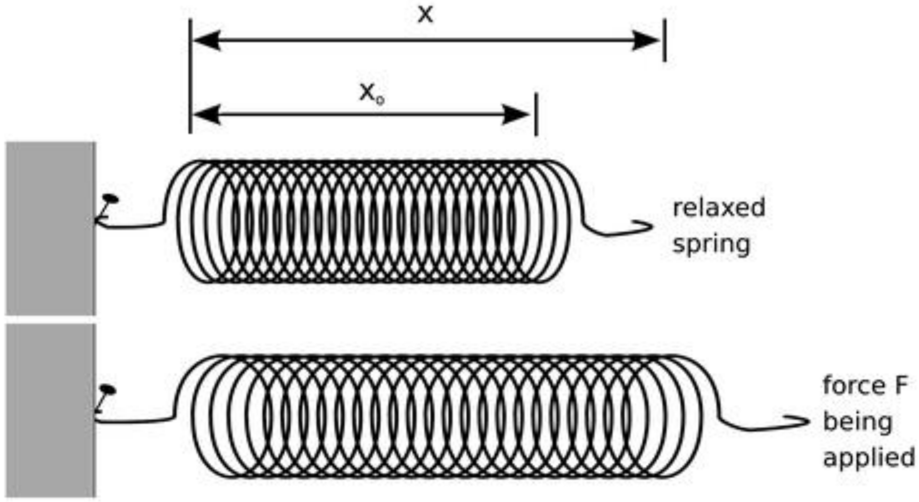
Let's you what we know about the moon to determine the weight of the Earth.

$$F = G\frac{mM}{r^2} = m\frac{v^2}{r} \Rightarrow M = \frac{v^2 r}{G} = \frac{(2\pi r / Period)^2 r}{G}$$

$$M = \frac{(2\pi \times 384403km / 27.15days)^2 \times 384403km}{6.672 \times 10^{-11} Nm^2 / kg^2}$$

$$M_{\oplus} = 5.974 \times 10^{24} kg$$

Springs – Hooke's Law F=-kx



There are other forces that we have talked about in physics 7A, for example the force of an extended or compressed spring (given by Hooke's Law).

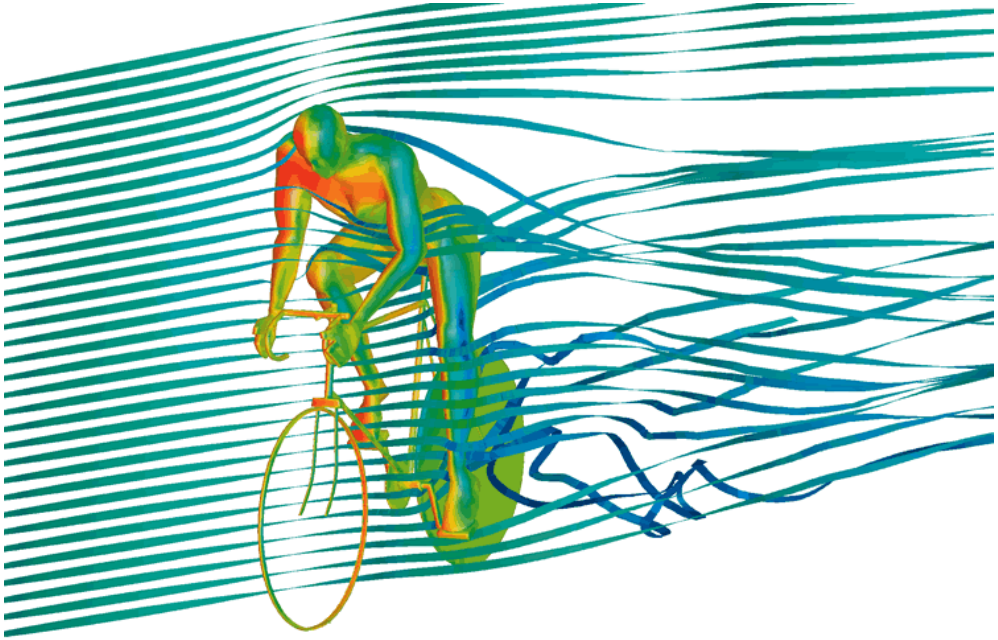
Fundamentally, like friction, this is an electric force coming from the microscopic compressions or stretches of the inter-atomic spacing in the metallic lattice of the spring steel.

Drag Forces

In fluid dynamics, ***drag*** (sometimes called ***air resistance*** or ***fluid resistance***) refers to forces that oppose the relative motion of an object through a fluid (a liquid or gas).

Drag forces act in a direction opposite to the oncoming flow velocity.

Unlike other resistive forces such as dry friction, which is nearly independent of velocity, drag forces depend on velocity.



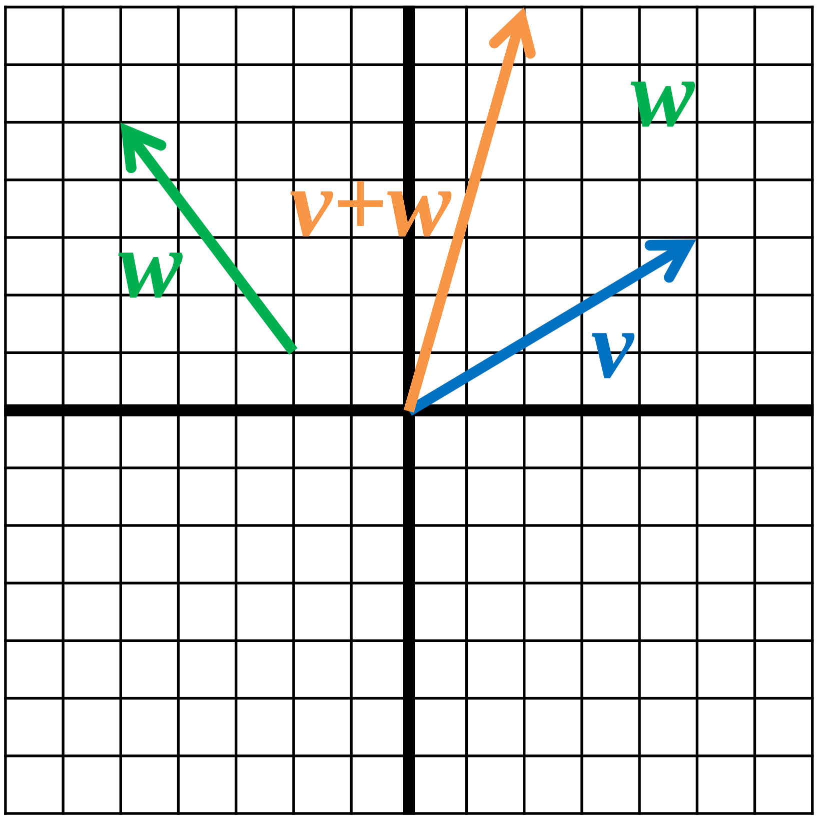
Announcements

DL Sections

Winter 2010 7B-1 (A/B) D/L Assignments & Job Responsibilities

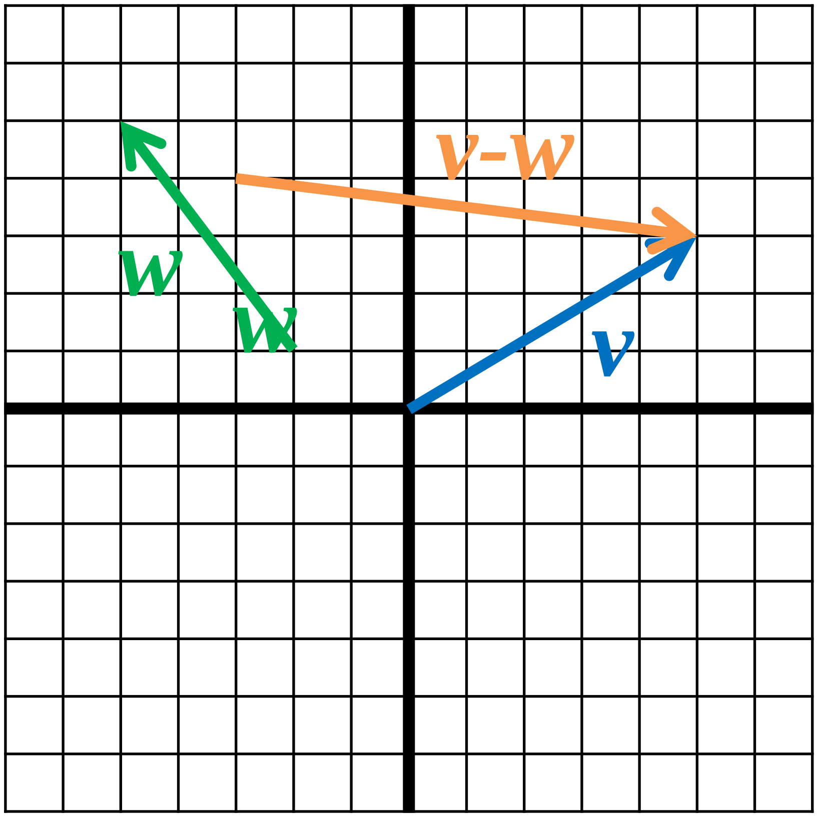
1	WF	10:30-12:50	2317 EPS	Marcus Afshar
2	MW	2:10-4:30	2317 EPS	Aaron Hernley
3	MW	4:40-7:00	2317 EPS	Rylan Conway
4	MW	7:10-9:30	2317 EPS	Rylan Conway
5	MR	8:00-10:20	2317 EPS	Robert Lynch
6	TR	10:30-12:50	2317 EPS	Aaron Hernley
7	R	2:10-4:30	2317 EPS	Justin Dhooghe
7	M	10:30-12:50	2317 EPS	Justin Dhooghe
8	TR	4:40-7:00	2317 EPS	Britney Rutherford
9	TR	7:10-9:30	2317 EPS	Britney Rutherford
10	TF	8:00-10:20	2317 EPS	Emily Ricks
11	TF	2:10-4:30	2317 EPS	Justin Dhooghe

Vector Addition – Geometric Method



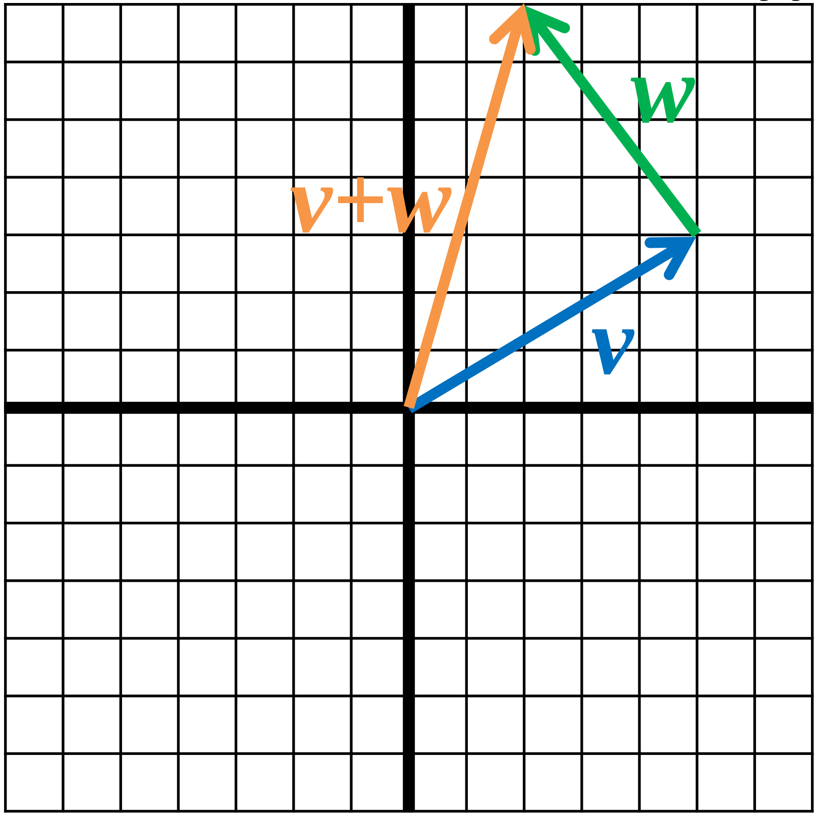
1. Place tail of ***w*** on tip of ***v***.
2. Connect tail of ***v*** to tip of ***w***.

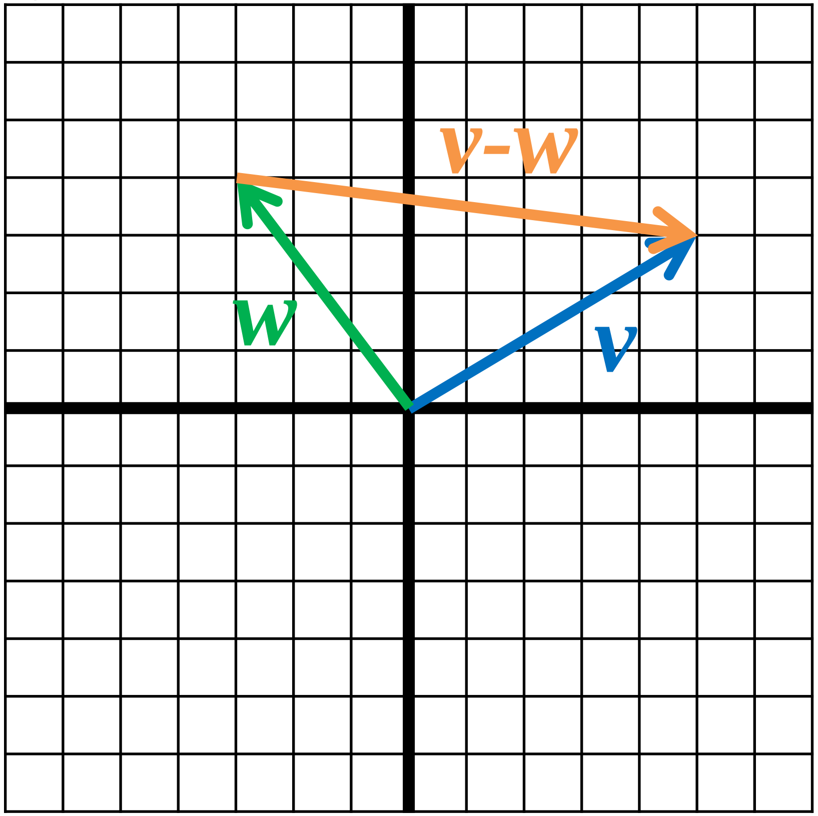
Vector Subtraction – Geometric Method



1. Place tail of ***w*** on tail of ***v***.
2. Connect tip of ***w*** to tip of ***v***.

Addition & Subtraction – Algebraic Method





$$\vec{v} + \vec{w} = (5,3) + (-3,4)$$
$$= (2,7)$$

$$\vec{v} - \vec{w} = (5,3) - (-3,4)$$
$$= (8,-1)$$

Vector Multiplication

- Multiplying a vector by a scalar:

- Multiplying a vector by another vector:
  - Dot product (a.k.a. scalar product)
  - Cross product (a.k.a. vector product)

Position

- Position Vector: Indicates location of an object with respect to an origin
  - Important: Position vector depends on origin
- Displacement: Change in position vector

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$

- Units: $[r] = m$

Velocity

- Velocity: Rate of change in position vector
  - Includes both speed of motion (magnitude of $\boldsymbol{v}$) and direction of motion (direction of $\boldsymbol{v}$)
- Two types of velocity:

$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t} \qquad \vec{v}_{ins} = \frac{d\vec{r}}{dt}$$

- Units: $[v] = m/s$

Acceleration

- Acceleration: Rate of change in velocity vector
- Two types of acceleration:

$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t} \qquad \vec{a}_{ins} = \frac{d\vec{v}}{dt}$$

- Units: $[a] = m/s^2$

Momentum

- Momentum: Product of mass and velocity vector

$$\vec{p} = m\vec{v}$$

- Why is momentum important? Like energy, momentum is conserved.
- Impulse: Change in momentum vector

$$\Delta\vec{p} = \vec{p}_2 - \vec{p}_1$$

- Units: $[p] = kg\ m/s$

Force

- Force: The cause or agent of acceleration resulting from the interaction of two objects
- A force always involves two objects:

$$\vec{F}_{A\ on\ B}$$

- Units: $[F] = kg\ m / s^2 = N$

Friction and Normal Forces

- Friction force:
- Normal force:

Force Diagram

- What are the forces acting on the block?