Review of Linear Transport Model and Exponential Change Model

Linear Transport Model

Starting with Ohm's Law ($\Delta V = -IR$), which we had derived from the Energy Density Model, we can rewrite this to solve for current ($I = -(1/R)\Delta V$

Using our definition of R from the previous slide [$R = \rho(L/A)$ or $(1/R) = k\,(A/L)$],
We get

$$I = -\mathrm{k}\,(A/L)\,\Delta V$$

If we let $\Delta V \rightarrow \Delta\phi$, then we get

$$I = -\mathrm{k}\,(A/L)\,\Delta\phi$$

Divide through by area A,

$$j = -\mathrm{k}\,(1/L)\,\Delta\phi$$

Let L become an infinitesimal

$$j = -k\,d\phi/dx \qquad \leftarrow \textit{Transport Equation}$$

Linear Transport

Which statement is not true about linear transport systems?

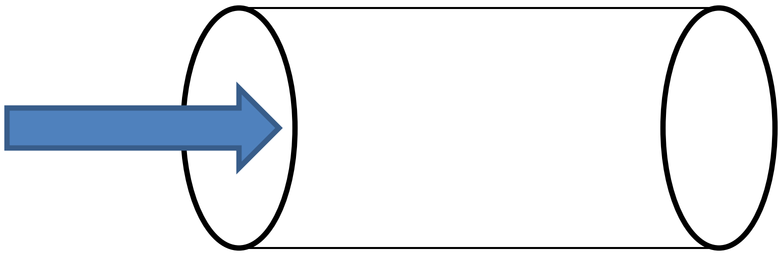
a) If you double the driving potential (voltage, temperature difference,...) , the current doubles.
b) If you double the resistance, the current is halved.
c) In linear transport systems, the driving potential varies linearly with the spatial dimension.
d) For both fluid flow and electrical circuits, the continuity equation requires that the current density is independent of position.
e) All of the other statements are true.

Application of Linear Transport Model

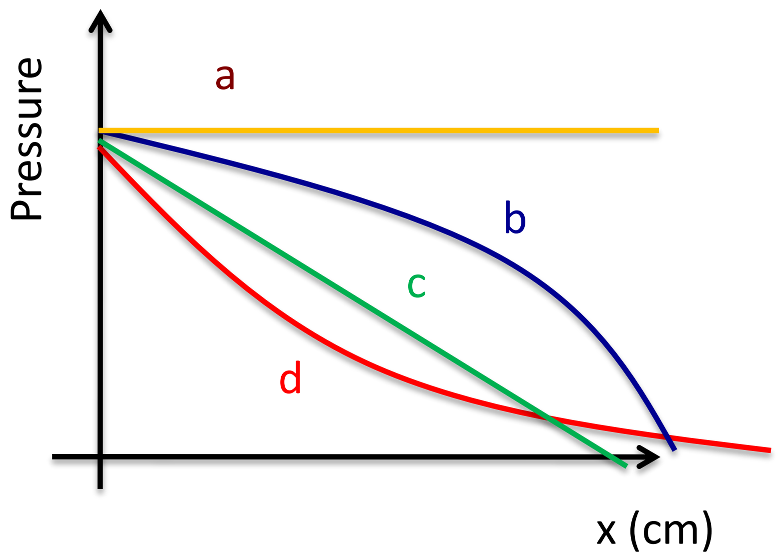
- Fluid Flow
  - ϕ=*Head* and j=*mass current density*
- Electric Current (Ohm's Law)
  - ϕ=*Voltage* and j=*charge current density*
- Heat Conduction
  - ϕ=*Temperature* and j=*heat current density*
- Diffusion (Fick's Law)
  - ϕ=*Concentration* and j=*mass current density*

Fluid Flow and Transport

$$\boxed{j = -kd\phi/dx}$$



Which curve best describes the pressure in the above pipe as a function of position along the direction of laminar flow?



The correct answer is "c"

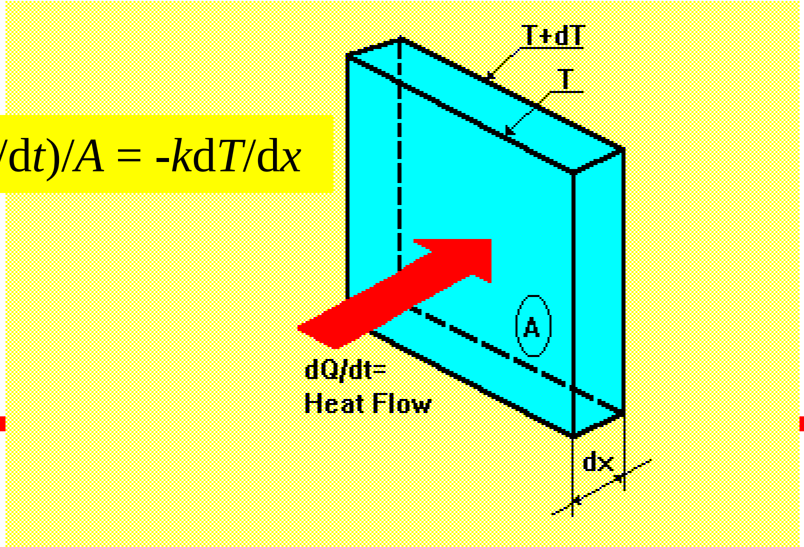
Heat Conduction – Fourier's Law

Suppose you lived in a 10' x 10' x 10' cube, and that the wall were made of insulation 1' thick. How much more insulation would you have to buy if you decided to expand your house to 20' x 20' x 20', but you did want your heating bill to go up?

a) Twice as much
b) Four times as much
c) Eight times as much
d) Sixteen times as much

Answer:
d) The area went up by a factor of four, therefore you will need four times as much thickness => sixteen times the total volume of insulation

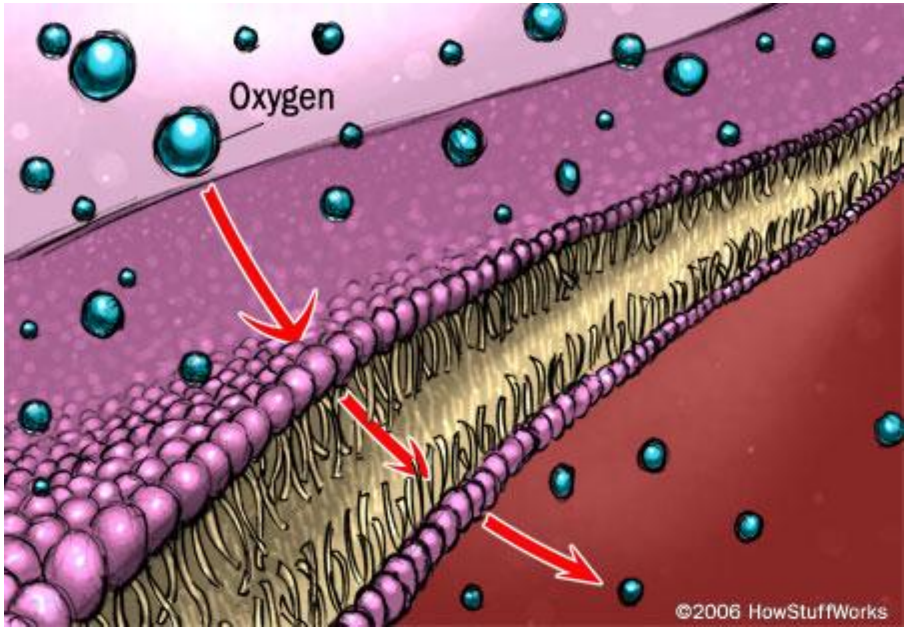
$$j_Q = (dQ/dt)/A = -k\,dT/dx$$



Diffusion – Fick's Laws

$j = -D\, dC(x,t)/dx$

Where j is the particle flux and *C* in the concentration, and D is the diffusion constant



Exponential Growth

- suppose rate of growth of a population is proportional to the size of the population:

$$\frac{d}{dt}N(t) = kN(t)$$

- N(t) = number in population as a function of time
- k = rate of growth (units: 1/time)

Exponential Growth

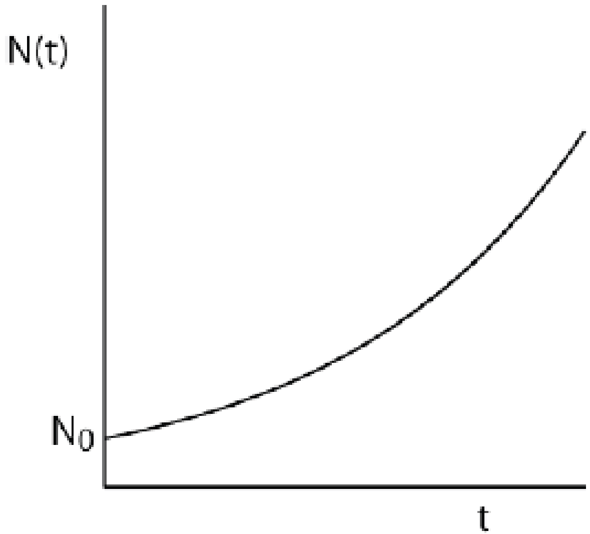
- want to solve equation for N(t)
- need function whose derivative w.r.t. time is that function multiplied by a constant:

$$\begin{aligned} N(t) &= Ce^{kt} \\ \frac{d}{dt}N(t) &= Cke^{kt} = kN(t) \end{aligned}$$

- exactly what we want!

Exponential Growth

- the behavior of N(t) is a rapidly increasing function of time:



$$N(t) = N_0 e^{kt}$$

- N_o is the value of N(t=0)

Examples

- bacteria colony
- interest-bearing fund (APR versus int.)
- global human population?
- Google stock price

Exponential Decay

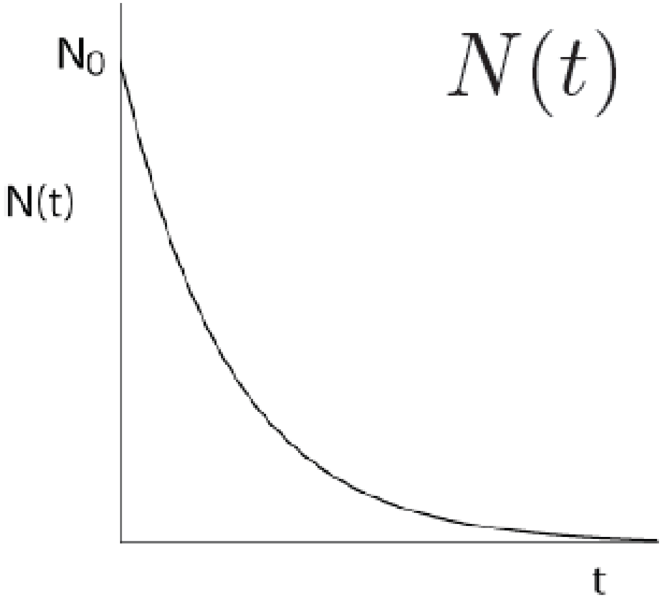
- suppose the rate of disappearance of members of a population is proportional to the number remaining
- this is true, for example, if any member has a random chance k of disappearing in a certain time interval
- the constant k here is the decay rate
- units of k: probability/time

Exponential decay

- equation is similar to growth case:

$$\frac{d}{dt}N(t) = -kN(t)$$

$$N(t) = N_o e^{-kt}$$



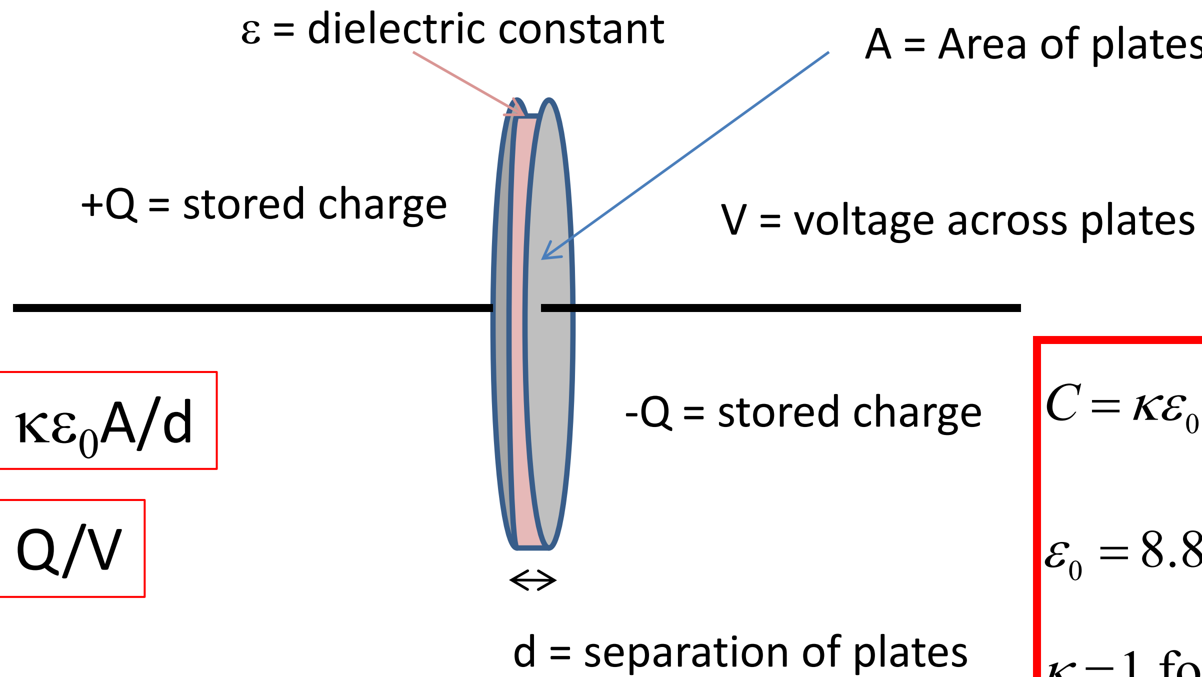
k > 0, but slope of N(t) is <u>always</u> negative!

N never gets to zero!

Examples

- radioactive nuclei in atoms
- water leaking from a tank
- charging/discharging capacitor
- approach to terminal velocity

The Parallel Plate Capacitor



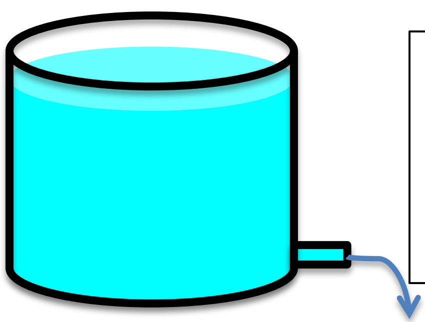
$\boxed{C = \kappa\varepsilon_0 A/d}$

$\boxed{C = Q/V}$

$$C = \kappa\varepsilon_0 \frac{A}{d} = [\mathrm{F}],\ \text{Farads}$$

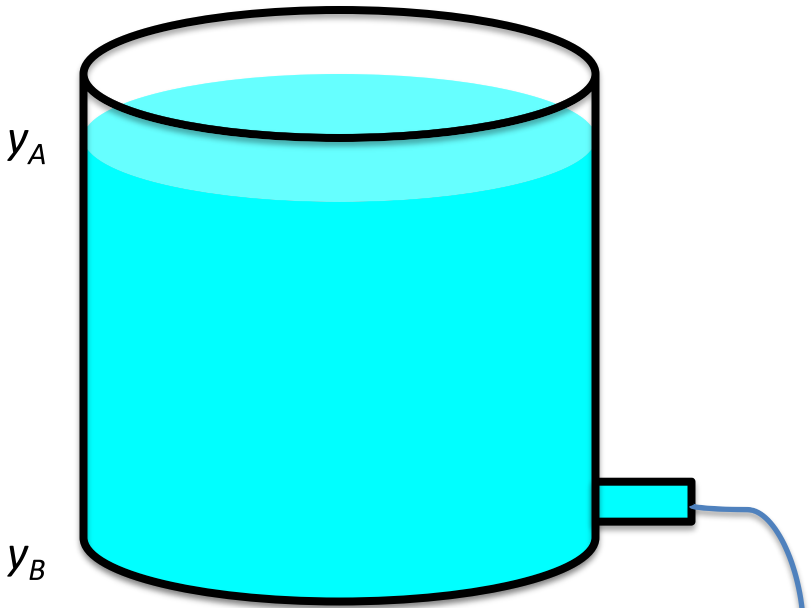
$$\varepsilon_0 = 8.85 \times 10^{-12}\left[\frac{\mathrm{As}}{\mathrm{Vm}}\right]$$

$$\kappa = 1 \text{ for air or vaccum}$$



Electrical *Capacitance* is similar to the cross sectional area of a fluid reservoir or standpipe. Electrical charge corresponds to amount (volume) of the stored fluid. And voltage corresponds to the height of the fluid column.

Non-Linear Phenomenon – Dependence on source



$$v_A = 0 \qquad v_B = ?$$

$$\frac{1}{2}\rho(v_B^2 - v_A^2) + \rho g(y_B - y_A) = 0$$

$$v_B = \sqrt{2gh}$$

- Note, for a realistic tank, the v_B depends on the height of the fluid in the tank. Thus the flow rate changes with time!
- More specifically we can say that the current depends upon the volume of fluid in the tank. $Current = \Delta Vol / \Delta t$

$$I = dVol / dt$$

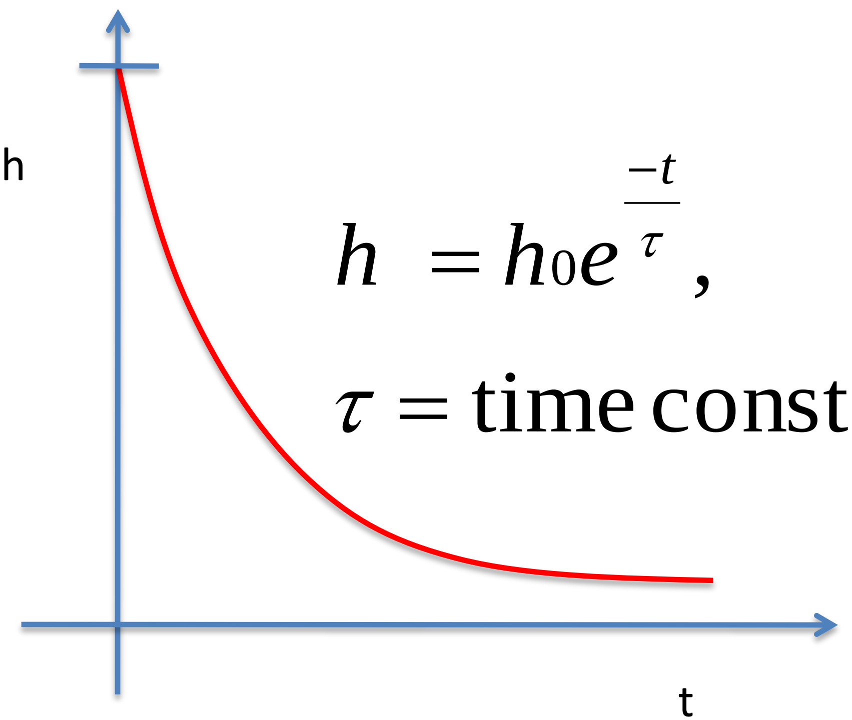
$$\frac{dV}{dt} = -aV$$

Note V is volume not potential and the negative sign is for decrease in volume with time.

- What function is it's own derivative?

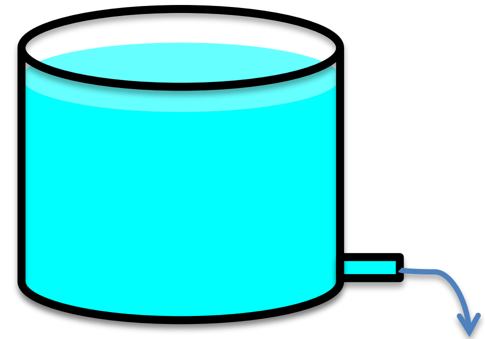
$$\boxed{\begin{aligned} y(t) &= y_0 e^{-at} \\ \frac{dy}{dt} &= -ay_0 e^{-at} = -ay(t) \end{aligned}}$$

Fluid Reservoir



$$h = h_0 e^{\frac{-t}{\tau}},$$

$$\tau = \text{time const}$$



What will increase the time constant?

Cross section area of the reservoir

Resistance of the outlet

Exponential Change in Circuits: Capacitors - Charging

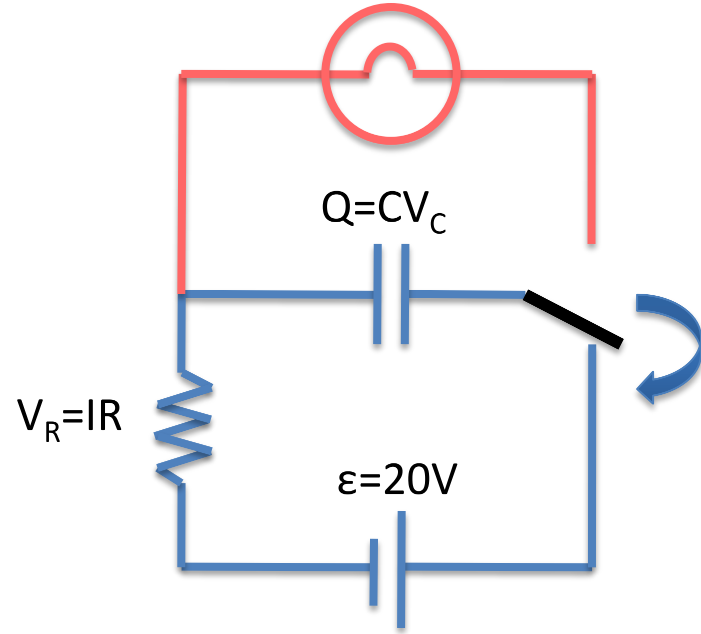
- we can write an equation, then, for the current:

$$\mathcal{E} - R\frac{dQ}{dt} = \frac{Q}{C}$$

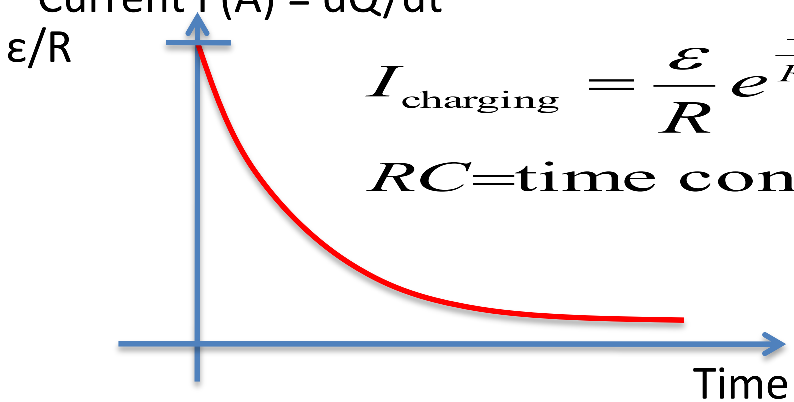
$$\frac{\mathcal{E}}{R} - \frac{Q}{RC} = \frac{dQ}{dt}$$

V = Q/C

I = dQ/dt

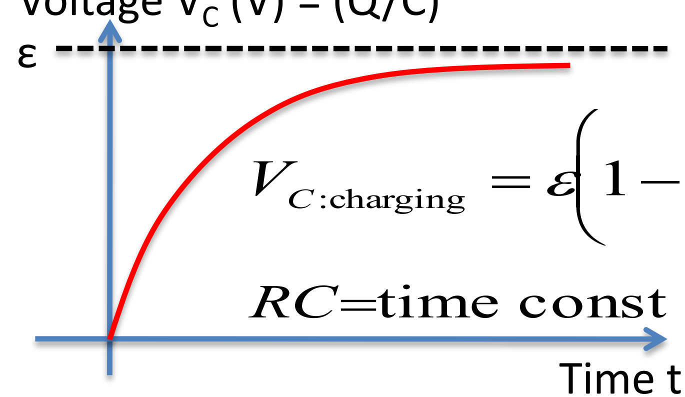


- solution: $\boxed{Q(t) = \mathcal{E}C(1 - e^{-t/RC})}$



$$I_{\text{charging}} = \frac{\mathcal{E}}{R} e^{\frac{-t}{RC}},$$

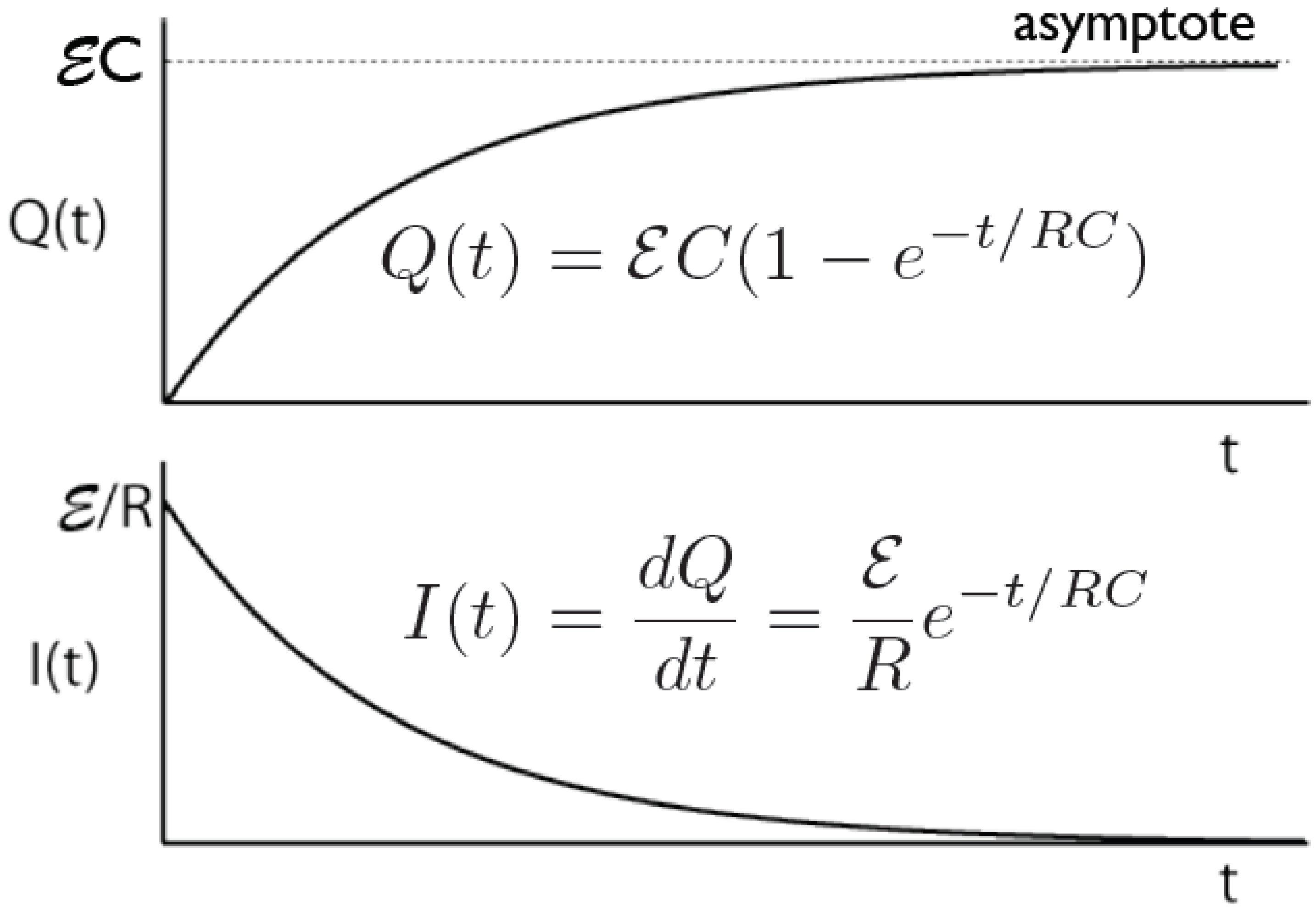
RC=time const



$$V_{C:\text{charging}} = \varepsilon\left(1 - e^{\frac{-t}{RC}}\right),$$

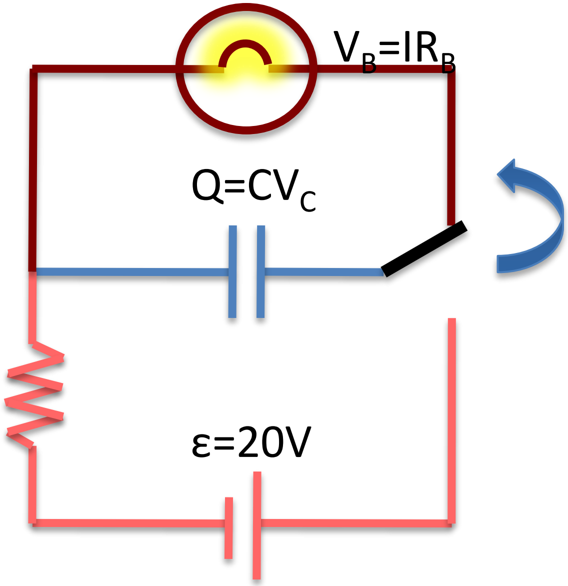
RC=time const

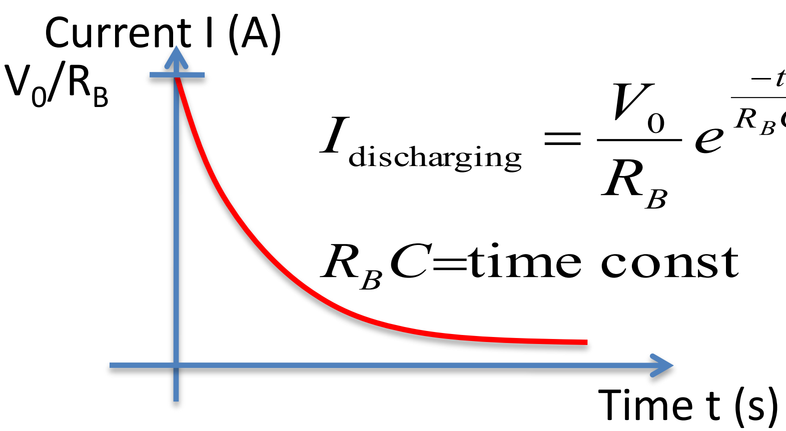
Charging a Capacitor



Exponential Change in Circuits: Capacitors - Discharging

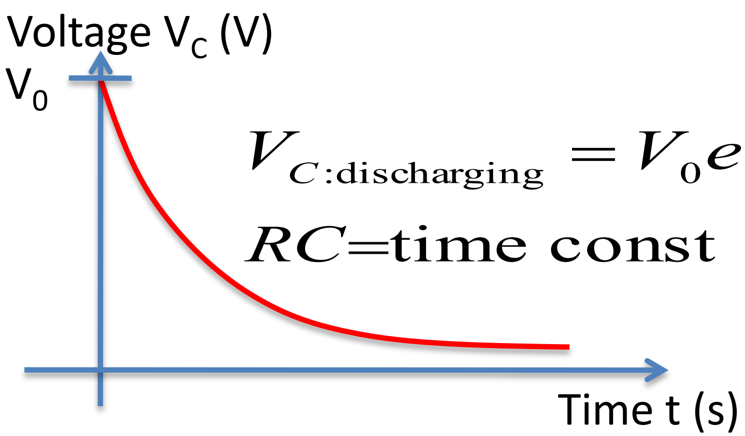
- Now we use the charge stored up in the battery to light a bulb
- Q: As the Capacitor discharges, what is the direction of the current? What happens after some time?
- Q: Initially what is the voltage in the capacitor V_0? What is the voltage in the bulb? What is the current in the circuit?
- Q: At the end, what is the voltage in the capacitor? What is the current in the circuit? What is the voltage of the resistor?





$$I_{\text{discharging}} = \frac{V_0}{R_B} e^{\frac{-t}{R_B C}},$$

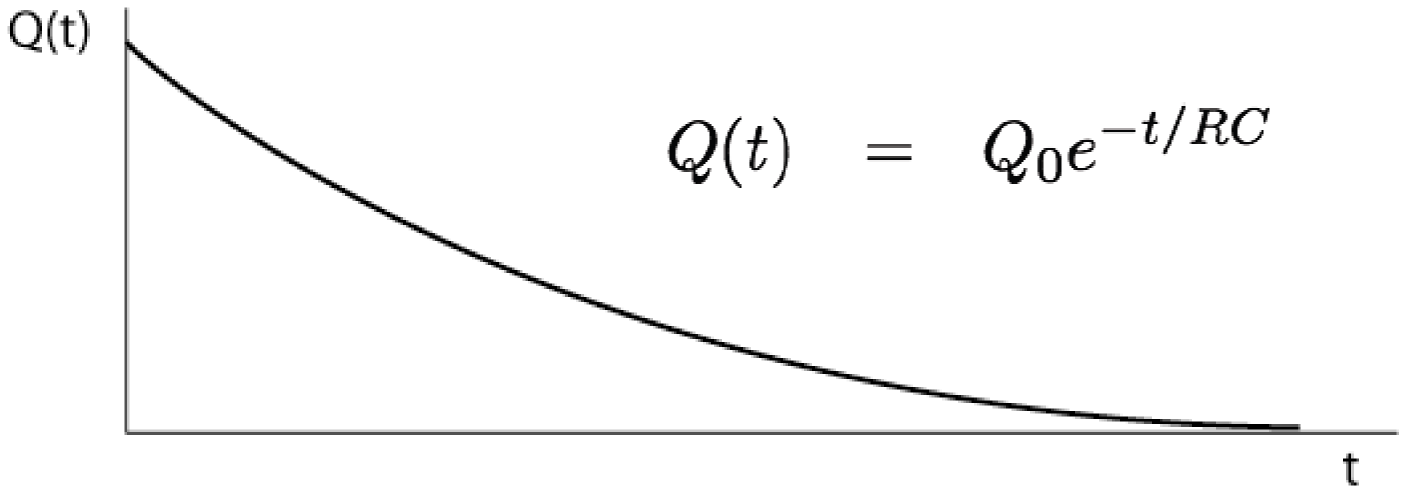
$$R_B C = \text{time const}$$



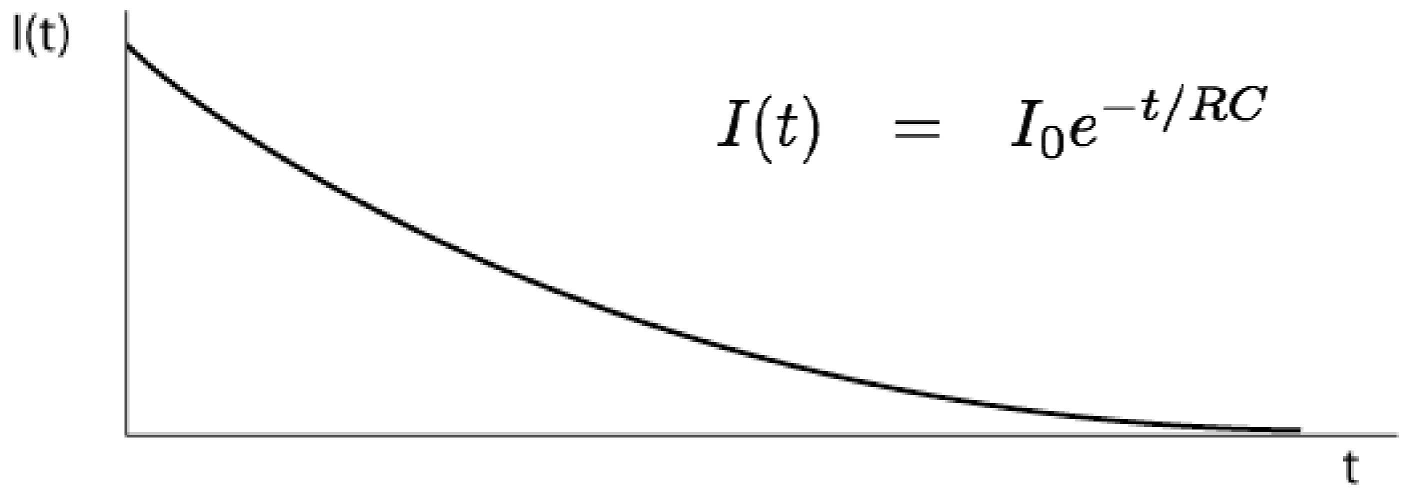
$$V_{C:\text{discharging}} = V_0 e^{\frac{-t}{RC}},$$

$$RC = \text{time const}$$

Discharging a Capacitor



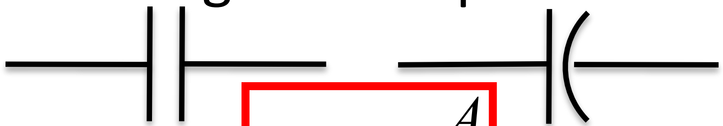
$$Q(t) = Q_0 e^{-t/RC}$$



$$I(t) = I_0 e^{-t/RC}$$

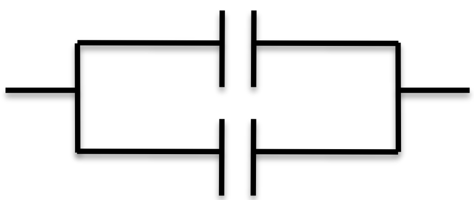
Capacitors in Series and Parallel

- Circuit Diagrams: Capacitors



$$C = \kappa \varepsilon_0 \frac{A}{d}$$

- Capacitors in parallel (~2xA)



$$C_{parallel} = C_1 + C_2 + \ldots$$

- Capacitors in series (~2xd)

$$\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$$

- Circuit Diagrams: Resistors

$$R = \rho \frac{L}{A}$$

- Resistors in Series (~2xL)

$$R_{series} = R_1 + R_2 + \ldots$$

- Resistors in parallel (~2xA)

$$\frac{1}{R_{parallel}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$$

Capacitors: Energy Stored in a capacitor

- Because resistors dissipate power, we wrote a an equation for the power dissipated in a Resistor:

$$\boxed{\begin{gathered} P = IV, \quad \text{using } V = IR: \\ P = I^2 R \quad \text{or} \quad P = \frac{V^2}{R} \end{gathered}}$$

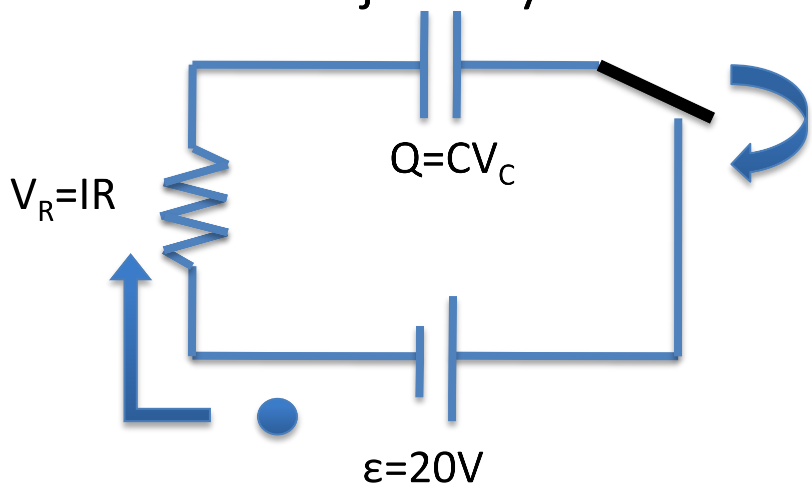
Note: Since *I* is same for resistors in series, identical resistors in series will have the same power loss. Since *V* is the same for resistors in parallel, identical resistors in parallel will have the same power loss

- Because capacitors are used to store charge and energy, we concentrate on the energy stored in a capacitor.
- We imagine the first and the last electrons to make the journey to the capacitor. What are their ΔPE's?

$\Delta PE_{first} = q\Delta V$, $\Delta V = 20$ $\Delta PE_{last} = q\Delta V$, $\Delta V = 0$

Thus on average for the whole charge:

$$\boxed{\begin{gathered} PE = \frac{1}{2} QV, \text{ using } Q = CV \\ PE = \frac{1}{2} CV^2 \end{gathered}}$$



Announcements

DL Sections

Winter 2010 7B-1 (A/B) D/L Assignments & Job Responsibilities

1	WF	10:30-12:50	2317 EPS	Marcus Afshar
2	MW	2:10-4:30	2317 EPS	Aaron Hernley
3	MW	4:40-7:00	2317 EPS	Rylan Conway
4	MW	7:10-9:30	2317 EPS	Rylan Conway
5	MR	8:00-10:20	2317 EPS	Robert Lynch
6	TR	10:30-12:50	2317 EPS	Aaron Hernley
7	R	2:10-4:30	2317 EPS	Justin Dhooghe
7	M	10:30-12:50	2317 EPS	Justin Dhooghe
8	TR	4:40-7:00	2317 EPS	Britney Rutherford
9	TR	7:10-9:30	2317 EPS	Britney Rutherford
10	TF	8:00-10:20	2317 EPS	Emily Ricks
11	TF	2:10-4:30	2317 EPS	Justin Dhooghe