Review of Circuits and the Linear Transport Model

Basic Circuit Theory

Potential (Voltage): V Volts (V)
Current: I Amps (A)
Resistance: R Ohms (Ω)
Capacitance: C Farads (F)
Power: P Watts (W)

Ohm's law: $V = IR$
Power: $P = IV = I^2R = V^2/R$

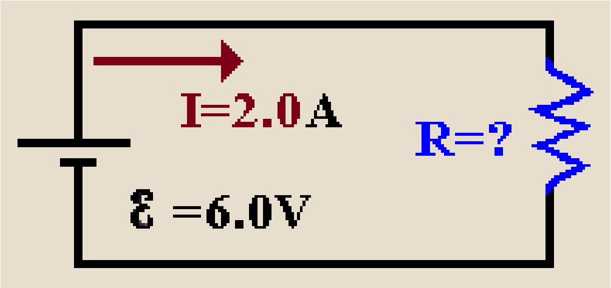
Example

Question

A battery of 6.0V is connected to a purely resistive lamp and a current of 2.0 A flows. All the wires are resistance-free. What is the resistance of the lamp?

Hints

Where in the circuit does the gain in potential energy occur?
Where in the circuit does the loss of potential energy occur?
What is Ohm's Law?

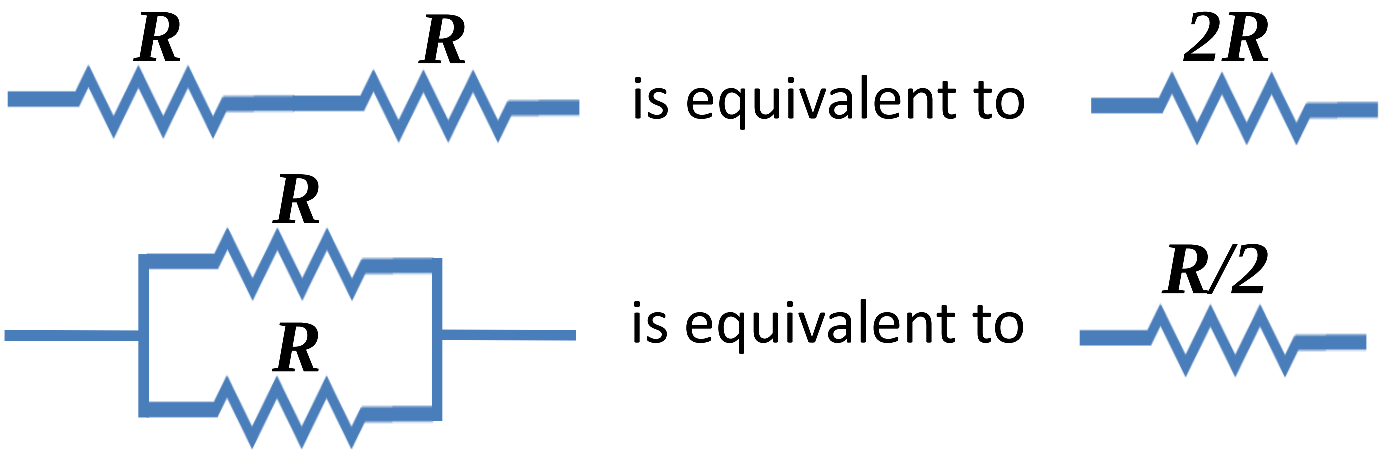


a) 12 W
b) 12 Ω
c) 3.0 Ω
d) (1/3) W
e) 36 Ω

Solution: $V = IR \rightarrow R = V/I$ = 6.0 V/2.0 A = 3.0 Ω

Series and Parallel

- Complicated circuits can be simplified.

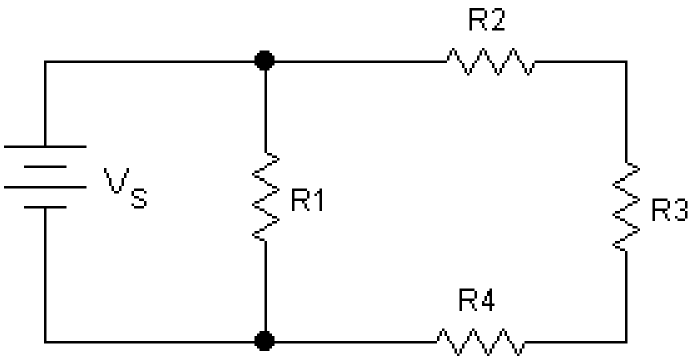


- Resistors in series: $R_{equiv} = R_1 + R_2 + R_3 + \cdots$
- Resistors in parallel: $R_{equiv} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \right)^{-1}$

Example

Consider the circuit show below. Vs = +12V, R1 = 6Ω, R2 = 1Ω, R3 = 2Ω, R4 = 3Ω. What is the current flowing through the battery?

a) 1A
b) 2A
c) 3A
d) 4A
e) 6A

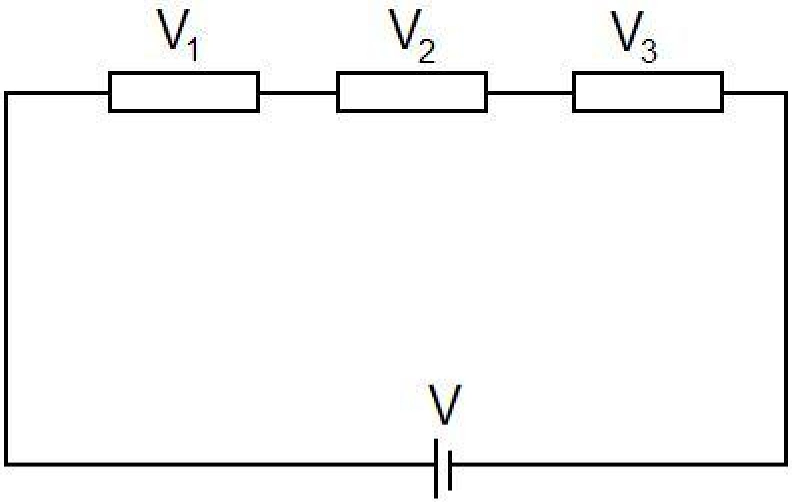


Solution: R2 + R3 + R4 = 6Ω. Two parallel 6Ω resistors have an equivalent resistance of 3Ω. The current through the battery we get from Ohm's law, $I = V/R$ = 12V/3Ω = 4A

The Loop Rule

For any closed loop that one can draw on a circuit, no matter how complex, the sum of the voltage drops must be equal to the sum of the voltage rises (forward biased batteries).

$$\Sigma\mathcal{E} = \Sigma\, I\, R_i$$



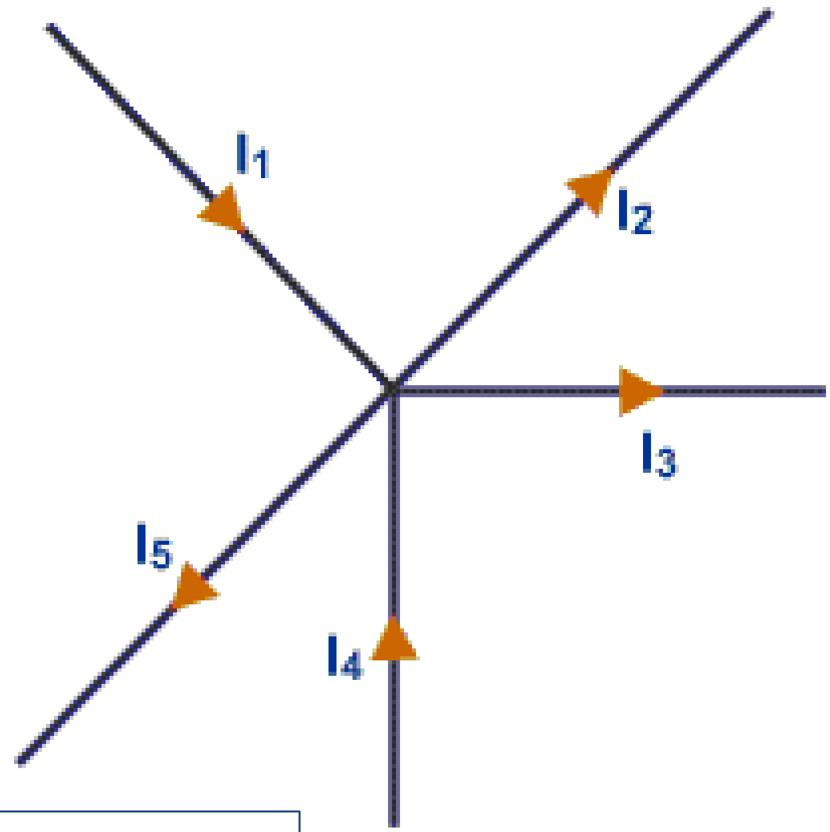
$$V = V_1 + V_2 + V_3$$

Conservation of energy

The Junction Rule

At any junction (or node), the sum of the incoming currents must be equal to the sum of the outgoing currents.

$$\Sigma I_{in} = \Sigma I_{out}$$



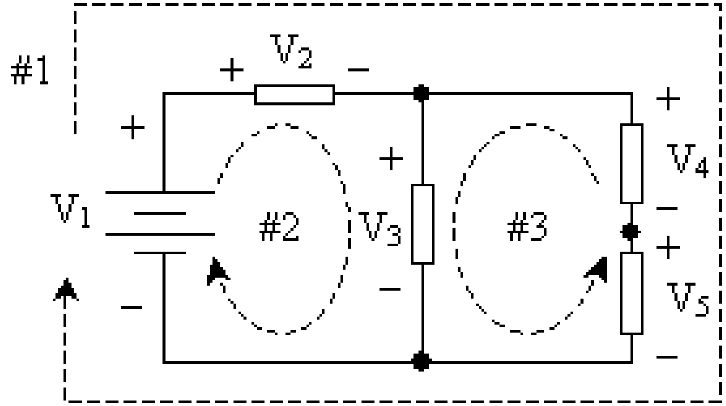
Conservation of charge

Example

Question

Find V5 given that V1=12V, R2 = 6Ω, R3 = 12Ω, R4 = 10Ω, R5 = 2Ω.

a) 1 V
b) 2 V
c) 3 V
d) (1/2) V
e) 6 V



Mesh #1: $-V_1 + V_2 + V_4 + V_5 = 0$
Mesh #2: $-V_1 + V_2 + V_3 = 0$
Mesh #3: $V_3 - V_5 - V_4 = 0$

Solution: V5 = 1V

Linear Transport Model

- Current density: The amount of material passing through an aperture of unit area per unit time ➔ $j = I/A$

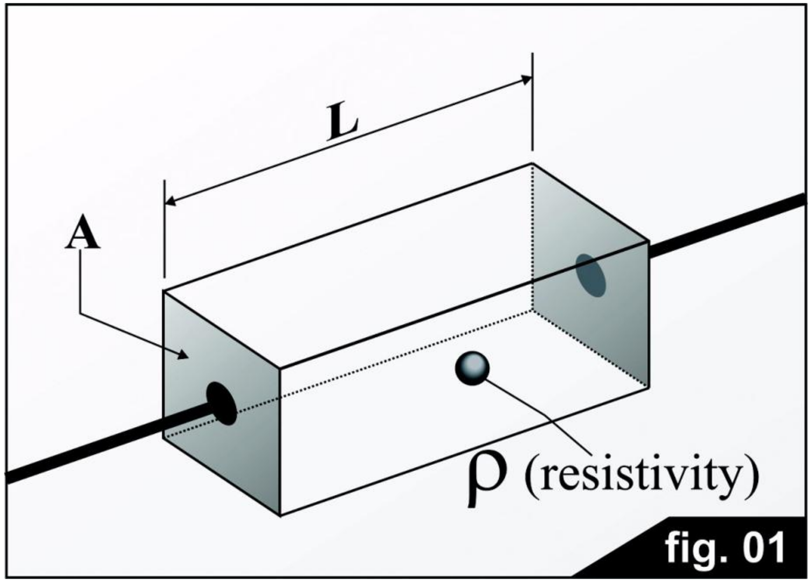
- Linear transport equation: Describes the flow of something (j) as a result of change in a *potenial* (ϕ) with position(i.e. the gradient of the potential)➔ $j = -k\mathrm{d}\phi/\mathrm{d}x$ The constant of proportionality, k, is the *conductivity*.

Ohm’s Law - Resistivity

The resistance of a piece of material is given by its intrinsic resistivity (ρ), times the Length (L), divided by the area (A).

$R = \rho\,(L/A)$

The resistivity (ρ) is the inverse of the conductivity (k or σ).



Linear Transport Model

Starting with Ohm's Law ($\Delta V = -IR$), which we had derived from the Energy Density Model, we can rewrite this to solve for current ($I = -(1/R)\Delta V$

Using our definition of R from the previous slide [$R = \rho(L/A)$ *or* $(1/R) = k\,(A/L)$], We get

$$I = -k\,(A/L)\,\Delta V$$

If we let $\Delta V \rightarrow \Delta\phi$, then we get

$$I = -k\,(A/L)\,\Delta\phi$$

Divide through by area A,

$$j = -k\,(1/L)\,\Delta\phi$$

Let L become an infinitesimal

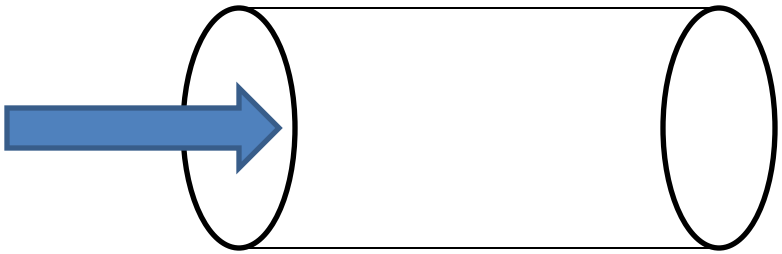
$$j = -k\,d\phi/dx \qquad \leftarrow \textit{Transport Equation}$$

Application of Linear Transport Model

- Fluid Flow
  - ϕ=*Head* and j=*mass current density*
- Electric Current (Ohm's Law)
  - ϕ=*Voltage* and j=*charge current density*
- Heat Conduction
  - ϕ=*Temperature* and j=*heat current density*
- Diffusion (Fick's Law)
  - ϕ=*Concentration* and j=*mass current density*

Fluid Flow and Transport

The resistance of pipes to fluid transport has a very strong dependence on the radius of the pipe.



$$R = (\text{Fluid properties}) \times (\text{geometric properties})$$
$$= (8\eta/r^2) \times (L/A)$$
$$= 8\eta L/(\pi r^4)$$

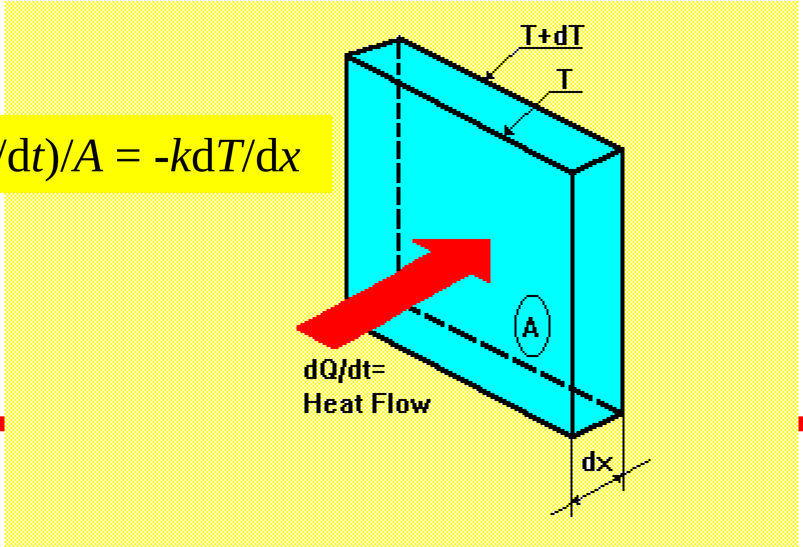
The Reynolds number is a dimensionless quantity that indicates when flow will transition from laminar to turbulent. This occurs for Reynolds numbers between 2000 and 3000.

Heat Conduction – Fourier's Law

Thermal conductivity, *k*, is the property of a material that indicates its ability to conduct heat. It appears primarily in Fourier's Law for heat conduction. Thermal conductivity is measured in watt per kelvin per meter ($W{\cdot}K^{-1}{\cdot}m^{-1}$). Multiplied by a temperature difference (in Kelvin, K) and an area (in square meters, m^2), and divided by a thickness (in meters, m) the thermal conductivity predicts the energy loss (in watts, W) through a piece of material.

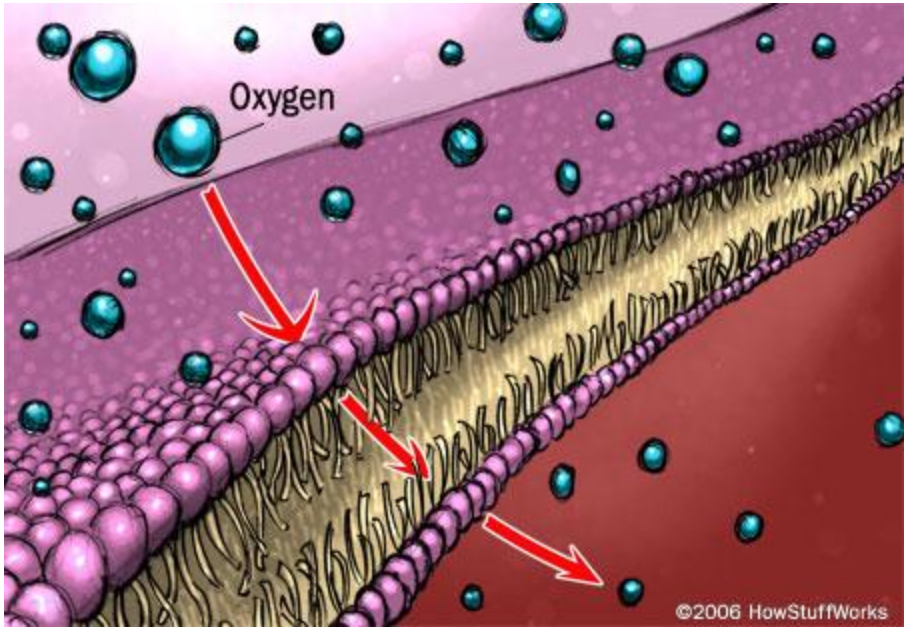
The reciprocal of thermal conductivity is *thermal resistivity*, usually measured in kelvin-meters per watt ($K{\cdot}m{\cdot}W^{-1}$). When dealing with a known quantity of material, its *thermal conductance* and the reciprocal property, *thermal resistance*, can be described.

$$j_Q = (dQ/dt)/A = -k\,dT/dx$$



Diffusion – Fick's Laws

Diffusion can be defined as the random walk of an ensemble of particles from regions of high concentration to regions of lower concentration. Diffusion is describes by Fick's Laws.



Fick's First Law relates the transfer of material to the gradient:

$$j = -D\, dC(x,t)/dx$$

Where j is the particle flux and C in the concentration, and D is the diffusion constant

From the Conservation of mass, we know that $dC/dt = -dj/dx$, if we combine this constraint with Fick's First Law, we get the Second Law:

$$dC/dt = D\, d^2C/dx^2$$

Announcements

DL Sections

Winter 2010 7B-1 (A/B) D/L Assignments & Job Responsibilities

1	WF	10:30-12:50	2317 EPS	Marcus Afshar
2	MW	2:10-4:30	2317 EPS	Aaron Hernley
3	MW	4:40-7:00	2317 EPS	Rylan Conway
4	MW	7:10-9:30	2317 EPS	Rylan Conway
5	MR	8:00-10:20	2317 EPS	Robert Lynch
6	TR	10:30-12:50	2317 EPS	Aaron Hernley
7	R	2:10-4:30	2317 EPS	Justin Dhooghe
7	M	10:30-12:50	2317 EPS	Justin Dhooghe
8	TR	4:40-7:00	2317 EPS	Britney Rutherford
9	TR	7:10-9:30	2317 EPS	Britney Rutherford
10	TF	8:00-10:20	2317 EPS	Emily Ricks
11	TF	2:10-4:30	2317 EPS	Justin Dhooghe