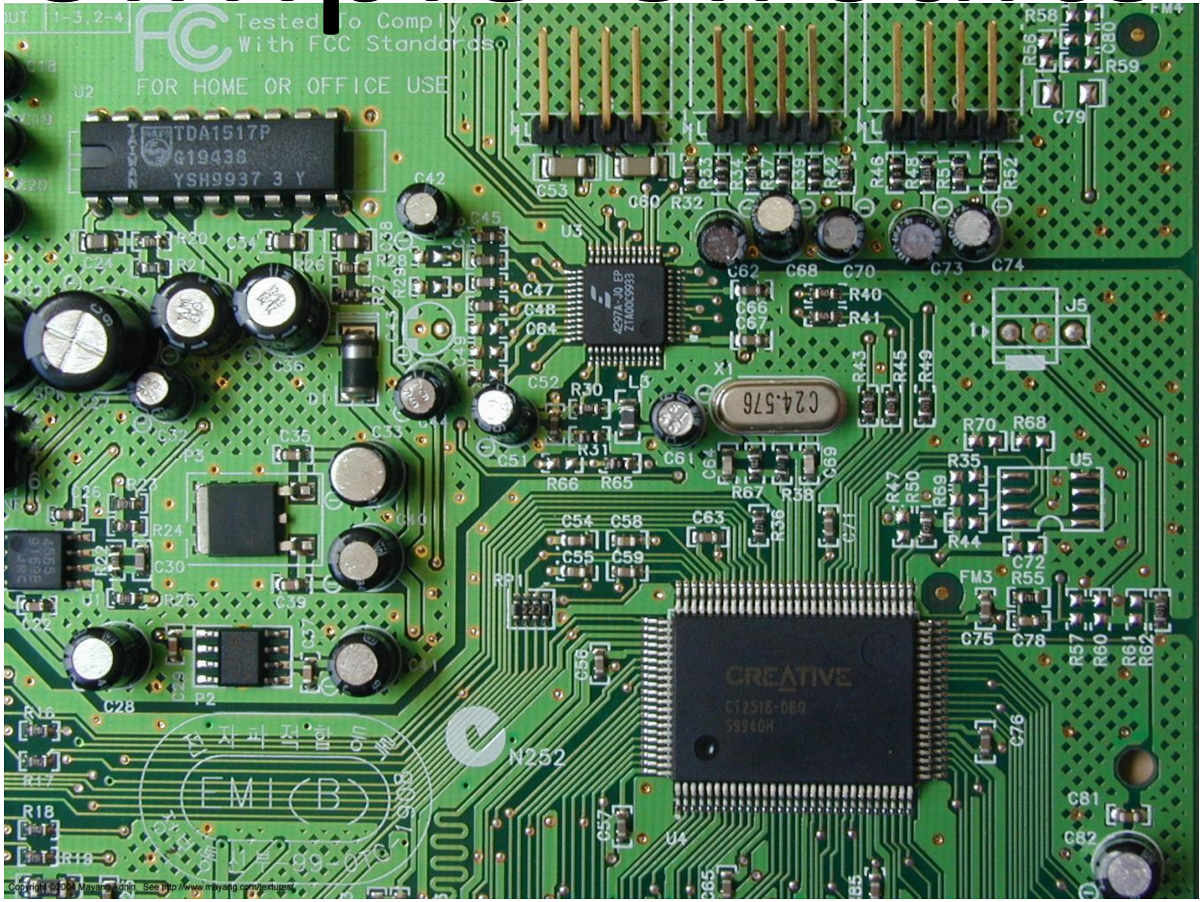
Simple Circuits



Conservation of Energy Density

In the First lecture, we started with energy conservation. We divided by volume (making conservation *intensive* rather than *extensive*) to get an energy density conservation equation.

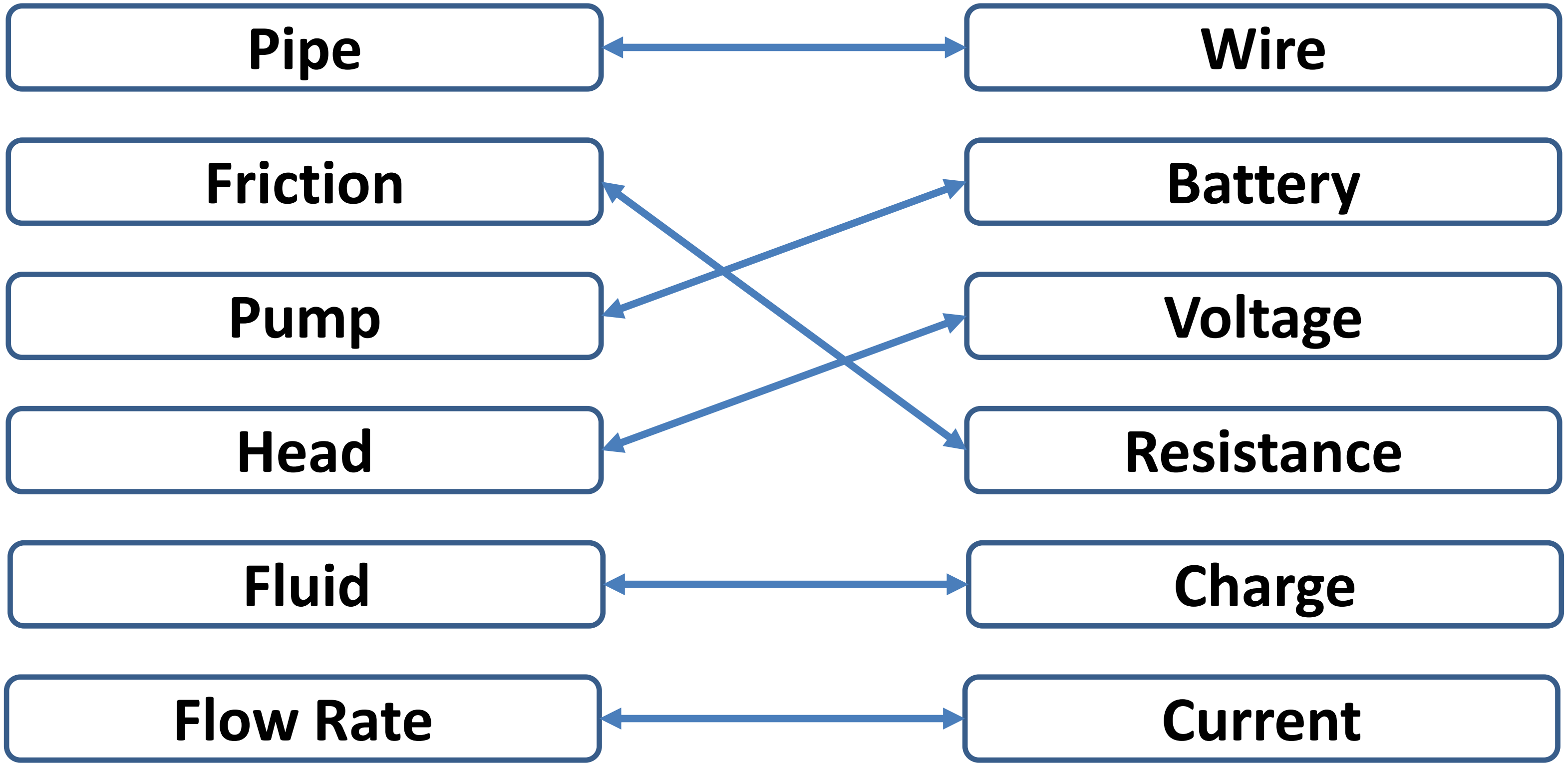
For the non-isolated system, we added in pumps and resistance from the pipes to get a more general equation which describes the general features of fluid systems:

$$\frac{\sum \Delta E_{External}}{V} = \frac{E_{Pump}}{V} - \mathrm{IR} \qquad \Rightarrow \qquad \Delta P + \rho g \Delta y + \frac{1}{2}\rho \Delta(v^2) = \frac{E_{Pump}}{V} - \mathrm{IR}$$

In this lecture, instead of dividing by volume, we will divide the energy by the electric charge. This again gives us an intensive energy density equation which now describes the behavior of electrical circuits.

Comparison of Fluids to Electricity

- Fluid systems and electrical circuits are analogous:



Pipe

Wire

Friction

Battery

Pump

Voltage

Head

Resistance

Fluid

Charge

Flow Rate

Current

Units of Electrical Circuits

Quantity	Unit	Definition	Dimensions
Electric Charge (*q*)	*Coulomb* (C)	1 e = 1.6 x 10^{-19} C	1 C
Energy (*E*)	*Joule* (J)	1 Nm	1 kg m^2 / s^2
Electric Potenial (*V*)	*Volt* (V)	1 V = 1 J/C	1 kg m^2 / s^2 C
Electric Resistance (*R*)	*Ohm* (Ω)	1 Ω = 1 V/A	1 kg m^2 / sC^2
Electric Current (*I*)	*Ampere* (A)	1A = 1 C/s	1 C/s
Power (*P*)	*Watt* (W)	1 W = 1 J/s	1 kg m^2 / s^3
Energy (*E*)	*kiloWatt-Hour* (kW-hr)	3.6 x 10^6 J	

What is a Circuit?

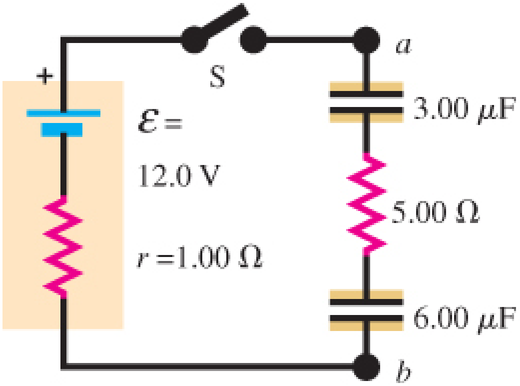
An electrical device that provides a path for electrical current to flow

Conservation of charge requires that the circuit must be **complete**, i.e. it must be continuous.

A circuit that does not have a complete conducting path is said to be an ***open circuit***.
In the diagram at the right, the switch *opens* the circuit.

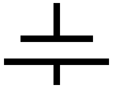
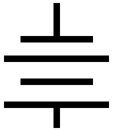
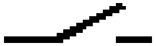
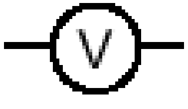
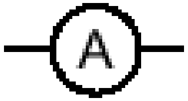
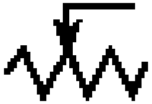
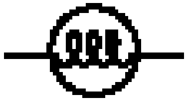
A component can be in a condition where the current has alternative and lower resistance conducting path. This is known as a ***short circuit***.

Often a circuit will connect to ***ground***. Ground is the reference voltage defined as the electric potential of the surface of the earth.



DEMO: Circuit Board

Circuit Components and Symbols

	cell	A cell is maintains a defined voltage across its two terminals
	battery	A battery is a collection of two or more cells
	switch	A switch can be set to open on close the circuit
	voltmeter	A voltmeter measures the potential between two points
	ammeter	An ammeter measures the current flowing through it
	resistor	A resistor impedes the flow of current
	variable resistor	A variable resistor has an adjustable resistance
	lamp	A lamp is a resistor that produces light

Electric Energy per Charge -- Voltage

- Voltage: Electrical potential energy per unit charge.
- In fluid systems:

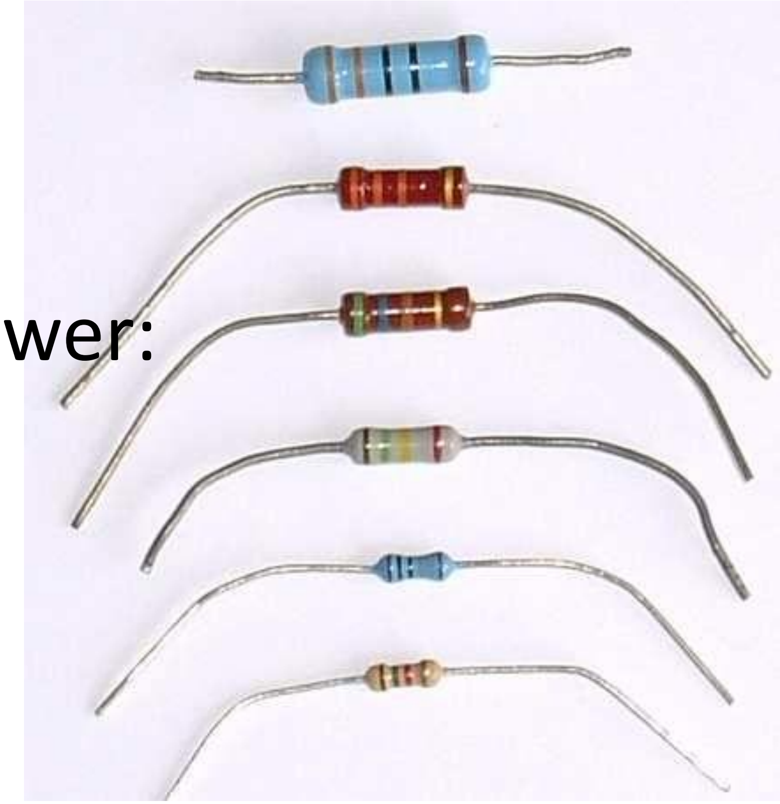
$$\Delta Head = \frac{E_{Pump}}{V} - IR$$

- In electrical circuits:

$$\boxed{\Delta Voltage = \varepsilon - IR}$$

Definition of a Resistor

- Resistor: Any circuit component that
  1. Opposes current
  2. Produces a voltage drop
  3. $\Delta V \propto I \Longrightarrow V = IR$
- Resistors use/dissipate power:



$$\boxed{P = I\Delta V}$$

DEMO: Resistor Model

A conductor with zero resistance is a wire

Batteries and Power Supplies

Batteries, generators, or power supplies are able to maintain a defined electric voltage (energy per charge) across its two terminals. To do this, these devices must convert energy from some other form.

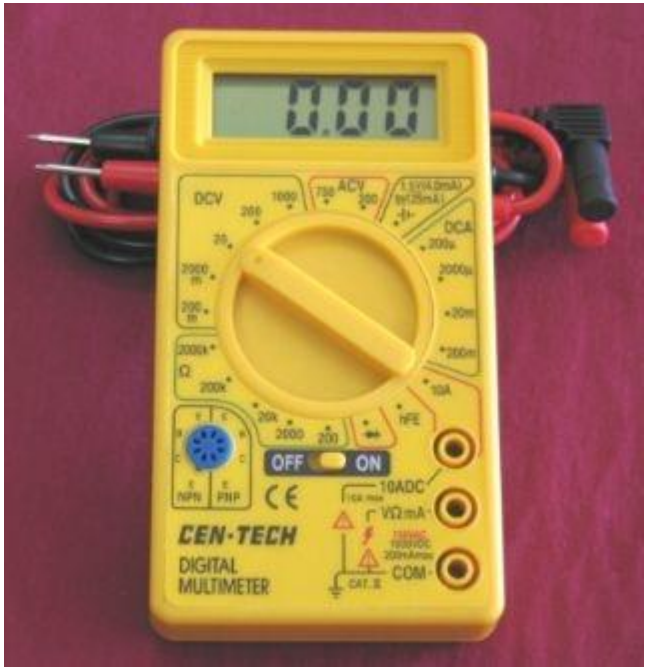
Batteries use chemical energy (E_{bond}) and convert this to electric energy.

Generators convert mechanical energy.

Power supplies convert the AC line power into another form of electric Energy.

Batteries have a characteristic electro-motive force ($\mathcal{E}$), measured in volts.

Meters



A ***voltmeter*** is an instrument used for measuring the electrical potential difference between two points in an electric circuit.

An ***ammeter*** measures the electric current flowing through its leads.

An ***ohmmeter*** uses a battery to run a small current through an unknown resistor. By measuring the battery voltage and the current and using Ohm's Law, the device determines the resistance.

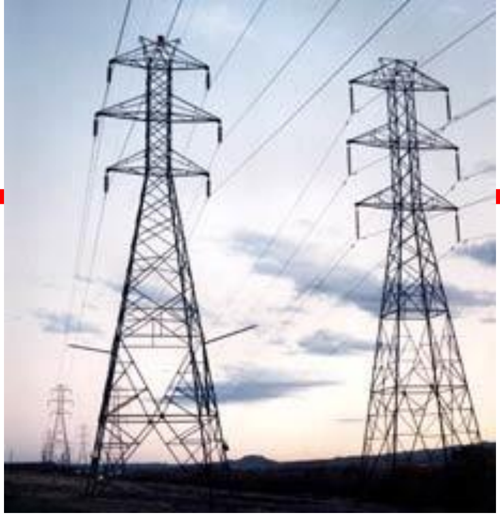
Ohm’s Law ($V = IR$)

In electrical circuits, **Ohm's law** states that the current through a component between two points is directly proportional to the potential difference or voltage across the two points, and inversely proportional to the resistance between them.

$$I = \frac{V}{R}$$

DEMO: Single Resistor and Meters

Electrical Power



In resistive circuits, electrical power is calculated using Joule's law:

$$P = V \cdot I$$

where *P* is the electric power, *V* the potential difference, and *I* the electric current.

In the case of resistive (Ohmic, or linear) loads, Joule's law can be combined with Ohm's law (*I* = *V*/*R*) to produce alternative expressions for the dissipated power:

$$P = I^2 R \qquad P = \frac{V^2}{R},$$

DEMO: Single Light Bulb

Capacitor

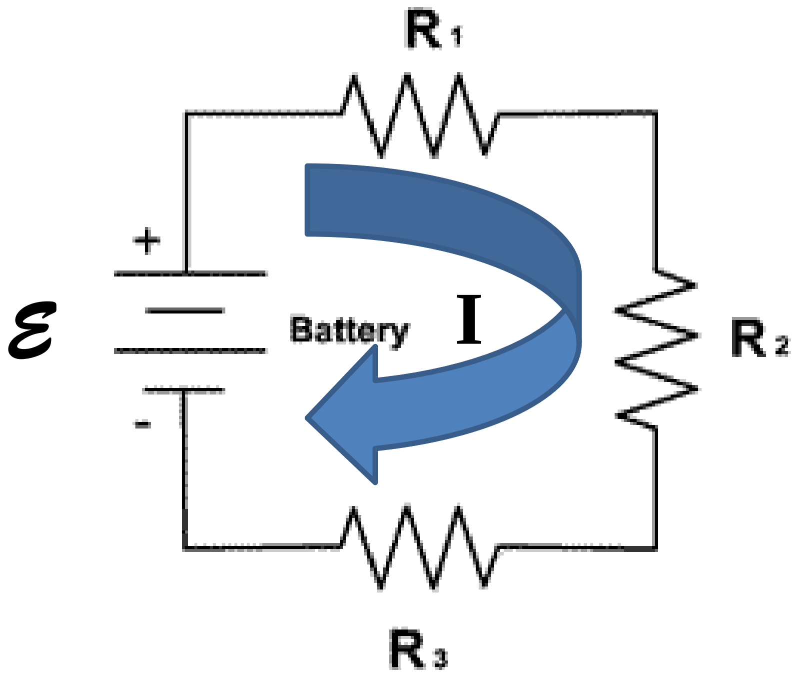
- Capacitor: Any circuit component that
  1. Stores charge
  2. Produces a voltage drop
  3. $\Delta V \propto Q$
- Capacitors can behave like rechargeable batteries

Series Circuits

A series circuit in one in which there is only a single conducting path. There are no branches and all components come one after another.

The current will flow around the circuit form the positive to the negative terminal of the battery.

All the components will experience the same current, however each will have a voltage drop determined by the size of its resistor.

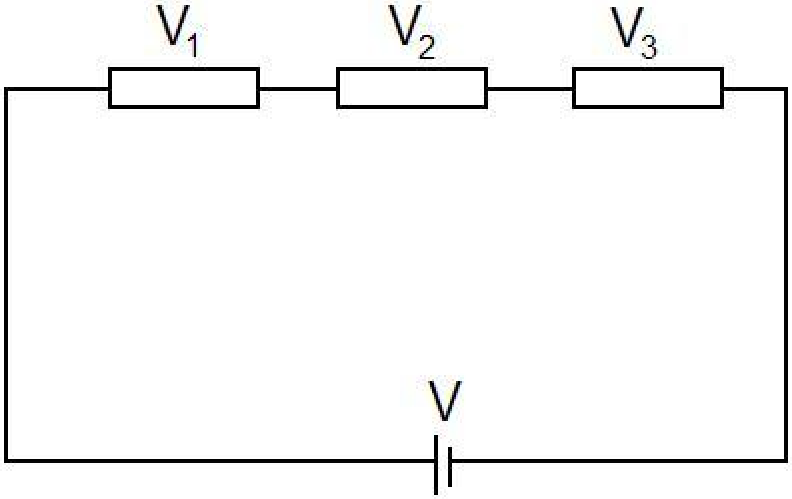


$$\mathcal{E} = IR_1 + IR_2 + IR_3$$

The Loop Rule

For any closed loop that one can draw on a circuit, no matter how complex, the sum of the voltage drops must be equal to the sum of the voltage rises (forward biased batteries).

$$\Sigma\mathcal{E} = \Sigma\, I\, R_i$$

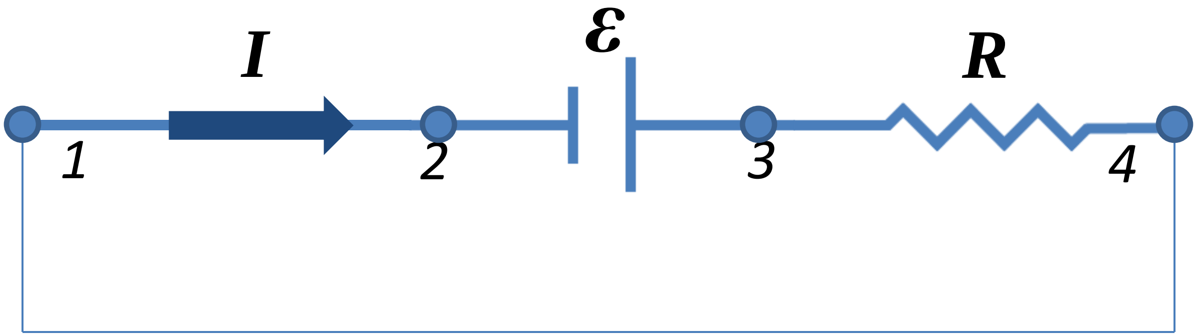


$$V = V_1 + V_2 + V_3$$

Conservation of energy

Simple Flow Exercise

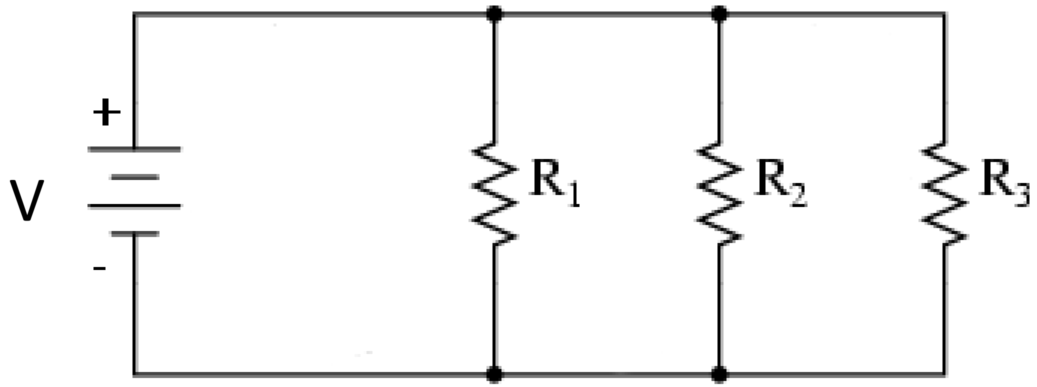
- Suppose current I flows through point 1 and consider a battery with emf ε and resistor with resistance R.
  - Calculate the current through point 2, 3 and 4.
  - Calculate voltage change between points 1&2, 2&3 and 3&4.
  - Calculate power used/dissipated by resistor.



Parallel Circuits

A parallel circuit is one in which the leads of the components are joined by a common wire which is then connected across the battery or other voltage source.

The current will be split between the parallel components, however they will all have the same voltage drop.

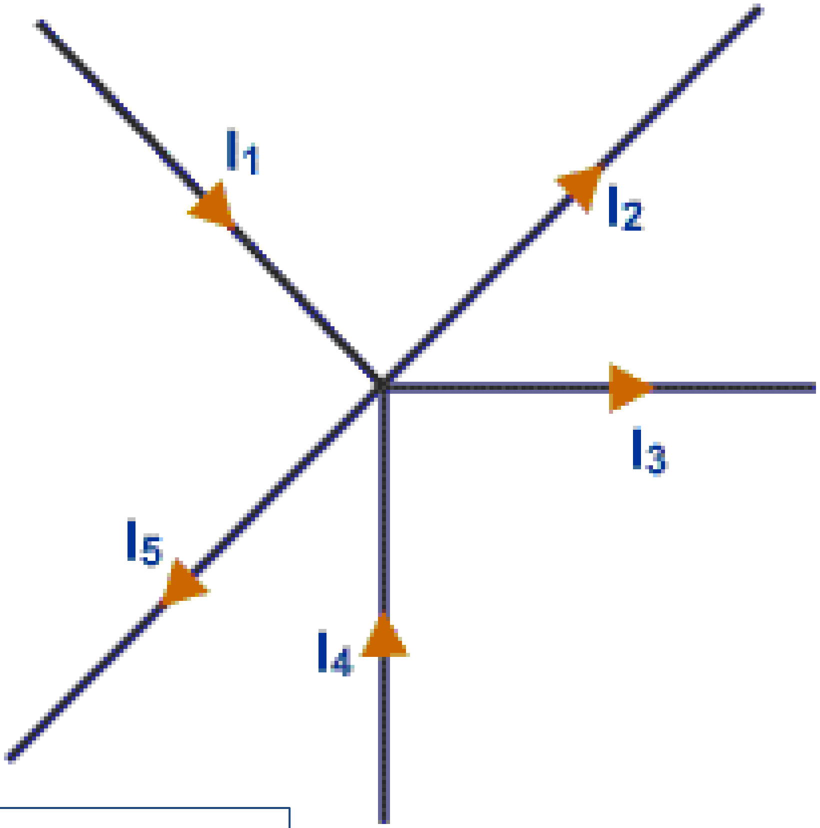


$$V = I_1R_1 = I_2R_2 = I_3R_3$$

The Junction Rule

At any junction (or node), the sum of the incoming currents must be equal to the sum of the outgoing currents.

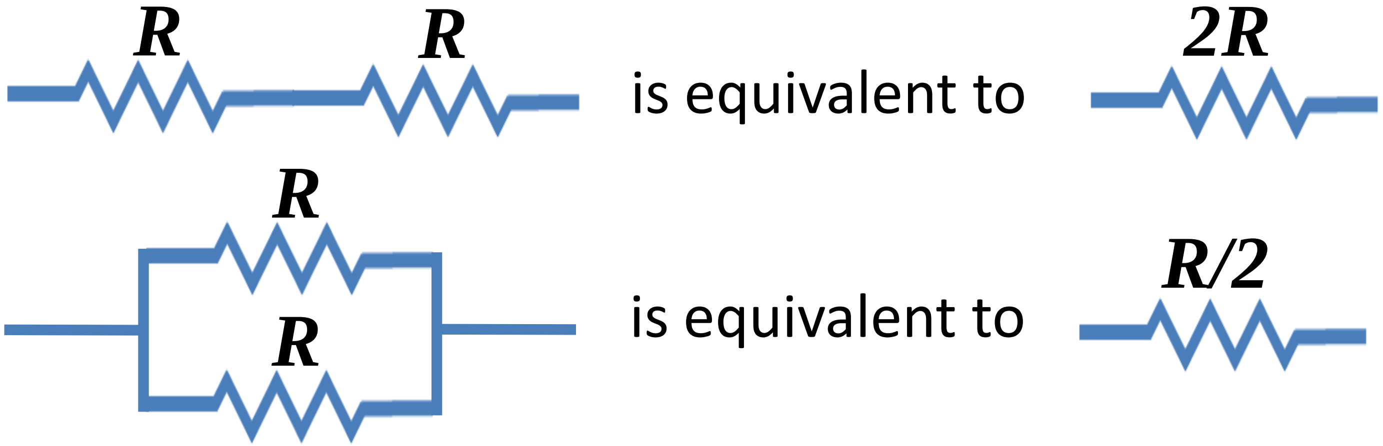
$$\Sigma I_{in} = \Sigma I_{out}$$



Conservation of charge

Series and Parallel

- Complicated circuits can be simplified.



is equivalent to

is equivalent to

- Resistors in series: $R_{equiv} = R_1 + R_2 + R_3 + \cdots$
- Resistors in parallel: $R_{equiv} = \left(\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \cdots \right)^{-1}$

DEMO: Series Light Bulbs

Household Electrical Power

Household power is delivered as AC power.

In the North America, standard electrical outlets have 110 V_{rms} at 60 Hz.

Much of the rest of the world uses 220 V at 50 Hz.

Transmission lines carry power from plants at several thousand volts. This is stepped down at a series using a series of transformers. The final stage is in your breaker box at the power meter. There is a center tapped transformer. Taking either lead to ground gets the 110 V. For more energy demanding appliances, one can wire both leads to get 220 V.

DEMO: Utility Box

Announcements

Remember the MLK day Holiday.
Section 01 (Marcus) does not meet on Friday. No sections meet on Monday or Tuesday.

Death by Electricity

It’s the current that kills not the voltage