
Simple Harmonic
Motion

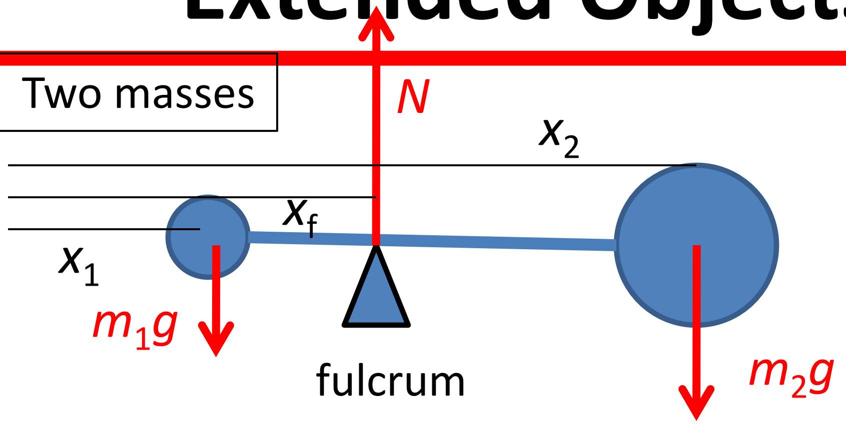
Announcements

- Final exam will be next Wednesday 3:30-5:30
  - A Formula sheet will be provided
  - Closed-notes & closed-books
  - One single-sided 8 ½ X 11 Formula Sheet allowed
  - Bring a calculator
  - Bring your UCD ID (*picture and student number*)
- Practice problems – Online
- Formula sheet – Online
- Review Sessions – Online

Final Exam Room	Last Name Begins With:
198 Young	N - Z
1100 Social Sciences	C - M
55 Roessler	A - B

Extended Objects - Center of Gravity

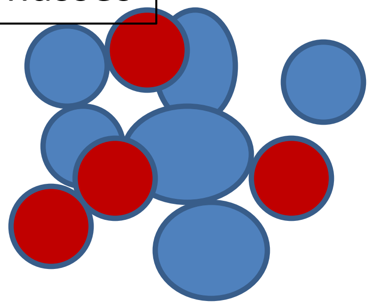
Two masses



Balance forces and Torques

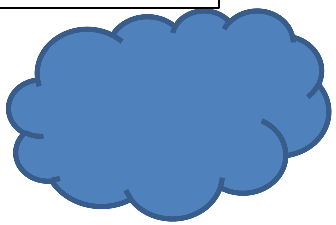
$$x_{cm} = x_f = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

Discrete masses



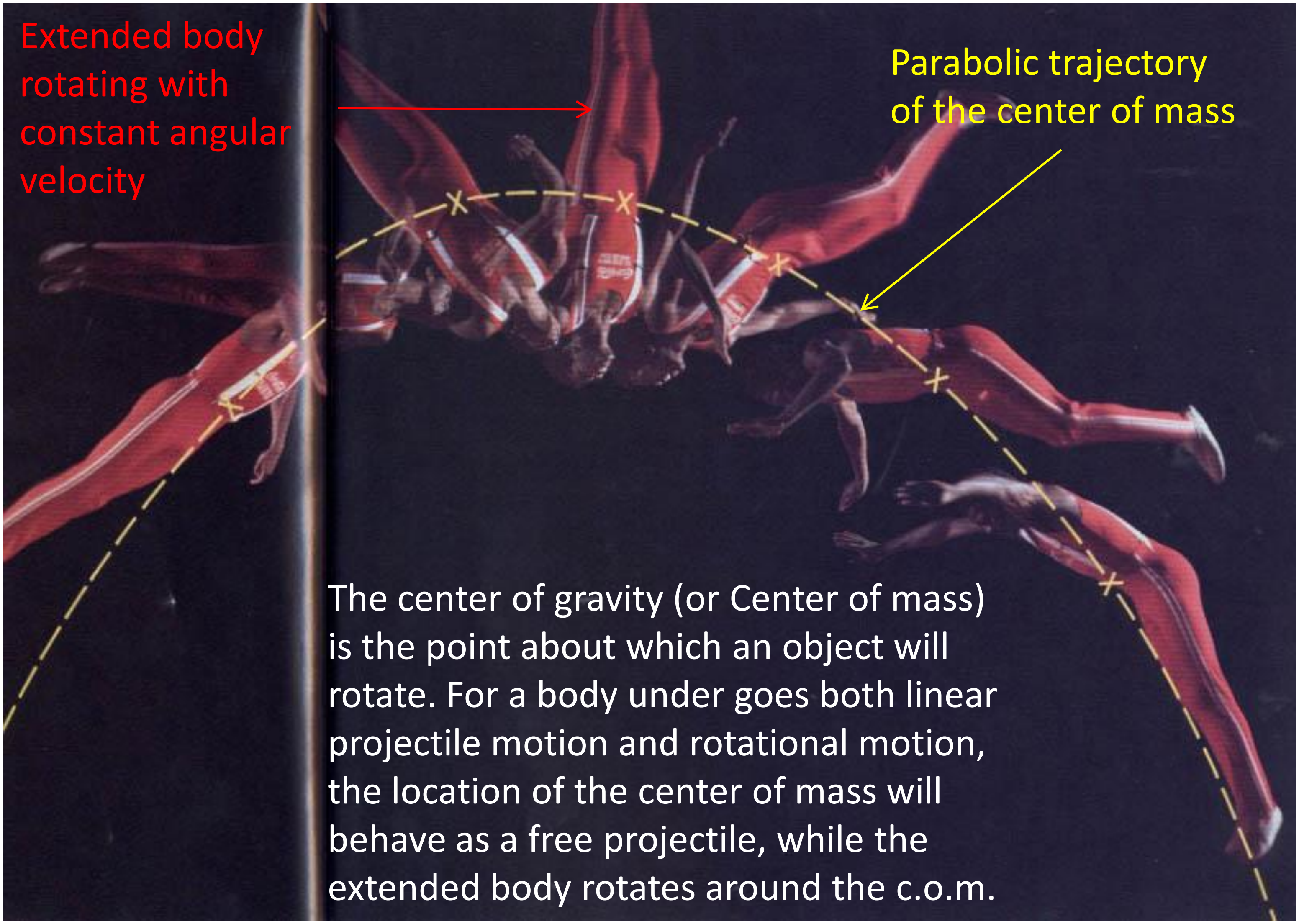
$$x_{cm} = \frac{\Sigma m_i \vec{r}_i}{\Sigma m_i}$$

Distribution of mass

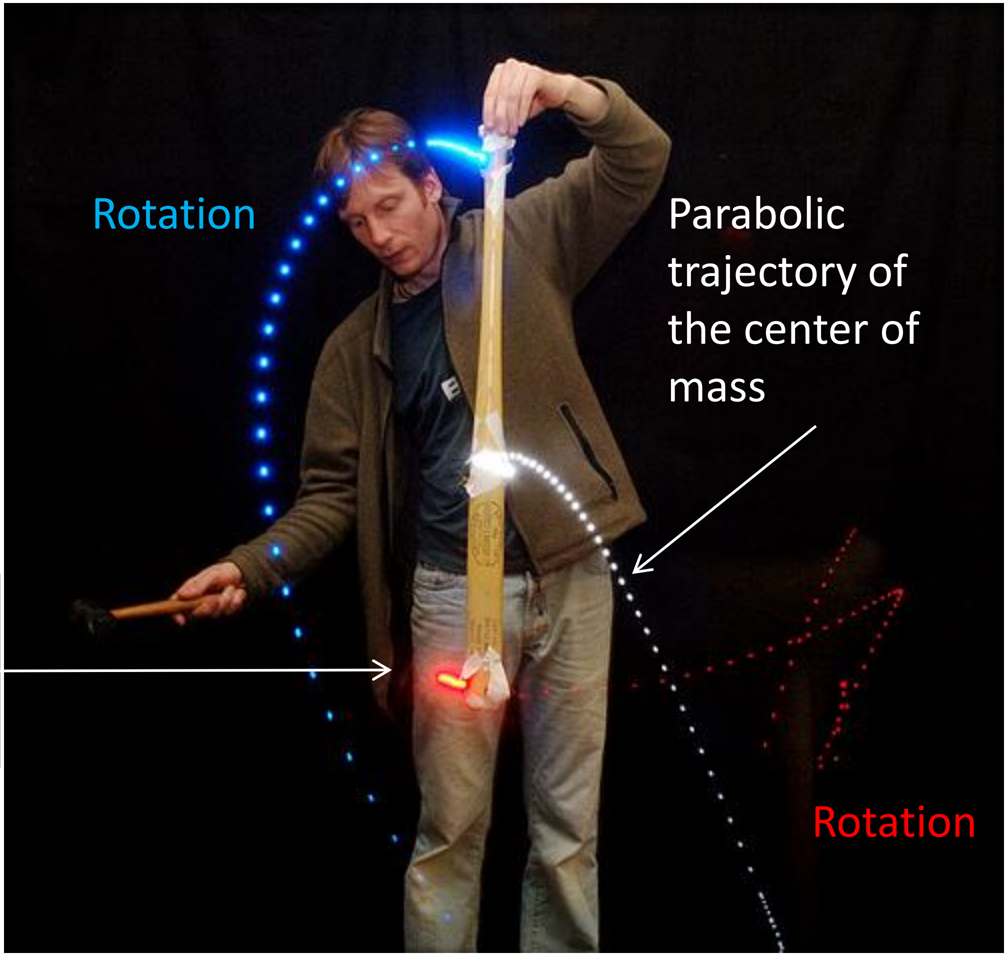


$$x_{cm} = \frac{1}{M} \int \rho(\vec{r}) \vec{r} dV$$

Center of Gravity



Center of Gravity



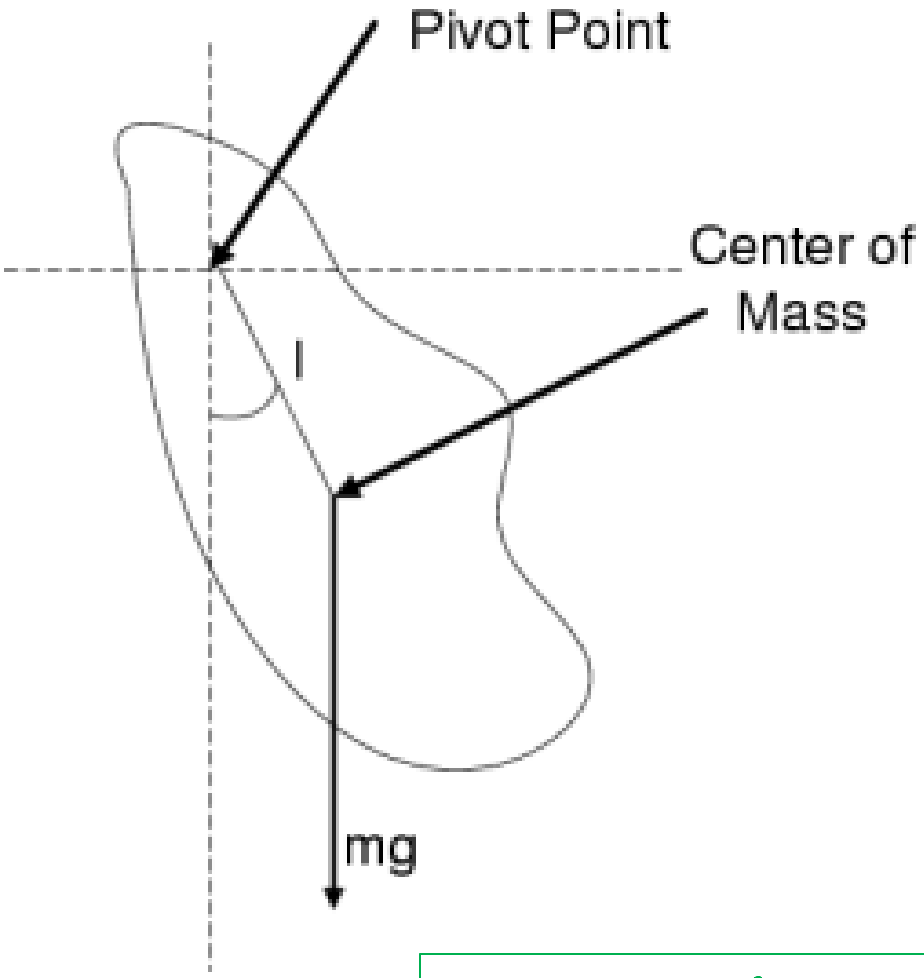
Linear Impulse
and
Angular Impulse

Rotating Off Axis vs On Axis

When can a body rotate about an arbitrary pivot point and when must it rotate about its center of gravity?

First, consider a *free* body (define *free* to mean no contact forces – i.e. Normal forces or frictions). For an object to rotate, there must be Centripetal forces – since these are all *internal* forces, they must add to zero. ➔The body **must** rotate about its center of gravity.

For a body with an *external* fixed pivot point, normal forces at the pivot can provide an *external* centripetal force.



Demo: Center of Gravity

Rotating Off Axis

When rotating off axis, in a uniform gravitational field:

Is the motion circular? ➔ Yes

Is the angular velocity uniform? ➔ No

Is there a net torque? ➔ Yes

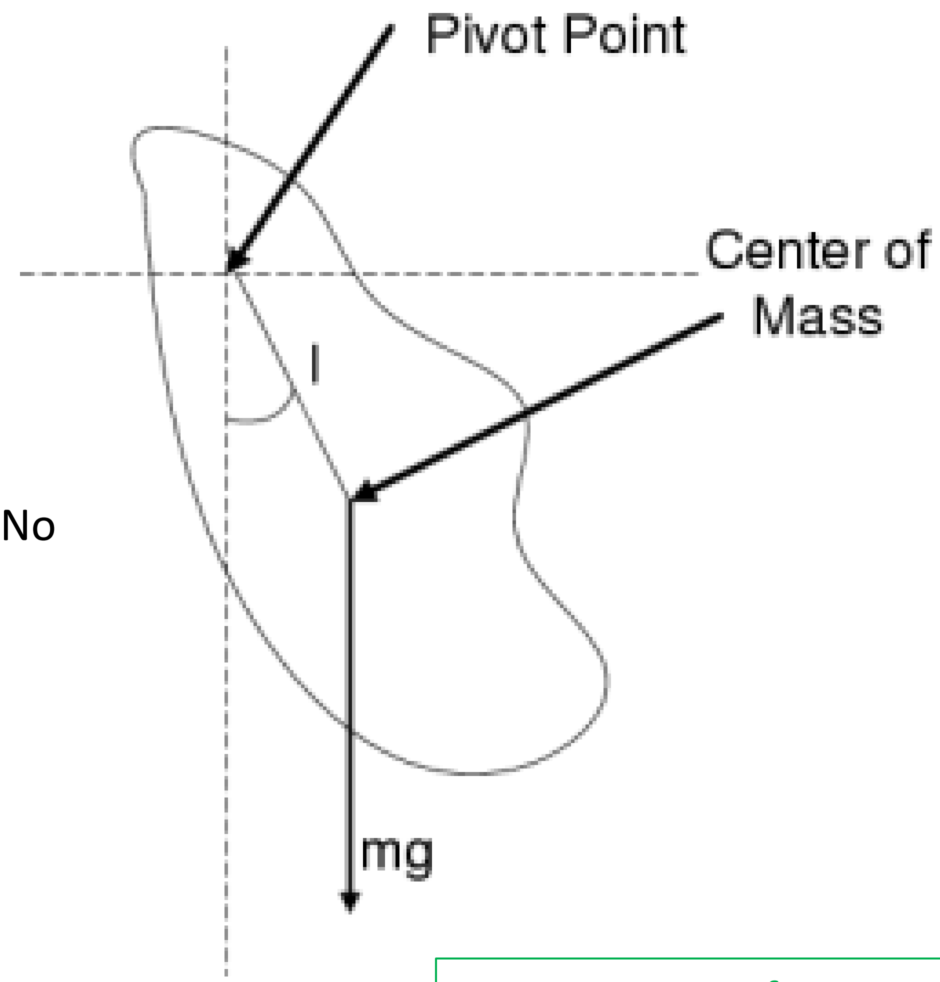
Is the angular acceleration uniform? ➔ No

Is the torque uniform? ➔ No

Is there a net linear motion? ➔ Yes

Is there a net force? ➔ Yes

Is it uniform? ➔ No



Demo: Center of Gravity

Oscillating Off Axis

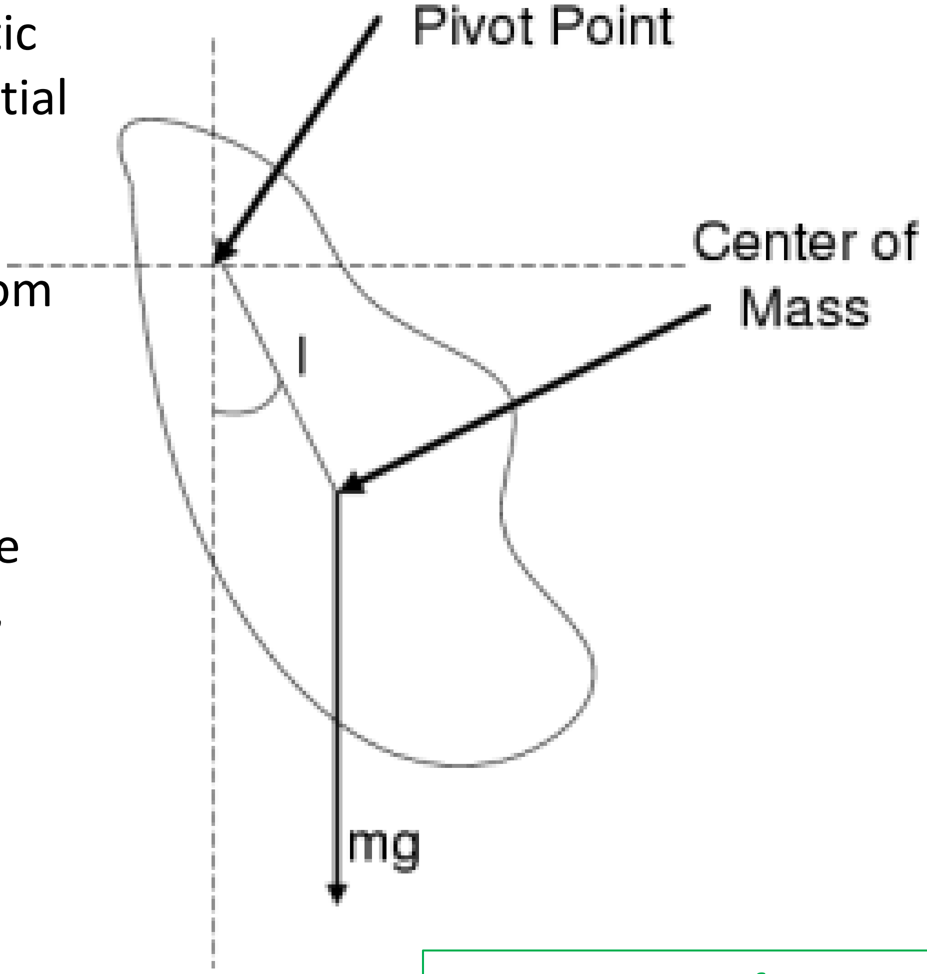
There is an oscillation between kinetic energy (½ mv^2 and ½ $I\omega^2$) and potential energy (mgh).

Where it KE maximum? ➔ The bottom

Where is PE maximum? ➔ The top

What happens is $KE_{max} < \Delta PE$? ➔ The motion is no longer circular! Instead, the body will oscillate.

Can we describe oscillatory motion?



Demo: Center of Gravity

Oscillatory Motion

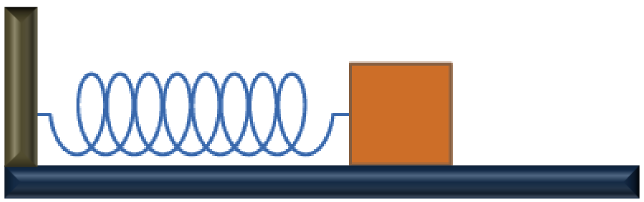
Oscillation: Periodic displacement of an object from an equilibrium point

Periodic or Oscillatory Motion

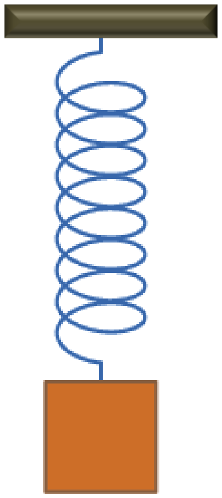
- Equilibrium Position: The position at which all forces acting on an object sum to zero.
- Restoring Force : Force driving the object towards equilibrium point
  - if restoring force is proportional to displacement => S.H.M.
  - i.e. Hooke's Law ➔ $F_{restore} = -kx$
- Period (T) : Interval of time for each repetition or cycle of the motion. Frequency ($f = 1/T$)
- Amplitude (A) : Maximum displacement from equilibrium point
- Phase (ϕ): Describes where in the cycle you are at time t = 0.

Examples of Oscillating Systems

Horizontal Mass-Spring



Vertical Mass-Spring



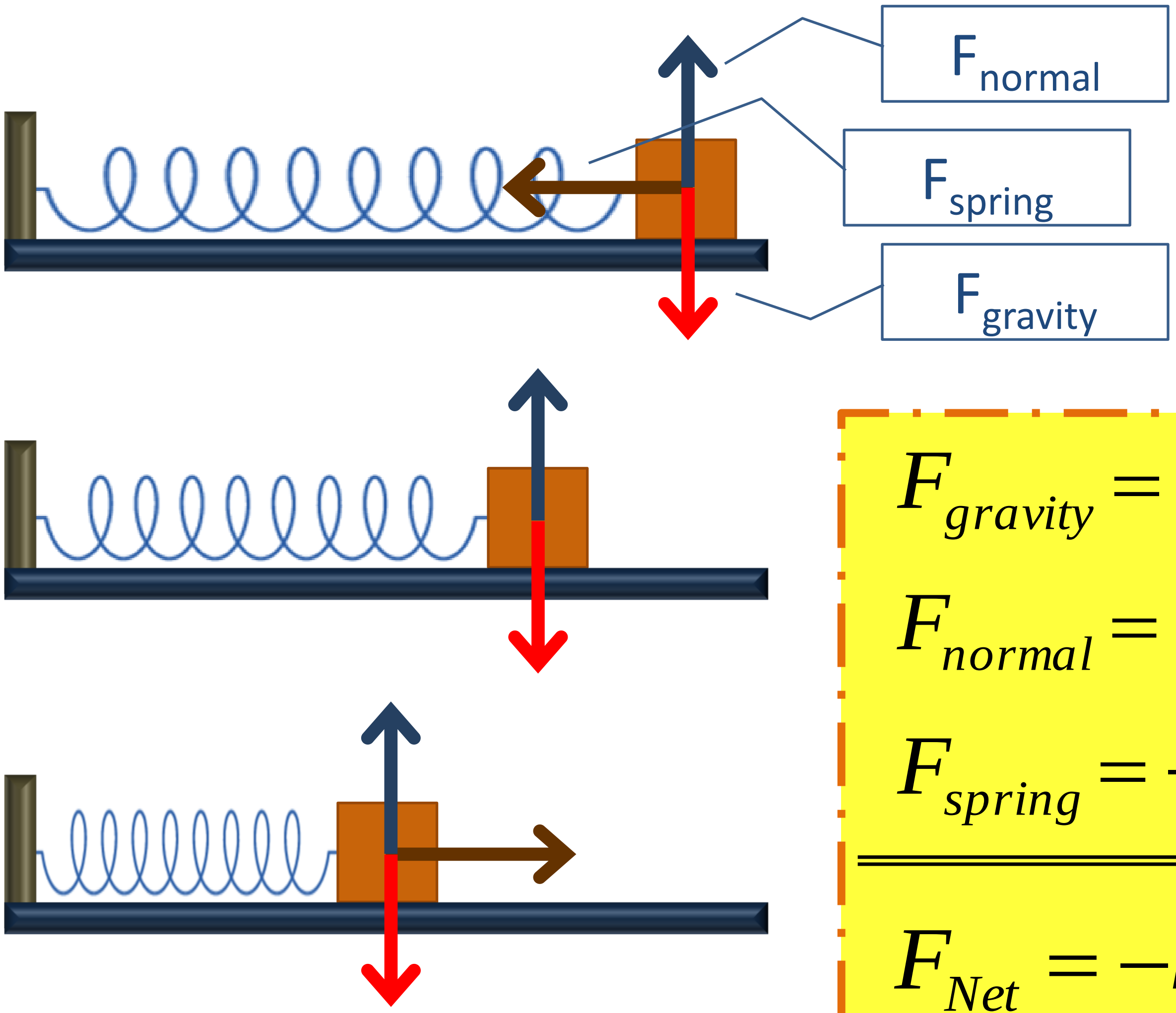
Pendulum

Horizontal Mass-Spring

Spring Extended:

Spring in Equilibrium:

Spring Compressed:



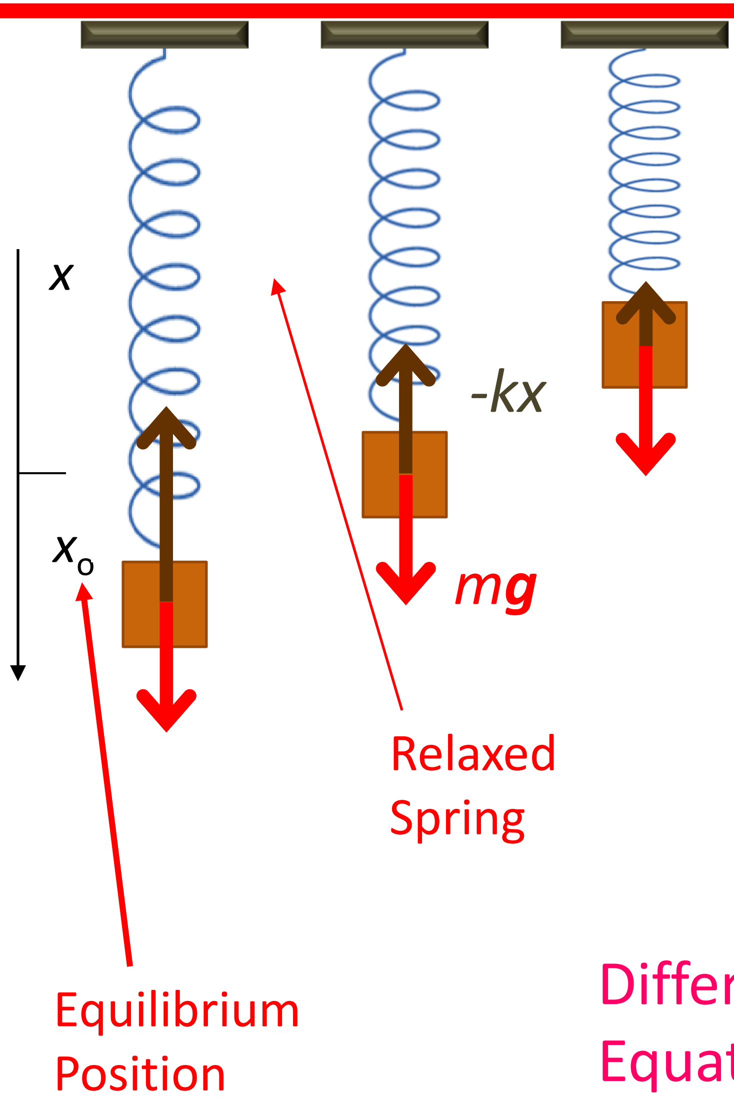
$$F_{gravity} = -mg$$

$$F_{normal} = mg$$

$$F_{spring} = -kx$$

$$F_{Net} = -kx$$

Vertical Mass on Spring System



Force from a Spring:

$$\boldsymbol{F}_{spring} = -k\boldsymbol{x} \quad \text{(upwards)}$$

At Equilibrium:

$$\Sigma \boldsymbol{F} = -kx_0 + mg = 0$$

Away from Equilibrium:

$$\Sigma \boldsymbol{F} = -kx + mg = -k(x - x_0) = -kx$$

Newton's 2nd Law ($F=ma$): Let $x = (x-x_0)$

$$\boldsymbol{F}_{tot} = -kx(t) = ma(t) = m\,(d^2x(t)/dt^2)$$

Differential Equation...

DEMO: Spring w/ mass

How to solve this differential equation?

Differential Equation for Simple Harmonic Oscillation

$$-kx(t) = m\frac{d^2x(t)}{dt^2}$$

$$\frac{d^2x(t)}{dt^2} = -\frac{k}{m}x(t)$$

Do we know any function whose second derivative is itself times a constant?

$$\frac{d}{dt}\sin bt = +b\cos bt$$

$$\frac{d}{dt}\cos bt = -b\sin bt$$

$$\frac{d^2}{dt^2}\sin bt = -b^2\sin bt$$

$$\frac{d^2}{dt^2}\cos bt = -b^2\cos bt$$

➔ Let $x(t) = \sin(\sqrt{\frac{k}{m}}t)$

Period:
sine function repeats when
$bt = 2\pi$
therefore,
$T = 2\pi/b$

$$T = 2\pi\sqrt{m/k}$$

General Solution: $x(t) = A \sin(2\pi t/T + \phi)$ or $x(t) = A \cos(2\pi t/T + \phi)$

Simple Harmonic Motion

- Simple Harmonic Motion: Oscillatory motion in which the restoring force is proportional to displacement

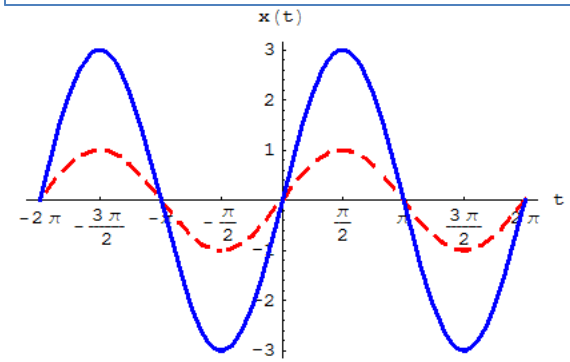
Restoring Force = Constant* X *Displacement

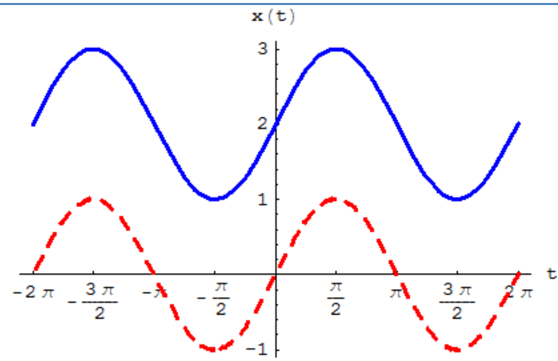
- Displacement vs. Time:

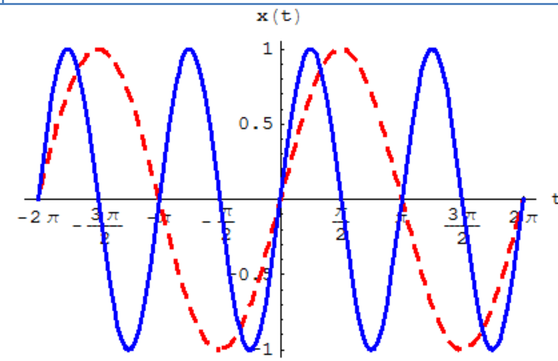
$$x(t) = A\sin(\frac{2\pi}{T}t + \phi) + B$$

Practice With SHM Equation

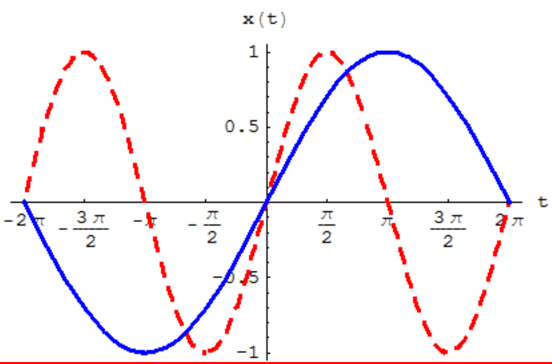
$\sin(t)$, $3\sin(t)$	$\sin(t)$, $\sin(t)+2$	$\sin(t)$, $\sin(2t)$

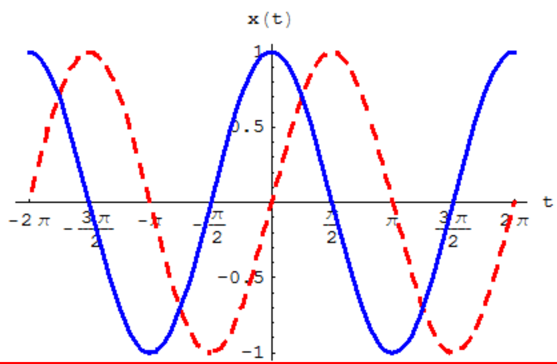


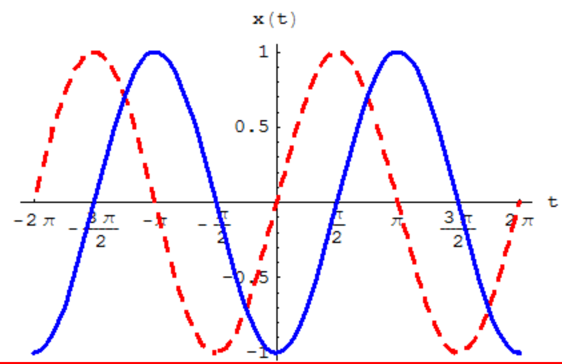




$\sin(t)$, $\sin(0.5\,t)$	$\sin(t)$, $\sin(t + 0.5\,\pi)$	$\sin(t)$, $\sin(t - 0.5\,\pi)$

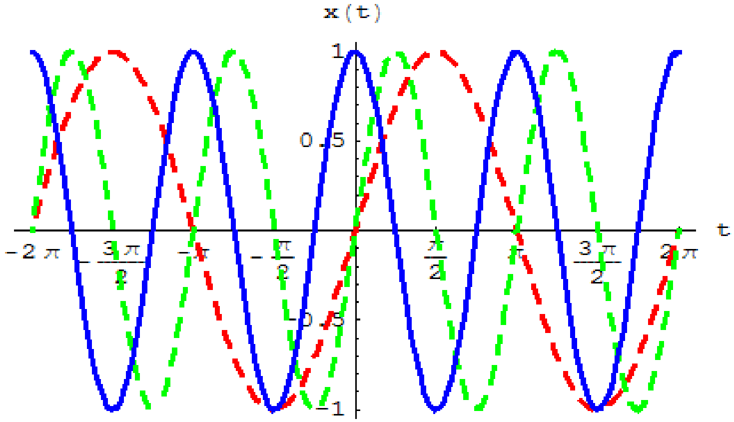


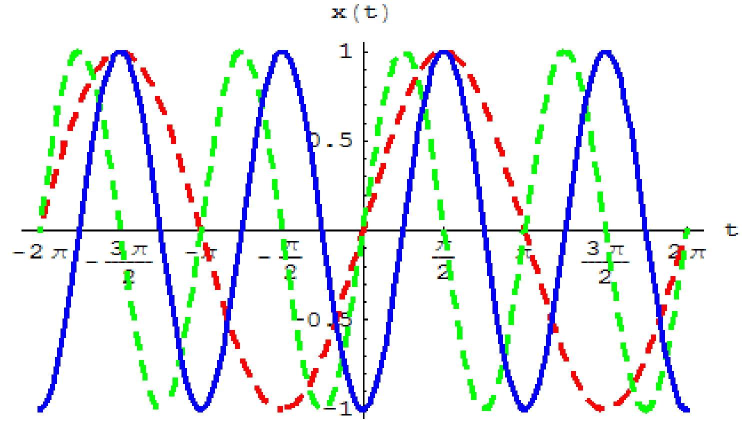




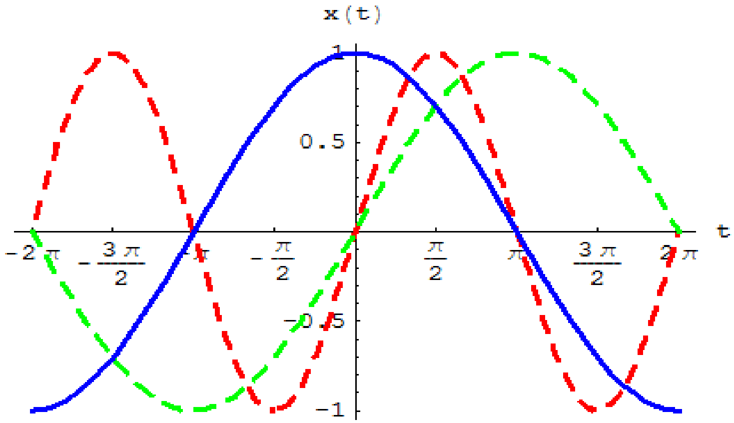
Practice With SHM Equation

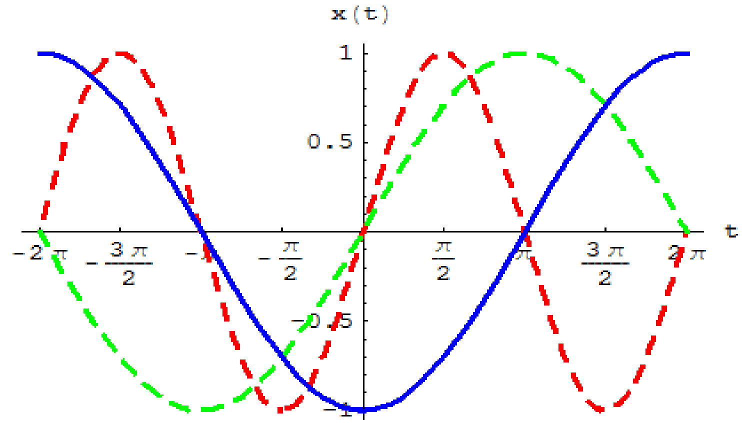
sin(t) , sin(2t) , sin(2t+0.5 π)	sin(t) , sin(2t) , sin(2t-0.5 π)



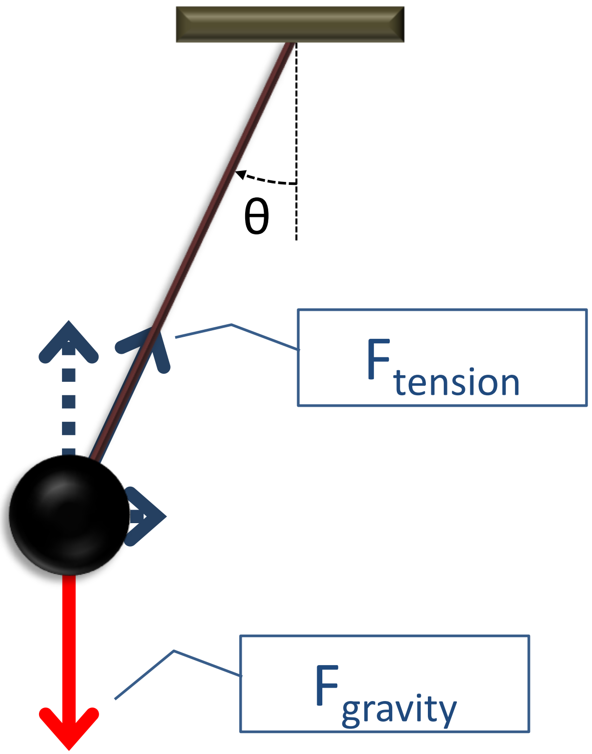


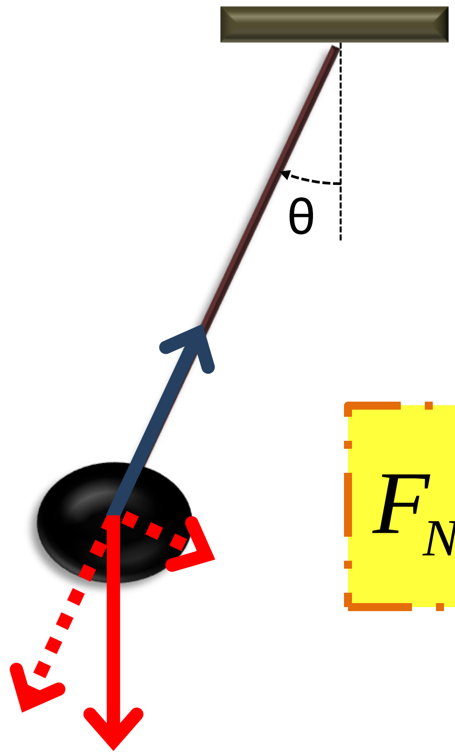
sin(t),sin(0.5t),sin(0.5t+0.5 π)	sin(t),sin(0.5t),sin(0.5t-0.5 π)





Pendulum System





$$F_{Net} = mg\sin\theta$$

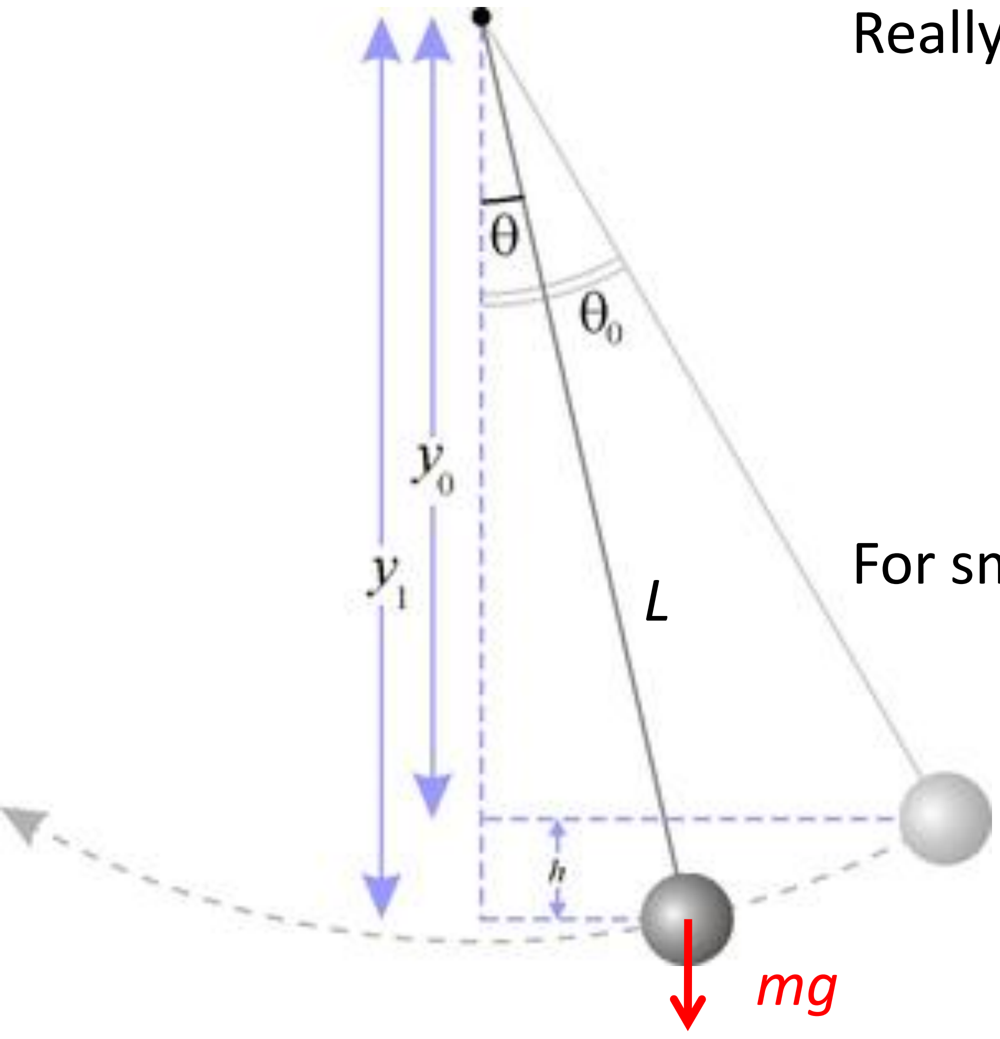
$$F_{tensionX} = F_{tension}\sin\theta$$

$$F_{tensionY} = F_{tension}\cos\theta$$

$$F_{gravityT} = F_{gravity}\sin\theta = mg\sin\theta$$

$$F_{gravityR} = F_{gravity}\cos\theta = mg\cos\theta$$

Pendulum System



Really a rotational problem…

$\tau = F_{tan}\, L = mg\sin\theta\, L$

$\tau = I\,\alpha \;= mR^2\,\alpha$

$-mgL\sin\theta = mL^2\, d^2\theta/dt^2$

For small θ, $\sin\theta = \theta$ (good to about 10 degrees)

$-mgL\theta(t) = mL^2\, d^2\theta/dt^2$

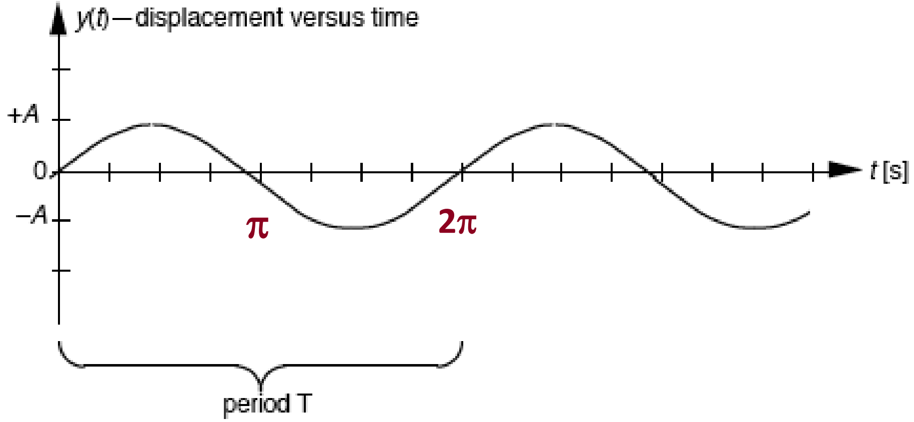
$-(g/L)\theta(t) = d^2\theta/dt^2$

$$T = 2\pi\sqrt{L/g}$$

SHM - Recap

The generalized solution is of the form:

$$y(t) = A\sin(\frac{2\pi}{T}t + \phi) + B$$



Pendulum: $T = 2\pi\sqrt{\frac{l}{g}}$

Mass/Spring: $T = 2\pi\sqrt{\frac{m}{k}}$

How is the frequency *f* of the oscillations related to the period T?

Period: $T = 1/f = 1/(1/\text{sec}) = \text{sec}$

Energy in SHM

Consider mass on a spring:

$$W = \int F \bullet dl = \int kx dx = \frac{1}{2}kx^2$$

$PE = (1/2)\, k\, x^2$

$KE = (1/2)\, mv^2 \quad v = dx/dt$

$x(t) = A\, \mathbf{sin}(2\pi t/T + \phi)$

$PE = (1/2)\, kA^2\, \mathbf{sin}^2\, (2\pi t/T + \phi)$

$KE = (1/2)\, m\, [(2\pi/T)A\cos(2\pi t/T + \phi)]^2$

remember… $(2\pi/T) = \text{sqrt}(k/m)$ => $(2\pi/T)^2 = k/m$

$KE_{peak} = PE_{peak}$

=> $KE = (1/2)\, A^2\, k\, \mathbf{cos}^2\, (2\pi t/T + \phi)$

$E_{total} = PE + KE = (1/2)kA^2[\underbrace{\mathbf{sin}^2\, (2\pi t/T + \phi) + \mathbf{cos}^2\, (2\pi t/T + \phi)}_{1}]$

➔ Energy is conserved with time!
And Energy is proportional to A^2

Topics (1/4)

- Fluids
  - Continuity Eq.
  - Energy Density Eq.
  - Steady-state Flow
  - Flow Line
  - Pressure
- Electrical Circuits
  - Batteries
  - Resistors
  - Capacitors
  - Power
  - Voltage
  - Current
  - Equivalent Circuits
- Linear Transport Model
  - Linear Transport Eq.
  - Current Density
  - Exponentials

Topics (2/4)

- Vectors
  - Addition
  - Subtraction
  - Cartesian Coords.
  - Polar Coords.
- Position Vector
- Velocity Vector
- Acceleration Vector
- Momentum Vector
  - Conservation of Momentum
- Force Vector
  - Friction
  - Normal
  - Gravity
  - Spring

Topics (3/4)

- Newton's Laws
  - Force & Acceleration
  - Force & Momentum
- Collisions
  - Momentum Cons.
  - Elastic vs. Inelastic
- Rotational Motion
  - Ang. Velocity
  - Ang. Acceleration
  - Ang. Momentum
  - Torque
- Right Hand Rule
- Conservation of Ang. Momentum
- Moment of Inertia
  - Rotational Kin. Energy

Topics (4/4)

- Oscillations
  - Period
  - Amplitude
  - Mass-spring system
  - Pendulum
  - Simple Harmonic Motion
  - Phase Constant

Evaluations

Practice With SHM Equation

- - - - - $\sin(t)$

- - - - - $\sin(t - \frac{\pi}{2})$

——— $\sin(\frac{1}{2}t - \frac{\pi}{2})$

- - - - - $\sin(\frac{1}{2}t)$