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MULTIPLICITY DISTRIBUTIONS IN NUCLEUS-NUCLEUS COLLISIONS AT HIGH ENERGIES

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The average and the dispersion of multiplicity distributions in nucleus-nucleus collisions are calculated assuming that the inelastic collision of two nuclei is an incoherent composition of collisions of individual nucleons. The average multiplicity is assumed to be proportional to the number of "wounded nucleons" i.e. the nucleons which underwent at least one inelastic collision. For the sake of comparison the average and the dispersion of the number of collisions is also discussed. Our calculations indicate that in nucleus-nucleus collisions, the amplification of various characteristics of the nucleon-nucleon interaction is far grater than in hadron-nucleus collisions.

1. Introduction

In this paper we discuss multiplicity distributions of particles produced in collisions of two nuclei at high energies. Our basic assumption is that the inelastic collision of two nuclei can be described as an incoherent composition of the collisions of individual nucleons. This picture is an old one. It proved useful in the description of nucleon-nucleus collisions [1–5]. It seems therefore natural and interesting to extend it further to the nucleus-nucleus collisions in the hope of obtaining useful estimates of experimental characteristics such as multiplicities, dispersions and their dependences on nuclear parameters.

It should be stressed that in this approach the collective effects which may occur in nuclei are neglected. The existence of such effects in e.g. nucleon-nucleus interactions is controversial [6,7]. It is therefore interesting to extend the investigation of this problem to nucleus-nucleus interactions and our calculations should prove useful by giving predictions for the cases where collective effects are absent. Should future experiments prove that collective effects are indeed absent in inelastic collisions * our scheme can be used to analyze the properties of elementary collisions which are otherwise difficult to see. For example, it may be possible to separate different components in the elementary production processes which get differently amplified in the processes of multiple collisions.

* We neglect altogether the diffractive production processes.

In the case of nucleon-nucleus collisions a fundamental role is played by the number of collisions (ν) of the incident nucleon with the nucleons in the target nucleus [1]. However, the generalization of this concept to nucleus-nucleus collisions is not unique.

In this paper we propose to describe the nucleus-nucleus collisions in terms of the number of "wounded" nucleons (w) i.e. the number of nucleons which underwent at least one inelastic collisions in this process. For instance in the case of nucleon-nucleus collisions there are ν "wounded" nucleons in the target nucleus and one "wounded" incident nucleon. Consequently, in this case, there is a simple relation between ν and w

$$w = \nu + 1 \ . \tag{1.1}$$

Thus either of them can be used.

In the nucleus-nucleus collisions, however, there is no unique relation between ν and w. Therefore a choice has to be made and our conjecture is that, physically, the more relevant variable is w. The motivation for this choice comes from the interpretation of the available data on nucleon-nucleus interactions [8]. The average multiplicities in collisions of a high-energy nucleon with a target nucleus of mass number A follow approximately the formula

$$\bar{n}_A = \tfrac{1}{2}(\bar{\nu} + 1)\,\bar{n}_{\mathrm{H}} = \tfrac{1}{2}\bar{w}\bar{n}_{\mathrm{H}} \ , \tag{1.2}$$

where $\bar{n}_{\mathrm{H}}$ is the average multiplicity in nucleon-nucleon collisions, and $\bar{\nu}(\bar{w})$ is the average number of collisions (of wounded nucleons).

This formula suggests that the incident nucleon contribution to $\bar{n}_A$ is the same as the contribution of each hit target nucleon and equals *approximately* * $\tfrac{1}{2}\bar{n}_{\mathrm{H}}$. Thus, there seems to be no difference whether a nucleon is hit once or several times. This observation justifies the relevance of w.

For nucleus-nucleus collisions this picture implies that the average multiplicity in a collision of two nuclei with the mass numbers A and B is

$$\bar{n}_{AB} = \tfrac{1}{2}\bar{w}\bar{n}_{\mathrm{H}} \ , \tag{1.3}$$

while for nucleon-nucleus collisions one could also use $\bar{\nu}$ (as indicated in eq. (1.2)) here it is no longer possible.

The main purpose of this paper is to explore the consequences of the model in which the multiplicity distributions for nucleus-nucleus collisions are given by incoherent superposition of distributions provided by each wounded nucleon. We calculated average multiplicity and dispersion in this model. We found that the expected nuclear effects are rather dramatic, particularly for collisions of two heavy nuclei. Consequently, we feel that experimental investigation of heavy nuclei collisions at

* It may appear that this argument depends critically on the accuracy of eq. (1.2). We show later that this is not the case (see sect. 4).

high energies may indeed be useful for establishing: (a) which aspects of elementary collisions get amplified in nuclear interactions and (b) whether collective phenomena play an important role in nuclear collisions at high energies.

The collisions of two nuclei were already discussed by many authors. The extension of the Glauber model was used to describe elastic, quasi-elastic and total cross sections [9,10]. Production processes were also discussed in this framework [10].

Average multiplicities are discussed in sect. 2 and dispersion in sect. 3. In sect. 4 we consider the stability of the obtained results with respect to variation of the parameters of the model. Our conclusions are listed in the last section. The derivation of the formulae for the average number of wounded nucleons is given in appendix A. The dispersion of the number of wounded nucleons and of the number of collisions is derived in appendix B.

2. Average multiplicities

As explained in the introduction the average number of particles produced in inelastic nucleus (mass number A) – nucleus (mass number B) collision is

$$\bar{n}_{AB} = \tfrac{1}{2}\bar{w}\bar{n}_{\mathrm{H}} . \tag{2.1}$$

Thus calculation of $\bar{n}_{AB}$ reduces to calculating the average number of wounded nucleons $\bar{w}$. Since our basic assumption is that the inelastic collisions of two nuclei can be described as an *incoherent* composition of individual nucleons, we compute $\bar{w}$ using probability calculus.

In appendix A we show that the number of wounded nucleons in the collision of

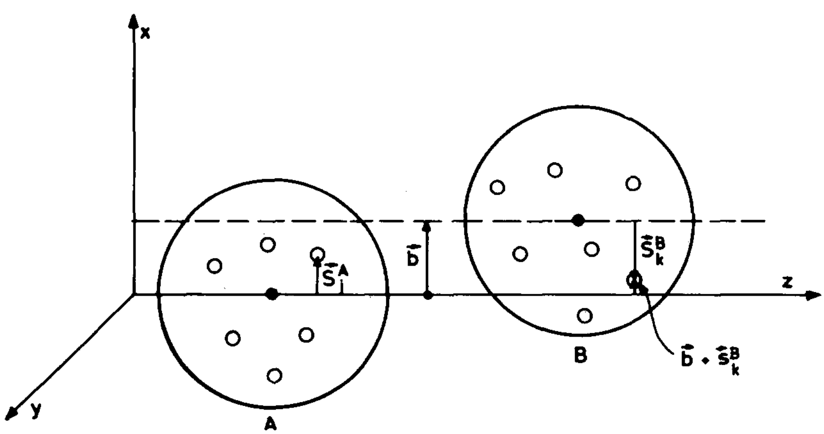

Fig. 1. Geometry of the nucleus-nucleus collision.

A and B is the sum of wounded nucleons in the nucleus A and the nucleus B:

$$\bar{w}_{AB} = \bar{w}_A + \bar{w}_B \,, \tag{2.2}$$

where

$$\bar{w}_A = \frac{A\sigma_B}{\sigma_{AB}} \quad \text{and} \quad \bar{w}_B = \frac{B\sigma_A}{\sigma_{AB}} \,. \tag{2.3}$$

Here σ_A is the nucleon-nucleus A production cross section, σ_B is the nucleon-nucleus B production cross section and σ_{AB} is the production cross section for the collision of nucleus A with nucleus B. The explicit formulae for σ_A, σ_B and σ_{AB} are given in appendix A.

When $B = 1$, (2.2) reduces to

$$\bar{w}_{1A} = \frac{1}{\sigma_A}(A\sigma_H + \sigma_A) = 1 + \frac{A\sigma_H}{\sigma_A} = 1 + \bar{\nu} \,, \tag{2.4}$$

in accordance with eq. (1.2). There, we have used the well-known expression for the average number of collisions $\bar{\nu} = A\sigma_H/\sigma_A$ [5], [11], where σ_H is the nucleon-nucleon production cross section.

In fig. 2 we show $R = \frac{1}{2}\bar{w}_{AB}$ for various A and B nuclei and σ_H = 30 mb. It is seen that in first approximation R is a function of the product AB. Furthermore, R

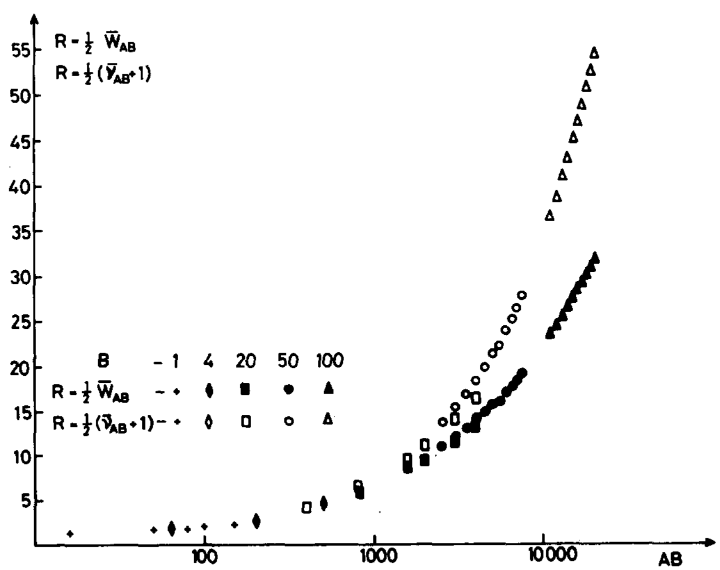

Fig. 2. Average number of wounded nucleons (black symbols) and average number of collisions (open symbols) *versus* AB. σ_H = 30 mb.

increases rather rapidly for AB greater than ~400 (this is far beyond the region attainable in hadron-nucleus collisions). For comparison we also plot points for $R_\nu = \frac{1}{2}(\bar{\nu} + 1)$ which fit nicely hadron-nucleus collisions. The following nuclear densities were used:

For $A > 16$:

$$\rho(r) = \rho_0 \left(1 + \exp\left[\frac{r - R}{c}\right]\right)^{-1} , \tag{2.5}$$

where $R = 1.07\, A^{1/3}$ fm, $c = 0.545$ fm.

For $A = 4$:

$$\rho(r) = \left(\frac{1}{\pi R^2}\right)^{3/2} \exp\left(\frac{-r^2}{R^2}\right) , \qquad R = 1.37 \text{ fm} . \tag{2.6}$$

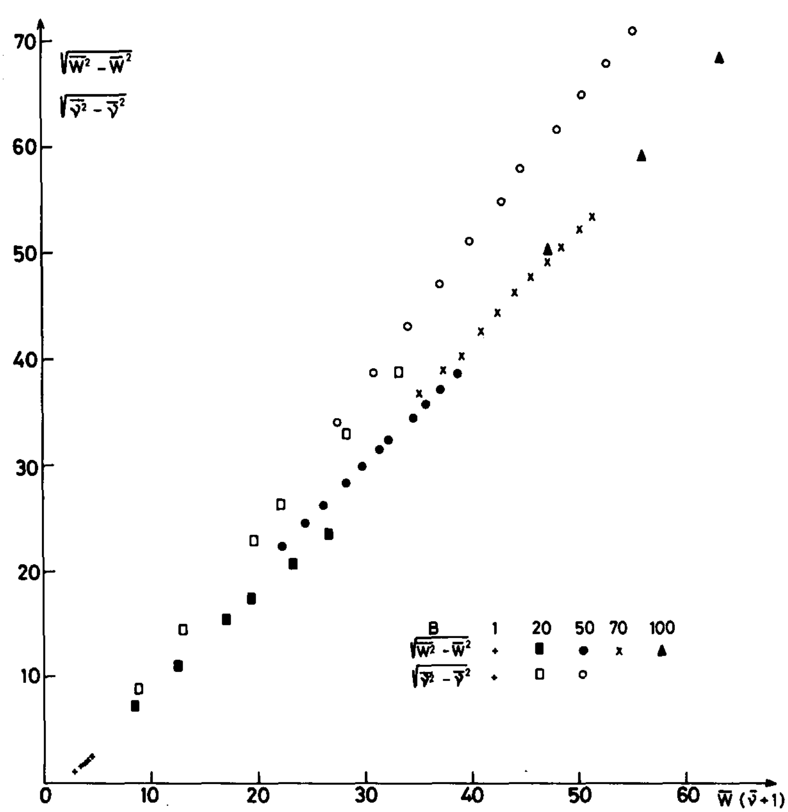

Fig. 3. Dispersion *versus* average for (a) number of wounded nucleons (black symbols) and (b) number of collisions (open symbols). σ_H = 30 mb.

3. Dispersions

In our model each wounded nucleon contributes independently to the observed multiplicity distribution. Consequently, for a fixed number of wounded nucleons, the dispersion is given by the formula

$$D^2(w) = w\,\tfrac{1}{2}D_H^2\,, \tag{3.1}$$

where D_H is the dispersion in the nucleon-nucleon interaction. Thus, the observed dispersion can be computed from the formula

$$D^2 = \tfrac{1}{2}\bar{w}D_H^2 + \tfrac{1}{4}[\overline{w^2} - \bar{w}^2]\,\bar{n}_H^2\,. \tag{3.2}$$

Here we employed the following expression, implied by our model, for average multiplicity with a given number of wounded nucleons:

$$\bar{n}(w) = \tfrac{1}{2}w\bar{n}_H\,. \tag{3.3}$$

Hence the problem of computing D reduces to evaluation of $\overline{w^2} - \bar{w}^2$. This calculation is given in appendix A. The numerical results are shown in fig. 3 where dispersion of the number of wounded nucleons w is plotted *versus* the average. The striking feature is that the observed dependence is approximately linear with the slope $\sim$1. Similar linear behaviour is seen for dispersion of the number of collisions, but its slope is greater ($\sim$1.3).

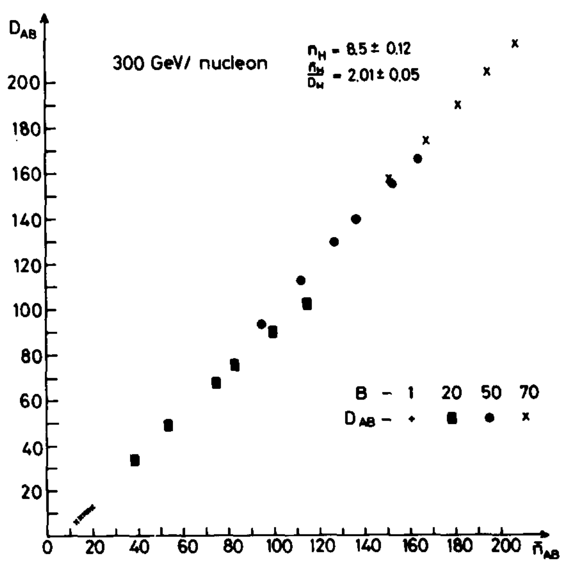

Fig. 4. Dispersion of multiplicity distribution *versus* average multiplicity for nucleus-nucleus collisions at lab energy 300 GeV/nucleon. σ_H = 30 mb, β = 0.5.

In fig. 4 we plot D_{AB} *versus* $\bar{n}_{AB}$ for 300 GeV obtained from eq. (3.2) using $\bar{n}_{\mathrm{H}} = 8.5$ and $\bar{n}_{\mathrm{H}}/D_{\mathrm{H}} = 2.0$. Note that points corresponding to the hadron-nucleus collisions are all concentrated in the lower left corner of the diagram. This illustrates the extent of the extrapolation involved in our model.

4. Stability of the results

In this section we discuss two possible modifications of our model which are suggested by the analyses of the data for nucleon-nucleus collisions in refs. [1, 3]. We shall show that although they may be important in the case of nucleon-nucleus interactions the nucleus-nucleus collisions are far less sensitive to them.

The first observation is that all modifications of our model must preserve symmetry of the process of the nucleon-nucleon interaction. The simplest realization of such symmetry is to assume that both nucleons contribute $\frac{1}{2}n_{\mathrm{H}}$ of the multiplicity, as expressed in eq. (1.3). However, as discussed in refs. [1] and [3] reasonable fits to the data on nucleon-nucleus collision may be obtained with the formula

$$\bar{n}_A = (\beta\bar{\nu}_A + 1 - \beta)\bar{n}_{\mathrm{H}} = (\beta\bar{w}_A + 1 - 2\beta)\,\bar{n}_{\mathrm{H}}\ , \tag{4.1}$$

where $\beta \lesssim \frac{1}{2}$. Now it is clear that $\beta < \frac{1}{2}$ means that only 2β fraction of $\bar{n}_{\mathrm{H}}$ gets multiplied in the collision whereas $(1 - 2\beta)$ does not. It is obvious that such a modification changes the predicted average multiplicity (since it is proportional to β). However, the relation between dispersion and average multiplicity seems to be unaffected by this modification of the model. We calculated dispersions for nucleus-nucleus collisions using eq. (4.1) (instead of eq. (1.3)) with $\beta = 0.4$. We found that such corrections introduce insignificant changes to the relation between D and $\bar{n}_A$ (plotted in figs. 3 and 4) wherever the average number of wounded nucleons exceeds ~10, that means in the majority of cases.

Another possible modification is to change eq. (3.2). For example in ref. [3] the Poisson multiplicity distribution is assumed for nucleon-nucleon interactions.With this assumption the formula (3.2) should be replaced by

$$D^2 = \tfrac{1}{2}\bar{w}\,\bar{n}_{\mathrm{H}} + \tfrac{1}{4}(\overline{w^2} - \bar{w}^2)\,\bar{n}_{\mathrm{H}}^2 . \tag{4.2}$$

We have calculated dispersions with this formula and again found the modifications unsignificant for nucleus-nucleus collisions. This is easily understood if one realizes that the first term in (4.2) is only a small correction to the leading second term provided the number of wounded nucleons exceeds ~10.

Thus other reasonable modifications of eq. (3.2) should also not change our conclusion for nucleus-nucleus interactions.

5. Conclusions

The standard description of hadron-nucleus interactions, in which inelastic collisions are incoherent compositions of the collisions of the incident hadron with individual nucleons in the target nucleus, is extrapolated to nucleus-nucleus collisions. The average multiplicities are sensitive to the details of the elementary nucleon-nucleon process and therefore can be used to fix the parameters of the model. On the other hand the relation between dispersion and average multiplicity depends almost exclusively on the mechanism of the amplification of the production process in consecutive collisions. It can be used therefore to distinguish between different mechanisms of multiplication of particles.

Appendix A

Formula for the average number of wounded nucleons.

The number of wounded nucleons in the collision of A and B is the sum of wounded nucleons in the nucleus A and the nucleus B. Thus it is enough to compute the average number of wounded nucleons in one nucleus e.g. B. Let us denote the probability of collision of the nucleon i from B with anyone of the nucleons of A with a given configuration $s_1^A, \dots s_A^A$ by

$$p(s_i^B; A; s_1^A, \dots s_A^A) \equiv p_A(s_i^B) , \tag{A.1}$$

where the variables are defined in fig. 1. The probability that the nucleons $s_{i_1}^B, \dots s_{i_w}^B$ collide and $s_{i_{w+1}}^B, \dots s_{i_B}^B$ do not is

$$p_A(s_{i_1}^B) \dots p_A(s_{i_w}^B) \cdot [1 - p_A(s_{i_{w+1}}^B)] \dots [1 - p_A(s_{i_B}^B)] .$$

After integrating over the configurations of B we get for the probability of having w_B wounded nucleons in B:

$$P(w_B; B; A; s_1^A, \dots s_A^A; b)$$
$$= \binom{B}{w_B} [1 - \bar{p}(A; s_1^A, \dots s_A^A; b)]^{B-w_B}$$
$$\times [\bar{p}(A; s_1^A, \dots s_A^A; b)]^{w_B} . \tag{A.2}$$

Here

$$\bar{p}(A; s_1^A, \dots s_A^A; b) = \int d^2 s_i^B \, p(s_i^B; A; s_1^A, \dots s_A^A) D_B(b - s_i^B) \tag{A.3}$$

with

$$D_B(s) = \int_{-\infty}^{+\infty} \mathrm{d}z\, \rho_B(s, z)\,,$$

where $\rho_B(s, z)$ is the single nucleon probability distribution in the nucleus B (normalized to unity) which can be identified with the probability density from the single particle wave function. We assume all nucleons to be "equivalent" in the sense that all the one-nucleon probabilities are the same (this simplifying assumption can be removed at the expense of complicating, inessentially, the algebra which we want to avoid).

We are interested only in production processes, hence we should subtract the probability that none of the nucleons got wounded $[P(w_B = 0)]$. Therefore, we normalize our probabilities as follows:

$$\begin{aligned}
\mathrm{Norm} &\equiv \int \mathrm{d}^2 b\,[1 - P(w_B = 0)] \\
&= \int \mathrm{d}^2 b\,[1 - \int \mathrm{d}^2 s_1^A \ldots \mathrm{d}^2 s_A^A\, \mathrm{d}^2 s_1^B \ldots \mathrm{d}^2 s_B^B \\
&\times D_A(s_1^A) \ldots D_A(s_A^A)\, D_B(s_1^B - b) \ldots D_B(s_B^B - b) \\
&\times \prod_{i=1}^{B} \{1 - p(s_i^B; A; s_1^A, \ldots s_A^A)\}] = \sigma_{AB}\,,
\end{aligned} \tag{A.4}$$

which is, in fact, just a cross section for production in a collision of the nucleus A with the nucleus B (for more details see the footnote in appendix B).

So, the average number of wounded nucleons in B is

$$\begin{aligned}
\sigma_{AB}\bar{w}_B &= \int \mathrm{d}^2 b\, \mathrm{d}^2 s_1^A \ldots \mathrm{d}^2 s_A^A \\
&\times \bar{w}_B(B, A; s_1^A, \ldots s_A^A; b)\, D_A(s_1^A) \ldots D_A(s_A^A)\,,
\end{aligned} \tag{A.5}$$

where

$$\begin{aligned}
\bar{w}_B(B, A; s_1^A, \ldots s_A^A; b) &= \sum_{w_B} w_B\, P(w_B, B, A, s_1^A, \ldots s_A^A; b) \\
&= \sum_{w_B=1}^{B} w_B \binom{B}{w_B} (1 - \bar{p})^{B-w_B}\, \bar{p}^{w_B} = B\bar{p}(A; s_1^A, \ldots s_A^A; b)\,.
\end{aligned}$$

So,

$$\sigma_{AB}\bar{w}_B = B\int d^2b\, d^2s_1^A \dots d^2s_A^A d^2s^B$$

$$\times D_A(s_1^A) \dots D_A(s_A^A)\, D_B(s^B - b)\, p(s^B; A; s_1^A, \dots s_A^A)\,. \tag{A.6}$$

However,

$$p(s^B; A; s_1^A, \dots s_A^A) = 1 - \prod_{i=1}^{A} [1 - \sigma(s^B - s_i^A)]\,, \tag{A.7}$$

where $\int d^2s\, \sigma(s) = \sigma_H$, σ_H being the nucleon-nucleon total inelastic cross section with diffraction production excluded. From (A.6) we get finally

$$\bar{w}_B = \frac{B\sigma_A}{\sigma_{AB}}\,, \tag{A.8}$$

where σ_A, the total inelastic nucleon-nucleus cross section, is

$$\sigma_A = \int d^2b\, d^2s^B\, D_B(s^B - b)\, \{1 - \prod_{i=1}^{A} [1 - \int d^2s_i^A D_A(s_i^A)\, \sigma(s^B - s_i^A)]\}$$

$$= \int d^2\, s^B \{1 - \prod_{i=1}^{A} [1 - \int d^2s_i^A\, D_A(s_i^A)\, \sigma(s^B - s_i^A)]\} \tag{A.9}$$

because $\int d^2b\, D_B(s^B - b) = 1$ for all s^B. Repeating the same calculation for A we get $\bar{w}_A = A\sigma_B/\sigma_{AB}$ and the complete expression for the number of wounded nucleons is thus

$$\bar{w}_{AB} = \frac{1}{\sigma_{AB}}(A\sigma_B + B\sigma_A)\,. \tag{A.10}$$

Appendix B

Generating function for multiplicity distributions

All averages discussed in this paper can be obtained from the following two generating functions:

$$F(x_1, \dots x_A; y_1, \dots y_B) = \prod_{i=1}^{A} \prod_{j=1}^{B} (1 - \sigma_{ij} + x_i y_j \sigma_{ij}) \tag{B.1}$$

generates all averages where the number of wounded nucleons is relevant, and

$$\phi(x) = \prod_{i=1}^{A} \prod_{j=1}^{B} (1 - \sigma_{ij} + x\sigma_{ij}) \tag{B.2}$$

generates all averages where the number of collisions is relevant.

In (B.1) and (B.2)

$$\sigma_{ij} = \sigma(b - s_i^A + s_j^B) , \tag{B.3}$$

where $\sigma(s)$ is normalized, as always, to σ_H:

$$\int d^2s\, \sigma(s) = \sigma_H .$$

Let us compute D_{AB}^2 from (B.1). From (3.2) we know that the problem reduces to computing

$$\overline{w^2} - \bar{w}^2 = (\overline{w_A^2} - \bar{w}_A^2) + (\overline{w_B^2} - \bar{w}_B^2) + 2\overline{w_A w_B} - 2\bar{w}_A \bar{w}_B . \tag{B.4}$$

The first two terms in (B.4) one obtains from immediate generalizations of (A.5). For instance we have

$$\sigma_{AB}\overline{w_B(w_B - 1)} = B(B - 1)\int d^2b\, d^2s_1^A \dots d^2s_A^A\, \bar{p}^2(A; s_1^A, \dots s_A^A; b)\, D_A(s_1^A)\dots D(s_A^A)$$

$$= B(B - 1)\int d^2b\, d^2s_1^A \dots d^2s_A^A\, d^2s_1^B\, d^2s_2^B$$

$$\times\, p(s_1^B, A; s_1^A, \dots s_A^A)\, p(s_2^B, A; s_1^A, \dots s_A^A)\, D_A(s_1^A) \dots D_A(s_A^A) . \tag{B.5}$$

Using the identity

$$p(s_1^B \dots)\, p(s_2^B \dots) = p(s_1^B \dots) + p(s_2^B \dots) - [p(s_1^B \dots) + p(s_2^B \dots) - p(s_1^B \dots) \cdot p(s_2^B \dots)] ,$$

we obtain

$$\sigma_{AB}\, \overline{w_B(w_B - 1)} = B(B - 1)\,(2\sigma_A - \sigma_{2A}) \tag{B.6}$$

and, *mutatis mutandis*,

$$\sigma_{AB}\, \overline{w_A(w_A - 1)} = A(A - 1)\,(2\sigma_B - \sigma_{2B}) . \tag{B.7}$$

Here $\sigma_A\,(\sigma_B)$ is the nucleon-nucleus $A(B)$ cross section and $\sigma_{2A}\,(\sigma_{2B})$ is a cross section of a two-nucleon object with the spatial shape of the $B(A)$ nucleus: $D_B(s)$ $[D_A(s)]$

is its density distribution. So,

$$\overline{w_A^2} - \bar{w}_A^2 = \frac{1}{\sigma_{AB}}[A^2(2\sigma_B - \sigma_{2B}) + A(\sigma_{2B} - \sigma_B)] - \frac{A^2\sigma_B^2}{\sigma_{AB}^2} ,$$

$$\overline{w_B^2} - \bar{w}_B^2 = \frac{1}{\sigma_{AB}}[B^2(2\sigma_A - \sigma_{2A}) + B(\sigma_{2A} - \sigma_A)] - \frac{B^2\sigma_B^2}{\sigma_{AB}^2} . \tag{B.8}$$

We evaluate $\overline{w_A w_B}$ using the generating function (B.1). Expanding F in powers of $x_1, \dots x_A, y_1, \dots y_B$ we find that the coefficients of various products give probabilities of all possible collisions. For instance the coefficient of $x_1^3 x_2^3 y_1^2 y_2^2 y_3^2$ term is the probability of a 6-fold collision in which *two* nucleons from A collided three times each and three nucleons from B collided twice each, hence 5 nucleons got wounded. So the number of wounded nucleons given by one term is equal to the number of different x_i's (w_A) and the number of different y_k's (w_B).

Let us denote

$$F(1, \dots 1; y_1, \dots y_B) = F(1, y) ,$$

$$F(1, \dots, x_i = 0, \dots 1; y_1, \dots y_B) = F_i(0, y) ,$$

$$F(1, \dots, x_i = 0, \dots 1; 1, \dots, y_k = 0, \dots 1) = F_{ik}(0, 0) . \tag{B.9}$$

$F(1, y)$ contains all the probabilities of F, while $F_i(0, y)$ contains all the probabilities of F except the ones with x_i to any power. Therefore $G_i(y) = F(1, y) - F_i(0, y)$ contains only these probabilities which are multiplied by x_i to any power and consequently $H(y) = \Sigma_i\, G_i(y)$ is the sum of all the probabilities with weights w_A and still multiplied by y_k's. But we want to have all the probabilities summed with weights $w_A w_B$. The following expression is the one we want

$$I = \sum_k [H(y_1 = 1, \dots y_B = 1) - H(y_1 = 1, \dots y_k = 0, \dots y_B = 1)]$$

$$= ABF(1, 1) - B \sum_{i=1}^{A} F_i(0, 1) - A \sum_{k=1}^{B} F_k(1, 0) + \sum_{i=1}^{A} \sum_{k=1}^{B} F_{ik}(0, 0) , \tag{B.10}$$

where we have re-traced our steps back to the original generating function F [(B.9) explains the notation].

$$\sigma_{AB}\overline{w_A w_B} = \int d^2b \int d^2s_1^A \dots d^2s_A^A\, d^2s_1^B \dots d^2s_B^B\, I(b, s_1^A, \dots s_B^B)$$

$$\times D_A(s_1^A) \dots D_A(s_A^A)\, D_B(s_1^B) \dots D_B(s_B^B) . \tag{B.11}$$

We normalize the probabilities similarly as in sect. 2. The expression for the norm symmetric in A and B variables is therefore

$$\mathrm{Norm} = \sigma_{AB} = \int \mathrm{d}^2 b \int \mathrm{d}^2 s_1^A \ldots \mathrm{d}^2 s_A^A\, \mathrm{d}^2 s_1^B \ldots \mathrm{d}^2 s_B^B$$

$$\times D_A(s_1^A) \ldots D_A(s_A^A)\, D_B(s_1^B) \ldots D_B(s_B^B)\, \{1 - \prod_{i=1}^{A} \prod_{j=1}^{B} [1 - \sigma(b - s_i + s_j)]\} \,. \tag{B.12}$$

From (B.1) we get

$$F(1,1) = 1, \qquad F_i(0,1) = \prod_{k=1}^{B} (1 - \sigma_{ik}) \,, \qquad F_k(1,0) = \prod_{i=1}^{A} (1 - \sigma_{ik}) \,,$$

$$F_{ik}(0,0) = \prod_{l \neq i}^{A} \prod_{j \neq k}^{B} (1 - \sigma_{lj}) \,. \tag{B.13}$$

The final result, after a straightforward algebra, is

$$\sigma_{AB} \overline{w_A w_B} = (\sigma_A + \sigma_B - S_{AB} - \sigma_\mathrm{H} N_{A-1} N_{B-1}) AB \tag{B.14}$$

with S_{AB}, N_{A-1} and N_{B-1} defined as follows

$$S_{AB} = \int \mathrm{d}^2 b\, \{1 - [\int \mathrm{d}^2 s\, D_A(s)(1 - \sigma_\mathrm{H} D_B(b - s))^{B-1}]\, [\int \mathrm{d}^2 s' D_B(s')$$

$$\times (1 - \sigma_H D_A(b + s'))^{A-1}]\} \,,$$

$$N_{A-1} = \int \mathrm{d}^2 s\, D_A(s)\, [1 - \sigma_\mathrm{H} D_A(s)]^{A-1} \,,$$

$$N_{B-1} = \int \mathrm{d}^2 s\, D_B(s)\, [1 - \sigma_\mathrm{H} D_B(s)]^{B-1} \,. \tag{B.15}$$

From (B.4), (B.8) and (B.15) we get the final formula

$$\overline{w^2} - \overline{w}^2 = \frac{1}{\sigma_{AB}}\, [A^2(2\sigma_B - \sigma_{2B}) + A(\sigma_{2B} - \sigma_B)] - \frac{A^2 \sigma_B^2}{\sigma_{AB}^2}$$

$$+\frac{1}{\sigma_{AB}}\,[B^2(2\sigma_A-\sigma_{2A})+B(\sigma_{2A}-\sigma_A)]-\frac{B^2\sigma_A^2}{\sigma_{AB}^2}$$

$$+\frac{2AB}{\sigma_{AB}}\left[\sigma_A+\sigma_B-S_{AB}-\sigma_{\rm H}\,N_{A-1}N_{B-1}-\frac{\sigma_A\sigma_B}{\sigma_{AB}}\right], \qquad \text{(B.16)}$$

which enables us to compute D^2_{AB} given by (3.2) *.

It is worth noticing that (B.16) reduces, in the case of the nucleon-nucleus A collision, to a compact and handy expression:

$$\overline{w_A^2}-\bar{w}_A^2=\frac{A^2\sigma_{\rm H}^2}{\sigma_A^2}\,[\sigma_A\int {\rm d}^2b\,D^2(b)-1]+\frac{A\sigma_{\rm H}}{\sigma_A}\,[1-\sigma_{\rm H}\int {\rm d}^2b\,D^2(b)]\,. \qquad \text{(B.17)}$$

From the generating function (B.1) one can also obtain the formula (2.2) for the average number of wounded nucleons. To this end one employs the function $H(y=1)$ which, after averaging over nuclear densities gives $\sigma_{AB}\bar{w}_A$.

The average number of collisions $\bar{\nu}$ and the dispersion $D^2=\overline{\nu^2}-\bar{\nu}^2$ we calculate from the generating function

$$\phi(x)=\prod_{i=1}^{A}\prod_{j=1}^{B}(1-\sigma_{ij}+x\sigma_{ij})\,. \qquad \text{(B.18)}$$

* In the actual numerical calculations we used the optical limit formula for σ_{AB}

$$\sigma_{AB}=\ {\rm d}^2b[1-\exp(-AB\sigma_{\rm H}\int {\rm d}^2s\,D_A(s)\,D_B(b-s))]$$

and the following formula for $\sigma_A(\sigma_B)$, $\sigma_{2A}(\sigma_{2B})$:

$$\sigma_A=\int {\rm d}^2b[1-(1-\sigma_{\rm H}D_A(b))^A]\,,$$

$$\sigma_{2A}=\int {\rm d}^2b\,{\rm d}^2b_1{\rm d}^2b_2\,D_B(b-b_1)\,D_B(b-b_2)\,[1-(1-\sigma_{\rm H}D_A(b_1)-\sigma_{\rm H}D_A(b_2))^A]$$

$$-\sigma_{\rm H}^2A[\int {\rm d}^2b\,D_B(b)^2]\int {\rm d}^2b[1-2\sigma_{\rm H}\,D_A(b)]^{A-1}\,,$$

which can be derived from the exact multiple scattering formula under only one assumption that the nucleon size is small compared to sizes of nuclei A and B.

Let us label the pair of ij with one index p, $1 \leqslant p \leqslant AB$, and re-write

$$\phi(x) = \prod_{p=1}^{AB} (1 - \sigma_p + x\sigma_p) = \sum_{\nu} \prod_{p}^{AB-\nu} (1 - \sigma_p) \prod_{p'}^{\nu} (x\sigma_{p'}) , \tag{B.19}$$

where Σ extends over all possible divisions of the set of AB indices into two groups.

$$\left.\frac{\partial\phi(x)}{\partial x}\right|_{x=1} = \sum_{\nu} \nu \prod_{p}^{AB-\nu} (1 - \sigma_p) \prod_{p'}^{\nu} \sigma_{p'} = \sum_{p=1}^{AB} \sigma_p . \tag{B.20}$$

This last expression is clearly the average number of collisions (each element of the first sum of (B.20) is a product of ν, the probability that $AB - \nu$ nucleons did not collide and the probability that ν nucleons did collide) *for a given configuration of the nucleons in the colliding nuclei.* So, to obtain the average number of collisions for two given nuclei we have to average $\partial\phi(x)/\partial x|_{x=1}$ over the nuclear densities and divide by σ_{AB}:

$$\bar{\nu} = \frac{AB \int \mathrm{d}^2b\, \mathrm{d}^2s^A\, \mathrm{d}^2s^B D_A(s^A)\, \sigma(b - s^A + s^B)\, D_B(s^B)}{\sigma_{AB}} = \frac{AB\sigma_{\mathrm{H}}}{\sigma_{AB}} . \tag{B.21}$$

Using the same generating function (B.18) we obtain the dispersion as follows:

$$\sigma_{AB}(\overline{\nu(\nu - 1)}) = \sigma_{AB} \left(\overline{\left.\frac{\partial^2\phi(x)}{\partial x^2}\right|_{x=1}} \right) = \sigma_{AB} \overline{\sum_{p}^{AB} \sigma_p \sum_{p'\neq p}^{AB} \sigma_{p'}}$$

$$= \int \mathrm{d}^2b\, \mathrm{d}^2s_1^A \ldots \mathrm{d}^2s_B^B\, D_A(s_1^A) \ldots D_B(s_B^B) \sum_{i,k} \sigma(b - s_i^A + s_k^B)$$

$$\times \sum_{l,n\neq i,k} \sigma(b - s_l^A + s_m^B) , \tag{B.22}$$

where $l, n \neq i, k$ means that the two pairs are different. We have therefore the following three expressions to compute (we assume $\sigma(s) = \sigma_{\mathrm{H}}\delta^2(s)$):

(i) $\quad i \neq l,\ k = n$,

$$\int \mathrm{d}^2b\, \mathrm{d}^2s_1^A \ldots \mathrm{d}^2s_B^B\, D_A(s_1^A) \ldots D(s_B^B) \sum_{i,k} \sigma(b - s_i^A + s_k^B) \sum_{l\neq i} \sigma(b - s_l^A + s_k^B)$$

$$= BA(A - 1)\, \sigma_{\mathrm{H}}^2 \times \int \mathrm{d}^2b\, D_A^2(b) ,$$

(*ii*) $i = l,\ k \neq n$,

$$\int \mathrm{d}^2 b\ \mathrm{d}^2 s_1^A \ldots \mathrm{d}^2 s_B^B\, D_A(s_1^A) \ldots D(s_B^B) \sum_{i,k} \sigma(b - s_i^A + s_k^B) \sum_{k \neq n} \sigma(b - s_i^A + s_n^B)$$

$$= AB(B-1)\, \sigma_\mathrm{H}^2 \times \int \mathrm{d}^2 b\, D_\mathrm{B}^2(b)\ ,$$

(*iii*) $i \neq l\ ,\ k \neq n$,

$$\int \mathrm{d}^2 b\ \mathrm{d}^2 s_1^A \ldots \mathrm{d}^2 s_B^B\, D_A(s_1^A) \ldots D(s_B^B) \sum_{i,k} \sigma(b - s_i^A + s_k^B) \sum_{\substack{l \neq i \\ k \neq n}} \sigma(b - s_l^A + s_n^B)$$

$$= AB(A-1)(B-1)\, \sigma_\mathrm{H}^2 \int \mathrm{d}^2 b\, \chi^2(b)\ ,$$

where $\chi(b) = \int \mathrm{d}^2 s\, D_A(b+s)\, D_B(s)$. The final formula is:

$$\sigma_{AB} D^2 = AB\, \sigma_\mathrm{H}^2 \left\{ (A-1)(B-1) \int \mathrm{d}^2 b\, \chi^2(b) + (B-1) \int \mathrm{d}^2 b\, D_B^2(b) \right.$$

$$\left. + (A-1) \int \mathrm{d}^2 b\, D_A^2(b) - \frac{AB}{\sigma_{AB}} + \frac{1}{\sigma_\mathrm{H}} \right\} . \tag{B.23}$$

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