Lecture 4 Motion and Kinematics
Review Turning Points
Interpreting Motion Graphs

Last time we left off talking about acceleration and turning points. Recall acceleration is what changes an initial velocity to a final velocity. A change in velocity implies acceleration. When acceleration and velocity point in the same direction, an object speeds up. When acceleration and velocity point in the opposite direction, an object slows down. So now as a review we’ll split the class in two and have you put motion diagrams on the board.

- A ball rolls up and down an incline
- A ball tossed up which comes down along the same path

Now that we’re clear on acceleration, we can expand our ideas about graphs. So far we’ve looked at velocity versus time graphs, but now we’ll look at how position, velocity, and acceleration graphs relate.

We’ve drawn the simplest graph possible here. This motion would represent an object with a constant position. Displacement $\Delta x=x_f-x_i$ is zero. Remember how the slope of a velocity graph is acceleration, it turns the slope of our position graph is velocity.

$slope = \frac{rise}{run} = \frac{\Delta x}{\Delta t}$ the units of slope in this case are m/s, units of velocity!
So let’s consider the velocity of this graph. $\Delta x = 0$ in this case, so $v = 0$. Now let’s draw the velocity vs. time graph even though it’s trivial.

![Graph showing a horizontal line for velocity over time, indicating constant zero velocity.](image)

Acceleration graph is also easy since there’s no change in velocity ($\Delta v = 0 \Rightarrow a=0$)

Now let’s try something a little more challenging:

[have someone walk across the room at constant velocity]

![Graph showing a straight line for position over time, indicating constant positive velocity.](image)

Let’s draw a velocity and acceleration graph to go with this position graph. The slope of a displacement graph gives you acceleration.

$$slope = \frac{rise}{run} = \frac{\Delta x}{\Delta t}$$

The slope here is constant and positive, so the velocity graph should be constant and positive.

![Graph showing a horizontal line for velocity over time, indicating constant positive velocity.](image)

The slope of the velocity graph gives us acceleration, in this case very easy.

$$slope = \frac{rise}{run} = \frac{\Delta v}{\Delta t}$$
Since velocity is constant $\Delta v = 0$, therefore $a = 0$.

Before we move onto other examples, let's summarize:

$position \xrightarrow{slope} velocity \xrightarrow{slope} acceleration$

Next example:
[have student walk at constant velocity from right to left]

Moving in the negative $x$-direction at a constant velocity - slope of position graph should be negative. Velocity graph is easy, constant but negative. Since velocity is constant $\Delta v = 0$, $a = 0$.

Next Example:
[have student start at a value of negative $x$ and walk to the right constant speed]
We have to start our position graph at a negative value, and it has to pass through \( x=0 \) with a positive slope. Velocity graph is easy – constant positive velocity. Acceleration again is easy, \( a=0 \). But we should move onto an example with a non-zero acceleration.

We have an object moving with an increasing positive velocity. Because the slope of the velocity graph is constant and positive, we can draw the acceleration graph accordingly.

The position is a little more challenging. The slope of the position graph gives you the velocity. So we need to draw a position graph that gives a line with positive slope. It turns out that a parabola is a good approximation for what we need. It makes sense if we think about the fact that velocity is increasing – in each time interval we need to cover an increasing bit of distance.

The slope of a parabola is a line. The way to describe a parabola is:
\[
x = at^2
\]
\[
slope = v = \frac{\Delta x}{\Delta t} = 2at
\]

Notice that the slope of the position is a line with a constant positive slope.

Now let’s try to make a motion diagram (simple at this point):

\[\bullet\bullet\bullet\bullet\bullet\bullet\bullet\]
It has a positive velocity, and velocity is increasing, so we are covering more distance over each time interval.

To review basic concepts concerning graphs and motion:

\[
\text{position} \xrightarrow{\text{slope}} \text{velocity} \xrightarrow{\text{slope}} \text{acceleration}
\]

The area of a velocity graph gives you displacement. The way to find displacement on a position graph is to take the difference in positions: \( \Delta x = x_f - x_i \)

One more basic concept to cover, instantaneous velocity and acceleration. The word instantaneous implies that it happens over a very small time interval. So let's do an example.

What's the instantaneous velocity of this object at 5s?

We want the velocity at a particular moment, so we can draw a tangent line at that point to find the slope, then you just find the slope of that line. That is how to find instantaneous velocity from a position graph or instantaneous acceleration from a velocity graph.

A couple of examples before moving on:
Do A and B ever have the same speed? If so, at what time or times?

Everyone take a couple of minutes to write an answer before we decide what the right answers are.