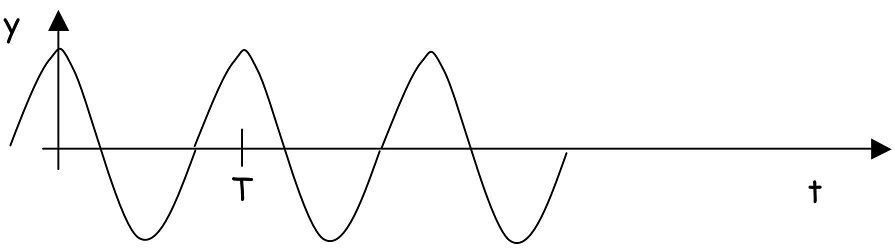
Lecture 21 Oscillations (Chapter 11)
Review of position, velocity, acceleration for an oscillator
Traveling Waves
Interference of Waves
Standing Waves

So remember last time we talked about how to graph the position, velocity, and acceleration for a simple harmonic oscillator. We began with the position function:

$$y(t) = A\cos(\frac{2\pi t}{T} + \phi) + B$$

In this equation A is the amplitude (max. displacement from equilibrium), T is the period (seconds per cycle), ϕ is the constant phase, and B is the equilibrium value.

The general cosine function starts at a maximum:

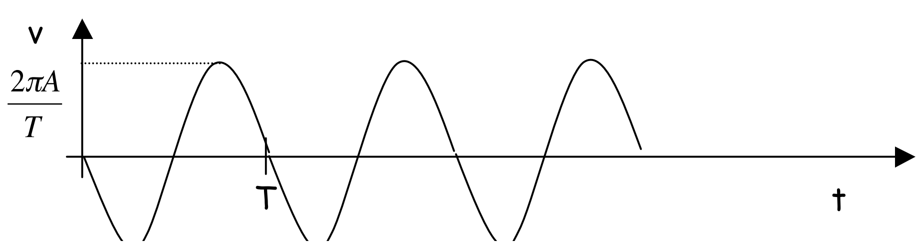


Recall, that the equilibrium value B tells us whether the graph is shifted up or down, and the constant phase ϕ, which tells us whether the graph is shifted left or right.

Now for the velocity function:

$$v(t) = -\frac{2\pi A}{T}\sin(\frac{2\pi t}{T} + \phi)$$

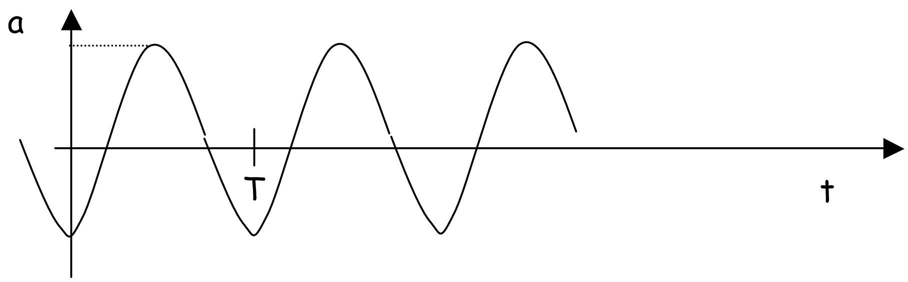
There's a couple of differences from Thursday:
the negative sign and the factor of 2π – it was brought to my attention that I had left these out! Now for a plot of velocity:



Finally, the velocity function:

$$a(t) = -\frac{4\pi^2 A}{T^2}\cos(\frac{2\pi t}{T} + \phi)$$

We can plot this as well:



Again, if we look at corresponding times on the graphs, they should correlate with the actual behavior of the system. For instance when the mass is at Amplitude (t = 0), its velocity is zero, and its acceleration is maximum.

Now back to the idea of a traveling wave. We began by asking the question, what would happen if we put a lot of oscillators together – we get a traveling wave. http://www.cbu.edu/~jvarrian/applets/waves1/lontra_g.htm Energy travels through a medium (like water, or a slinky) and the particles of that medium shake up and down.

A traveling wave has certain characteristics:

- it has a repeat distance called wavelength (λ)
- it has a velocity $v = \lambda f$ or λ/T

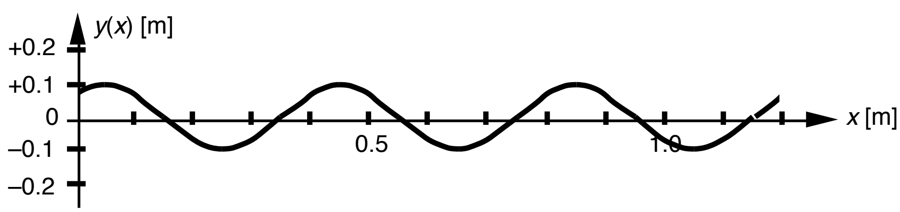
This is a good time to discuss some of the properties of a traveling wave and what they depend on.

	Depends on wave source	*Depends on medium*	*Depends on both*
amplitude A	X		
frequency f	X		
period T	X		
phase shift ϕ	X		

velocity		X	
wavelength λ			X
polarization type	X		

One thing we've pointed out is that a traveling wave has a repeat distance or wavelength. This gets to the nature of a traveling wave, versus the position graph that we look at for just one oscillator. They definitely do not have the same meaning! You can think of the traveling wave as taking a "snapshot" of what all the oscillators are doing while you freeze time. You can think of the position graph we've been looking at as what one single oscillator is doing as a function of time. They have the same shape, but they mean different things. Let's try an example:

A machine tugs sideways on an elastic rope with a frequency of 5 Hz. A graph of the transverse displacements of this rope wave at t = 10.0 s as a function of position is shown below.



(a) What is the velocity of waves on this rope?

(b) If the frequency of the machine tugging on the rope is doubled to f = 10 Hz, what is the new wavelength of waves on this rope? What is the new velocity of waves on this rope?

From the graph above, the wavelength λ of these transverse waves is 0.4 m. Thus the wave velocity along this rope is:

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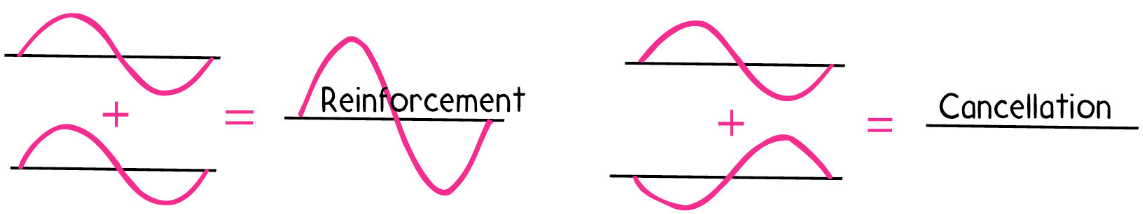
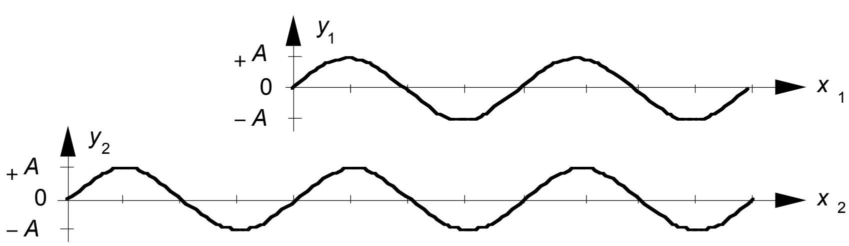
If the frequency of the SHM system transferring energy to the rope is doubled to 10 Hz, the wavelength of the waves traveling along the rope will change, *but not the wave velocity,* as depends *only* on the properties of the rope (tension and linear density), and not on the SHM system transferring energy to it. The new wavelength will be given by:

Now that we understand traveling waves, we can ask the question, 'what happens when you combine two traveling waves?' The answer is that we get something called interference. The two conditions we'll discuss are constructive and destructive interference.

When you have two waves that match crest to crest and trough to trough, you get an even bigger wave (constructive interference). When you have two waves that add crest on trough, then they cancel and you get destructive interference.



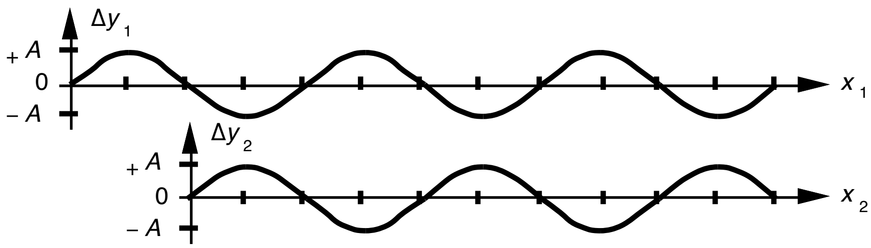
The way to figure out if the waves add constructively or destructively is to figure out their total phase difference. There are two contributions to this total phase difference. The first is called path length difference. We can see this easily with a couple of graphs.
Let's say we had two speakers emitting sound waves. If we took the first speaker and moved it a wavelength in front of the second we could represent the situation graphically like this.



$x_1 = 2\lambda$
$x_2 = 3\lambda$
$\Delta x = x_2 - x_1 = 1\lambda$

Clearly we can see that the troughs and crests match, so we get constructive interference. When you have a path difference of 1λ, 2λ, 3λ or any other whole number of wavelengths you get constructive interference.

It turns out that if we draw the same picture, but now the two waves are half a wavelength apart, you get destructive interference.



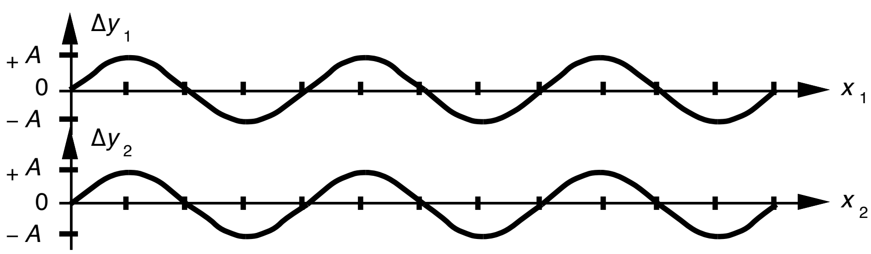
Notice that the crests and troughs overlap.

$x_1 = 3\lambda$

$x_2 = 2.5\lambda$

$\Delta x = x_2 - x_1 = -0.5\lambda$

Another factor influencing interference is constant phase difference. When the two have the same constant phase you get constructive interference.

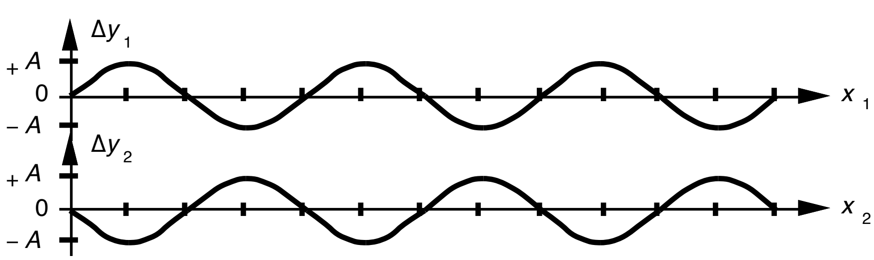


When you take the difference of the phases you get zero, you also get constructive interference.

$\phi_1 = 0$

$\phi_2 = 0$

$\Delta\phi = \phi_2 - \phi_1 = 0$



$\phi_1 = 0$

$\phi_2 = \pi$

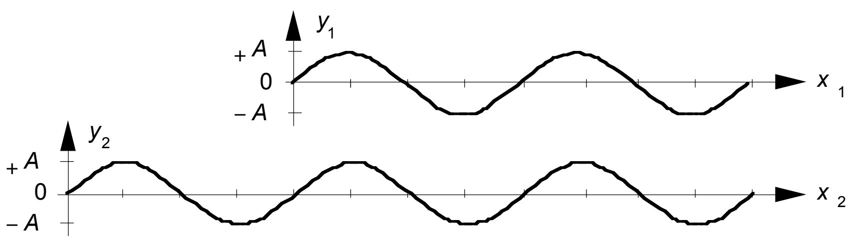
$\Delta\phi = \phi_2 - \phi_1 = \pi$

When you take the difference of the phases you get π, you also get destructive interference.

So, in summary – there are two ways to get either destructive or constructive interference: path length difference and phase difference. We can summarize this in an equation:

$$\Delta\Phi(x,t) = (\frac{2\pi\Delta x}{\lambda} + \Delta\phi)$$

The total phase difference tells us if we have constructive or destructive interference based on whether the difference is odd π[destructive] or even π[constructive]. So let's try an example.



$x_1 = 2\lambda$
$x_2 = 3\lambda$
$\Delta x = x_2 - x_1 = 1\lambda$

$$\Delta\Phi = \frac{2\pi\Delta x}{\lambda} + \Delta\phi$$

$$\Delta\Phi = \frac{2\pi(\lambda)}{\lambda} = 2\pi$$

So mathematically, we get constructive interference, and graphically we can see that we get constructive interference.