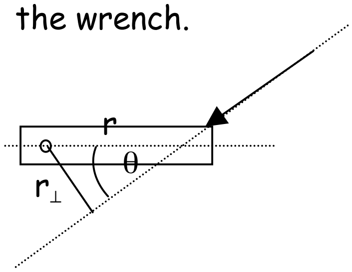
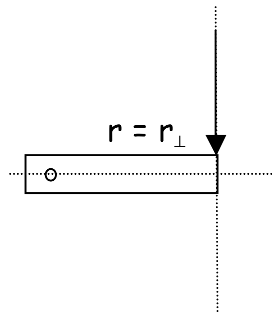
Lecture 19 Torque and Rotational Motion (Chapter 8)

Finish Torque

Angular Momentum

An example for practicing torque:
The drain plug on a car's engine has been tightened to a torque of 25 m•N. If a 0.15 m long wrench is used to change the oil, what is the minimum force need to loosen the plug? Assume the force makes a 30° with the length of the wrench.





So if we write down our equation for torque:

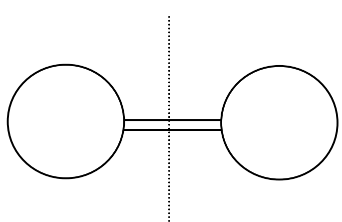
$$\tau = rF\sin\theta$$

$$\tau = 25m \bullet N$$

$$\tau = 0.15m(\sin 30^\circ)F = 25m \bullet N$$

$$F = 333N$$

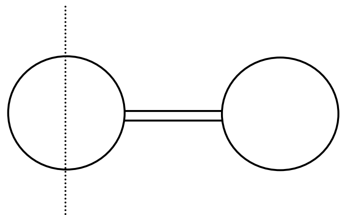
We've talked about rotational mass, i.e. moment of inertia before, but we've never tried a calculation with it. So let's try an example to figure out how to calculate it.
The formula for Rotational Inertia is: Σmr^2. This is if you have point masses in your system, like a barbell for instance. We'll do two different examples:



$m_1 = m_2 = 30kg$
$x_1 = x_2 = 0.50m$

$$\Sigma mr^2 = m_1x_1^2 + m_2x_2^2 = (30kg)(0.5m)^2 + (30kg)(0.5m)^2 = 15kg \bullet m^2$$

Now if we move the axis, it will change the calculation:



$m_1 = m_2 = 30kg$
$x_1 = 0$ $x_2 = 1m$

$$\Sigma mr^2 = m_1 x_1^2 + m_2 x_2^2 = (30kg)(0)^2 + (30kg)(1.0m)^2 = 30kg \bullet m^2$$

So if we look at Newton's Second Law, $\tau_{net} = I\alpha$ or $\alpha = \tau_{net}/I$, for a given torque the acceleration will be less for the second scenario because I is bigger. It would be tougher to get the second dumbbell to rotate because of where we've put the axis of rotation, though we haven't actually changed the masses!

But what if we don't have point masses to deal with? What if we have solid objects? It actually requires calculus to do this, so we've given you a table of objects to deal with (p.278). If you needed one of these on an exam, it would be provided to you. Notice that these all involve the mass and the radius in one form or another.

We have one final concept to cover with respect to Rotational Motion is Angular Momentum. Recall we covered linear momentum p = mv. By direct analogy angular momentum is $L = I\alpha$. It has units of $kg{\cdot}m^2/s$. Just as linear momentum can be conserved under certain conditions, so can angular momentum.
Also, recall $F_{net} = \Delta P/\Delta t$. If there's some net external force, then there's a change in momentum. By direct analogy, $\tau_{net} = \Delta L/\Delta t$. If there's some net external torque, then there's a change in angular momentum. If $\tau_{net} = 0$, then $\Delta L = 0$.

We can do an easy demonstration of conservation of momentum. If we let someone spin while holding arms out with weights we observe a change in speed when they bring the weights in. Let's see if we can explain this in terms of angular momentum.

When momentum is conserved:

$$\Delta L = 0$$

$$L_i = L_f$$

$$(I\omega)_i = (I\omega)_f$$

Take a minute and come up with an explanation for this. Compare explanations with other classmates.

We know that by changing I, ω has to change accordingly. Let's try a real problem:
A skater has a moment of inertia of 100 kg•m^2, when his arms are outstretched and a moment of inertia of 75 kg•m^2, when his arms are tucked in close to his chest. If he starts to spin at an angular speed of 2.0 rps (revolutions per second) with his arms outstretched, what will his angular speed be when they are tucked in?

$$\Delta L = 0$$

$$L_i = L_f$$

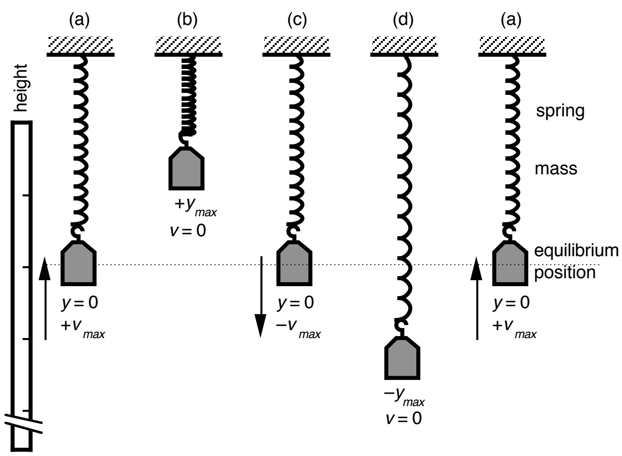
$$(I\omega)_i = (I\omega)_f$$

$$\omega_f = \frac{(I\omega)_i}{I_f} = \frac{100 \bullet 2.0}{75} = 2.7 rps$$

We're now going to move onto a new topic: oscillations. We can start by examining a system we have some familiarity with: a mass on a spring. Recall we learned about Hooke's Law, $F_s = -k\Delta x$. This means that when we have a mass on a spring, there are three factors affecting the force of the spring. First, the greater the displacement, the greater the force. Second, the stiffer the spring (k) the greater the force. Third, we know that the direction of the force opposes the displacement. If I displace the mass down, I feel a force upward (-). If I release the mass on the spring, it has a distinctive motion. Write down two adjectives to describe the motion.

It is called Simple Harmonic Motion. We see that it is smooth, not choppy, and it repeats itself over and over (harmonic). It has a period. We can

easily find the period of the motion by timing how long it takes to repeat its motion. We can also say other things about the motion.
Let's make a list:



$v_{\max}, v_{\min}, a_{\max}, a_{\min}, x_{\max}, x_{\min}$

One important point to make is that SHM requires a restoring force. This means a force that constantly tries to restore the system back to equilibrium. For the mass on the spring, we definitely have this. If we stretch the spring down, it pulls up toward equilibrium and vice versa!

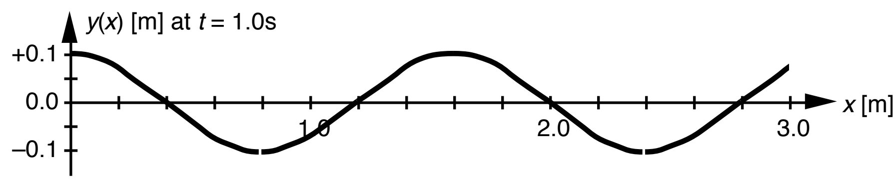
Again, we call the maximum displacement the Amplitude. It is determined by the initial conditions of the system (i.e. how far I decide to pull the spring down).

If we look at the motion of the mass on the spring over time, it has a distinctive motion. http://www.phy.ntnu.edu.tw/java/shm/shm.html (Figure 13.5, p.451) It is a sinusoidal (cosine or sine) curve. Notice that a sinusoidal curve has a period, i.e. it repeats its shape repeatedly. It also has an amplitude. Though your book uses a sine function, we're going to use a cosine function to describe the motion. The function tells us the position of the mass as a function of time.

$$y = A\cos\left(\frac{2\pi t}{T}\right) = A\cos(2\pi f t)$$

Notice that at t = 0s, y = A. Our system starts stretched at amplitude. When t = T/2, y = 0, our system is passing through equilibrium. So how would

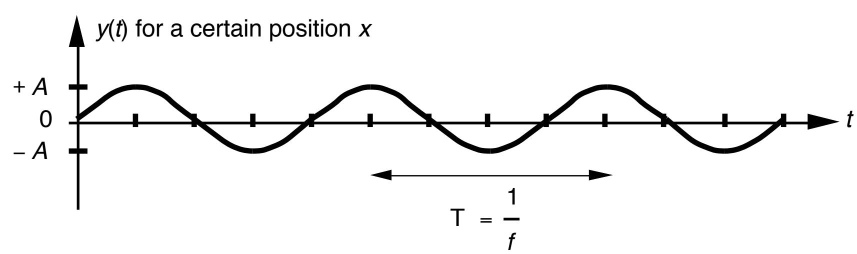
we draw this generic function? We know what it is generally going to look
his case),



We also have functions to describe the velocity and acceleration of the mass: (surprise, they're sinusoidal too!)
for the velocity:

$$v = \frac{A}{T}\sin(\frac{2\pi t}{T}) = v_{\max}\sin(\frac{2\pi t}{T})$$

The main difference between the position and velocity is that we have this sine function –



Notice we've started our function at zero instead of a maximum. When the position is at a maximum, the velocity is zero. When the velocity is at a max, the displacement is zero.

We can also talk about acceleration. The function is again a cosine function:

$$a = \frac{A}{T^2}\cos(\frac{2\pi t}{T}) = a_{\max}\cos(\frac{2\pi t}{T})$$

The maximum acceleration of the system is A/T^2. Also notice that the acceleration is a maximum when the spring is at its maximum stretch. This makes sense right, if you consider that the maximum force is also at that position.

Another system that exhibits SHM is a pendulum. We can talk about all the same properties for a pendulum, i.e. period, amplitude, etc. All the same sinusoidal motion equations apply. The main difference is that we express the position as θ (angle) instead of y (length). In this situation as in the mass-spring system we have a restoring force. For a pendulum the restoring force is gravity – for the mass-spring system, the restoring force is the force of the spring.