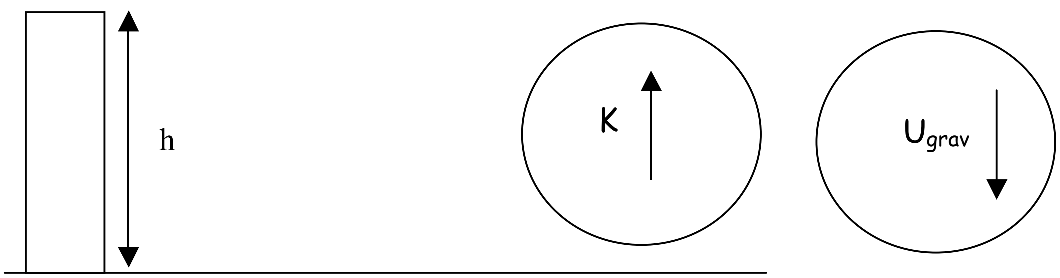
Lecture 15 Energy

More practice with conservation of energy
Conservation of Momentum (Elastic vs. Inelastic Collisions)

Now that we understand a little about conservation of energy, the best way to gain facility with it is to practice problems.

Example 1
Two balls, one twice as heavy as the other, are dropped from rest from the roof of a building. Just before hitting the ground, the heavier ball has
a. one-half  b. the same  c. twice  d. four times
the kinetic energy of the lighter ball.




$$\Delta PE + \Delta KE = 0$$

$$mg\Delta y + \frac{1}{2}m\Delta v^2 = 0$$

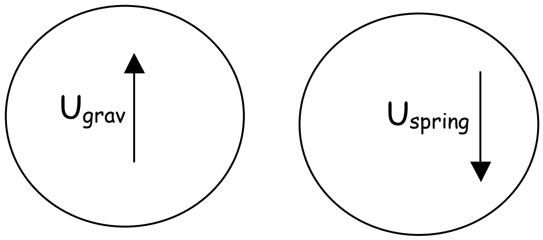
$$\frac{1}{2}\Delta v^2 = -g\Delta y$$

$$v_f^2 - v_i^2 = -2g\Delta y$$

$$v_f = \sqrt{-2g\Delta y}$$

Since both balls fall the same distance, they have to have the same final velocity. We have proven this through conservation of energy!

Example 2
A spring-loaded gun shoots a ball 12 m straight up into the air. The ball is shot again, but this time the spring is compressed only half as far. If air resistance and friction are negligible, the new height of the ball will be:
a. 3m  b. 6m  c. 12m  d. 24m  e. 48m
We can again use conservation of energy in this problem:




All the initial energy of compression goes into gravitational potential.

$$\Delta U_{grav} + \Delta U_{spring} = 0$$

$$\Delta U_{grav} = -\Delta U_{spring}$$

$$mg\Delta y = -\frac{1}{2}k\Delta x^2$$

$$\Delta y = -\frac{k\Delta x^2}{2mg} = 12m$$

But now if we compress it half as much $\Delta x_2 = \Delta x_1/2$ . So we can solve for the new $\Delta y$.

$$\Delta y_1 = -\frac{k\Delta x_1^2}{2mg} = 12m$$

$$\Delta y_2 = -\frac{k\Delta x_2^2}{2mg} = -\frac{k(\Delta x_1/2)^2}{2mg} = \frac{1}{4}\left(-\frac{k(\Delta x_1)^2}{2mg}\right) = \frac{1}{4}(12m) = 3m$$

So if you compress it half as far, you only get the ball to go a fourth as high. Notice we didn't even need all the numbers. We never knew the spring constant or the mass.

Something we haven't talked a lot about is reference level (where $U_{grav} = 0$). It actually doesn't matter where you put it. We've mostly considered the floor to be the reference level, but we could just as easily use the ceiling. What matters is $\Delta U$, and that stays the same no matter where $U_{grav} = 0$.

Let's do an example using different reference levels.

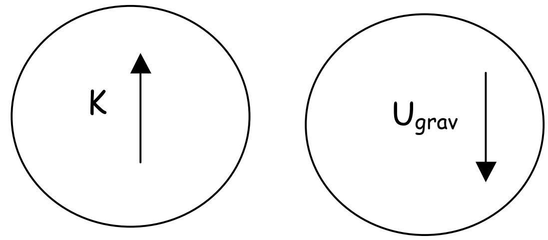
A ball (m = 0.2kg) is dropped from rest from a height of 2m. Find the speed of the ball just before it hits the ground.

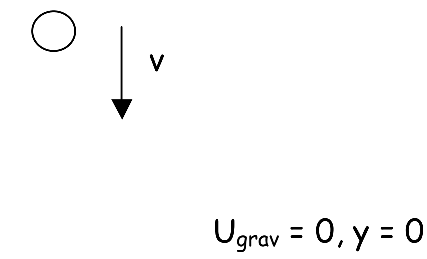
$$\Delta K + \Delta U = 0$$

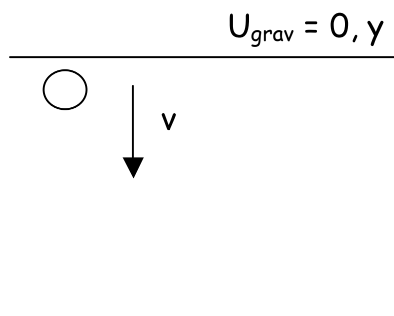
$$\Delta K = -\Delta U$$

$$\frac{1}{2}m\Delta v^2 = -mg\Delta y$$

$$v_f = \sqrt{-2g\Delta y}$$




$$\Delta y = y_f - y_i = 0m - 2m = -2m \qquad \Delta y = y_f - y_i = -2m - 0 = -2m$$

So we can see, regardless of choice of reference level $\Delta y$ is the same, which makes our $\Delta U$ the same.

$$\Delta K + \Delta U = 0$$

$$\Delta K = -\Delta U$$

$$\frac{1}{2}m\Delta v^2 = -mg\Delta y$$

$$v_f = \sqrt{-2g\Delta y} = \sqrt{-2(9.8m/s^2)(-2m)} = 6.3m/s$$

Another issue we haven't dealt with yet is what to do when a force like friction does work. Work is a transfer of energy, so if we're dealing with conservation of energy we have to include it somehow. I'm going to write down a more complete statement of conservation of energy now:

$$\Delta K + \Delta U + \Delta E_{therm} = W_{ext}$$

This formula now includes all the possible energy transformations and transfers that can occur. So far we've only analyzed systems where conservative forces and no external forces have been present. Now we'll introduce non-conservative forces and external forces.

Conservative and non-conservative forces are defined with respect to the work they do. A conservative force can do work to transfer energy from one system to another (gravity can do work to transfer energy from potential to kinetic, spring forces are also conservative) so that mechanical energy can be conserved. A non-conservative force can do work to transfer energy from the system to the environment (friction can do work to transfer energy to heat the environment) so that mechanical energy cannot be conserved. External forces (like a push or pull) transfer energy into the system, again mechanical energy cannot be conserved under an external force.

Let's go through some scenarios and see if we can list the basic elements:

- which objects are included in a system
- conservative interaction forces, non-conservative interaction forces, external forces
- Specify the value of $\Delta K$, $\Delta U$, $W_{nc}$, $W_{ext}$.
- Energy transformations
- Energy transfers

Pushing a block across a table (with friction) at constant speed.

objects in system: block

external force: push, non-conservative force: friction

$\Delta K = 0$, $\Delta U = 0$, $W_{nc} < 0$, $W_{ext} > 0$

The work done by the push is positive (energy going into system), the work done by friction is negative (energy leaving the system). These contributions are equal and opposite.

The block sliding to a halt after you release it.

objects in system: block

non-conservative force: friction

$\Delta K < 0$, $\Delta U = 0$, $W_{nc} < 0$, $W_{ext} = 0$

Energy is transferred from the kinetic energy system out to the environment through the work done by friction.

Picking up the block and placing it on a high ledge.
objects in system: block
external force: lifting force,
$\Delta K = 0$, $\Delta U > 0$, $W_c > 0$, $W_{nc} = 0$, $W_{ext} > 0$
There is no change in kinetic because initial and final velocity are zero. The positive work done by the lifting force is transferred into the gravitational potential energy system by the conservative force of gravity.

Recall that we discussed collisions briefly with conservation of momentum. But we never discussed the difference between elastic and inelastic collisions. But now that we know about energy, we can understand the difference.

In an elastic collision, the total <u>kinetic</u> energy of a system is conserved. For instance, two billiard balls colliding represents an elastic collision. Kinetic energy may be exchanged between the two balls, but $K_i = K_f$. (Figure 6.13 pp.197)

In an inelastic collision, the total <u>kinetic</u> energy of a system is not conserved. Energy is lost through work done by non-conservative forces. Heat, sound, or light may be generated. You may permanently deform one of the objects involved in the collision. You can think of a clay ball colliding with a billiard ball. In an inelastic collision $K_f < K_i$. (Figure 6.12 pp.195)

Now let's do an example working with an elastic collision:
A 0.30 kg object with a speed of 2.0m/s in the positive x-direction has a head-on elastic collision with a stationary 0.70kg object located at x = 0. What is the distance separating the objects 2.5s after the collision?

What we need to find is the velocity of each object after the collision. If this is an elastic collision (free of any external forces) we can say two things:

$P_i = P_f$

$K_i = K_f$

Now let's determine what we know and don't know:

$(v_a)_i = 2.0 m/s$

$(v_b)_i = 0 m/s$

$m_a = 0.30 kg$

$m_b = 0.70 kg$

$x_i = 0 m$

$\Delta t = 2.5 s$

Now we can expand our equations from above:

$K_i = K_f$

$$\frac{1}{2} m_a (v_a^2)_i + \frac{1}{2} m_b (v_b^2)_i = \frac{1}{2} m_a (v_a^2)_f + \frac{1}{2} m_b (v_b^2)_f$$

$$\frac{1}{2} m_a (v_a^2)_i = \frac{1}{2} m_a (v_a^2)_f + \frac{1}{2} m_b (v_b^2)_f$$

$P_i = P_f$

$$\left(m_a v_a\right)_i + \left(m_b v_b\right)_i = \left(m_a v_a\right)_f + \left(m_b v_b\right)_f$$

$$\left(m_a v_a\right)_i = \left(m_a v_a\right)_f + \left(m_b v_b\right)_f$$