Lecture 10 Impulse and Momentum

We are moving on from Force and Motion to new, but related subjects -- Impulse and Momentum.

The first major point to make is that from now on a lot of our discussion will revolve around laws of conservation. Physicists love when something is conserved, because that means that from the initial point to the final point of a problem, something is constant. For instance we can talk about the conservation of mass.

Let's do a quick example to demonstrate this. If I take two beakers, one with 100g of alcohol and one with 100g of water, we can predict the mass of the final state when I mix them together, right? It should be 200g. Let's try this out.

Indeed, in our initial state we have 200g (weighing the two quantities separately), then mixing them together in the final state (after mixing) we still have 200g! In this way we have proven to ourselves the conservation of mass. Mass is conserved, $m_i = m_f$.

Why did you expect mass to be conserved?

- your anecdotal experience tells you that mass is conserved
- there is no obvious mechanism for why mass would not be conserved

It turns out there are less obvious quantities that are conserved in nature as well. One that we'll discuss immediately is momentum. Something that we'll discuss in upcoming lectures is the conservation of energy. So now that we've dropped this name, momentum, several times already we should define it. Let's do some brainstorming, since you already have ideas about what momentum is. Take a couple of minutes to come up with some ideas that we'll list up on the board.

So one of the concepts we think of when we think of momentum is a tendency to keep going. We think of something big moving fast like a speeding bus, or a big lineman charging toward a quarterback. These ideas do relate to the actual physics definition of momentum.

The definition of momentum is: $\vec{p} = m\vec{v}$

From the definition we can note several obvious things:

- the variable we use to represent moment is p
- it is a vector quantity
- it has units of $kg \bullet \frac{m}{s}$
- it depends directly on mass and velocity, i.e. the bigger something is the more momentum it has, and the faster it goes the more momentum it has

So our previous ideas of momentum (i.e. the linebacker and the bus) are consistent with the actual definition of momentum.

We can do an example now using momentum:

Superman (m = 100 kg) is leisurely flying to New York City from California. If he flies at 120 m/s, what is his momentum?

$$\vec{p} = m\vec{v}$$
$$= 100kg(125\frac{m}{s}) = 12{,}500\, kg \bullet \frac{m}{s}$$

direction is east

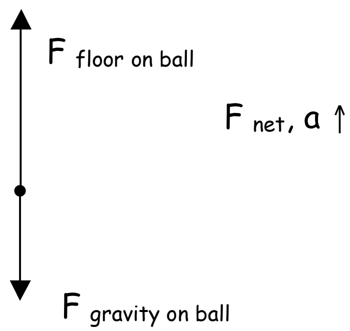
So now that we have defined momentum, we're ready to talk about something called impulse. Impulse is connected to the duration of a force. If Superman steps in front of a speeding bus and tries to stop it (change it's momentum in other words), he has to apply a force. Let's say he applies a big force. Then it doesn't take him as long to stop the bus. If he applies a smaller force, it takes him longer obviously. To summarize:

Δp	F	Δt
same	↑	↓
same	↓	↑

So to achieve the same change in momentum we have two variables to play with. If we increase the force it takes less time to stop the bus. If we decrease the force it takes longer.

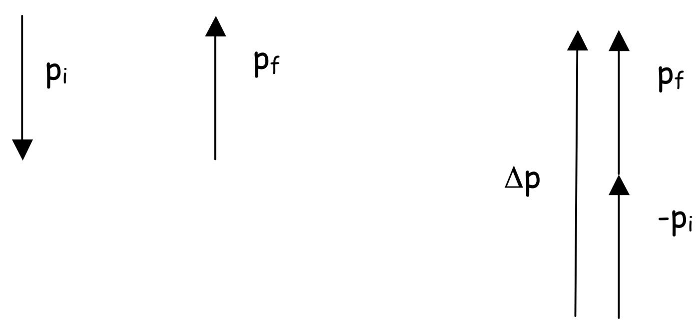
So now we can define impulse precisely: $\vec{J} = \vec{F}_{avg}\Delta t$. Impulse is a (vector) quantity that utilizes this relationship we've explored. But beyond that, and more importantly, it tells us about changes in momentum. In fact we can expand our definition: $\vec{J} = \vec{F}_{avg}\Delta t = \Delta\vec{p}$. The conclusion we can draw from this is that impulse, namely a force exerted over time causes changes in

momentum. This gives us a whole new way to think of old problems we analyzed with Newton's Laws.
Take a ball bouncing off of the ground. We can draw a force diagram for the ball: (at the point when the ball hits the floor)



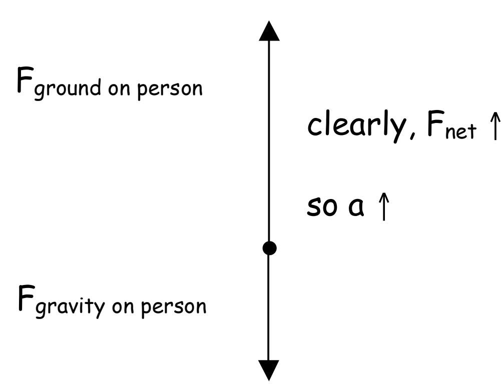
Clearly from a force standpoint the net force is pointed upward (since the ball has to accelerate up) and so the Normal force must be greater than the force of gravity.

Now let's look at this from an impulse standpoint. What is Δp? If we choose the moment before it hits the floor (p_i) and the moment after it leaves the floor (p_f), we can then find $\Delta p = p_f - p_i$.



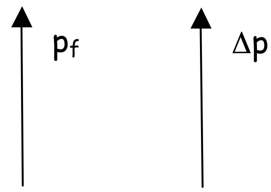
Recall that impulse is $F_{avg}\Delta t = \Delta p$. So these two pictures are consistent. We've determined that the force had to be up and we can clearly see that the change in momentum is also up. This makes great sense since the force and change in momentum are directly related by the definition of impulse, they should be in the same direction.

Another good example to analyze is someone jumping off of the ground. First, let's draw a force diagram:



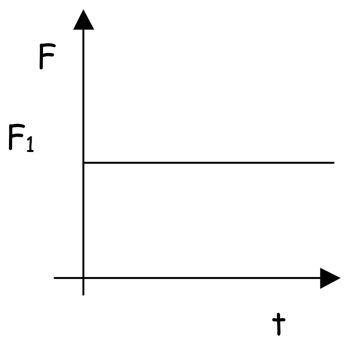
Now we want to think of this in terms of impulse:

$p_i = 0$,



Again, we see a direct correlation between change in momentum and force. Both pictures are consistent…

Another thing that we like to do is to plot Force vs. time. If we look at a simple graph:



Now let's solve for the area:

area = $F_1\Delta t$ ⟵ this is just impulse, or Δp!

So to summarize, the area under a force vs. time curve gives us impulse, or Δp. So let's try an example:

A golfer drives a 0.10 kg ball from an elevated tee, giving the ball an initial horizontal speed of 40 m/s. The club and ball are in contact for 1.0 ms. What is the average force exerted by the club on the ball during this time?

$F_{avg}\Delta t = p - p_o = mv - mv_o$