7.2

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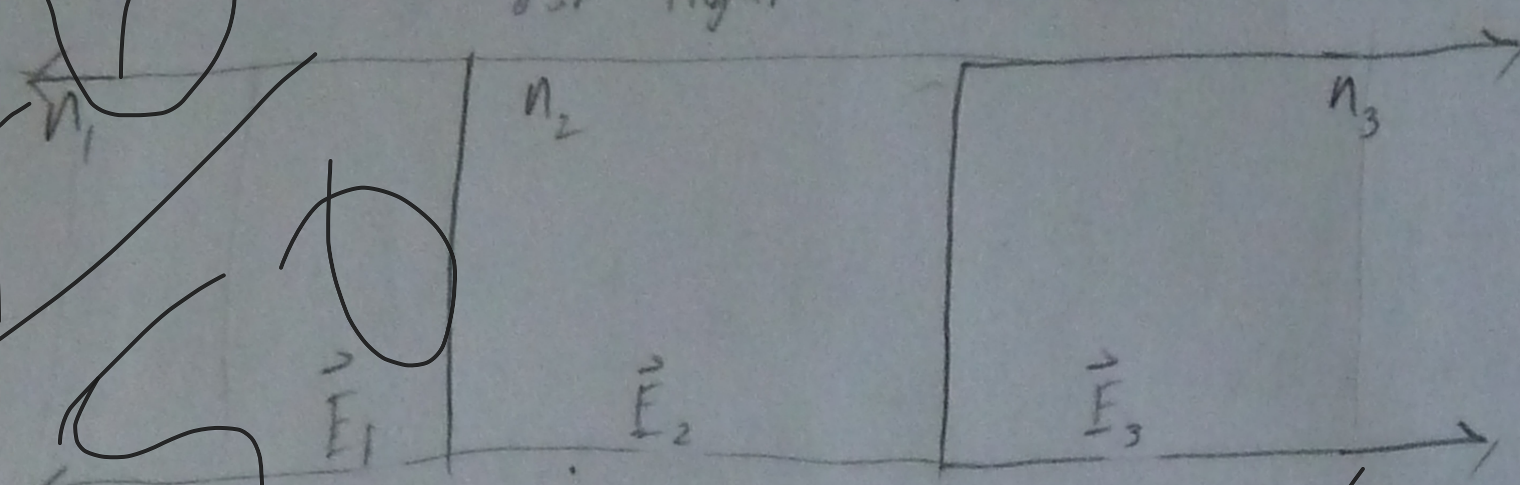
Assume without loss of generality that our light is polarized



why the same ω

$$\vec{E}_1 = \left(E_i e^{i(k_1 z - \omega t)} + E_r e^{-i(k_1 z + \omega t)}\right)\hat{e}_1$$

polarization vector

$$\vec{E}_2 = \left(E_+ e^{i(k_2 z - \omega t)} + E_- e^{-i(k_2 z + \omega t)}\right)\hat{e}_1$$

$$\vec{E}_3 = E_t e^{i(k_3 z - \omega t)}\hat{e}_1$$

where $K_1 = \frac{n_1}{c}\omega$ $\quad K_2 = \frac{n_2}{c}\omega$ $\quad K_3 = \frac{n_3}{c}\omega$

Our boundary conditions are

$E_1(0) = E_2(0)$ $\qquad \frac{1}{\mu_0}\left(K_1 E_i - K_1 E_r\right) = \frac{1}{\mu_0}\left(K_2 E_+ - K_2 E_-\right)$

$E_2(d) = E_3(d)$ $\qquad \frac{1}{\mu_0}\left(K_2 E_+ e^{ik_2 d} - K_2 E_- e^{-ik_2 d}\right) = \frac{1}{\mu_0}\left(K_3 e^{ik_3 d}\right)E_t$

(where time dependance cancelled out of all terms)

① $E_i + E_r = E_+ + E_-$

② $K_1(E_i - E_r) = K_2(E_+ - E_-)$

③ $E_+ e^{ik_2 d} + E_- e^{-ik_2 d} = E_t e^{ik_3 d}$

④ $K_2(E_+ e^{ik_2 d} - E_- e^{-ik_2 d}) = K_3 E_t e^{ik_3 d}$

7.2 cont.

combine (1) + (2) to eliminate E_r

(5) $2E_i = E_+ + E_- + \frac{k_2}{k_1}(E_+ - E_-) \Rightarrow E_+\left(1+\frac{k_2}{k_1}\right) = 2E_i - E_- + \frac{k_2}{k_1}E_-$

$$E_+ = \frac{1}{\left(1+\frac{k_2}{k_1}\right)}\left(2E_i + E_-\left(\frac{k_2}{k_1}-1\right)\right) = \frac{2E_ik_1 + E_-(k_2-k_1)}{k_1+k_2} \quad (5')$$

combine 3 + 4 to Eliminate E_τ

(6) $E_+e^{ik_2d} + E_-e^{-ik_2d} - \frac{k_2}{k_3}\left(E_+e^{ik_2d} - E_-e^{-ik_2d}\right) = 0$

$$E_+\left(e^{ik_2d} - \frac{k_2}{k_3}e^{ik_2d}\right) = -E_-\left(e^{-ik_2d} + \frac{k_2}{k_3}e^{-ik_2d}\right)$$

$$E_+ = -E_-\left(\frac{1+\frac{k_2}{k_3}}{1-\frac{k_2}{k_3}}\right)e^{-2ik_2d}$$

$$E_+ = -E_-\left(\frac{k_2+k_3}{k_2-k_3}\right)e^{-2ik_2d} \quad (6')$$

set 5' and 6' equal to eliminate E_+

$$\frac{2E_ik_1 + E_-(k_2-k_1)}{k_1+k_2} = E_-\left(\frac{k_2+k_3}{k_2-k_3}\right)e^{-2ik_2d}$$

$$\frac{2E_ik_1}{k_1+k_2} = E_-\left(\frac{k_2+k_3}{k_2-k_3}e^{-ik_2d} - \frac{k_2-k_1}{k_1+k_2}\right)$$

$$\frac{2E_ik_1}{k_1+k_2} = E_-\left(\frac{e^{-2ik_2d}(k_2+k_3)(k_1+k_2) - (k_2-k_1)(k_2-k_3)}{(k_2-k_3)(k_1+k_2)}\right)$$

7.2 cont.

$$E_- = \frac{2E_i k_1 (k_2 - k_3)}{e^{-2ik_2 d}(k_2+k_3)(k_1+k_2) - (k_2-k_1)(k_2-k_3)}$$

combine ① and ② to eliminate E_+

⑤ $E_i + E_r - \frac{k_1}{k_2}(E_i - E_r) = 2E_-$

$$E_r\left(1+\frac{k_1}{k_2}\right) = 2E_- - E_i\left(1-\frac{k_1}{k_2}\right)$$

$$E_r(k_2+k_1) = 2k_2 E_- - E_i(k_2-k_1)$$

$$E_r = \frac{2k_2}{k_2+k_1}E_- - \frac{(k_2-k_1)}{k_2+k_1}E_i$$ plug in E_-

$$= \frac{4k_2 k_1 (k_2-k_3)E_i}{(k_2+k_1)\left((k_2+k_3)(k_1+k_2)e^{-2ik_2 d} - (k_2-k_1)(k_2-k_3)\right)} - \frac{k_2-k_1}{k_2+k_1}E_i$$

$$E_r = \frac{E_i}{k_2+k_1}\left[\frac{4k_2k_1(k_2-k_3)}{(k_2+k_3)(k_1+k_2)e^{-2ik_2 d} - (k_2-k_1)(k_2-k_3)} - (k_2-k_1)\right]$$

Simplify please

7.2 cont

$$R = |E_r|^2 \qquad \text{with} \quad k_1 = \frac{n_1}{c}\omega \quad k_2 = \frac{n_2}{c}\omega$$

$$k_3 = \frac{n_3}{c}\omega$$

$$T = 1 - R$$

```
e[n1_, n2_, n3_, d_, w_] :=
```

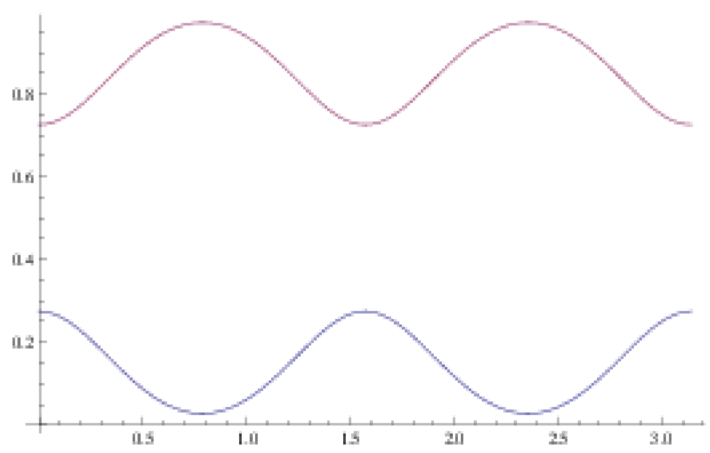
$$\mathrm{Abs}\left[\left(\frac{1}{(n1+n2)}\right)*\left(\frac{4*n1*n2*(n2-n3)}{\left((n2+n3)\,(n1+n2)\,e^{-2\,I\,w\,n2\,d/c}-(n2-n1)\,(n2-n1)\right)}-(n2-n1)\right)\right]^2$$

```
c = 1
```

1

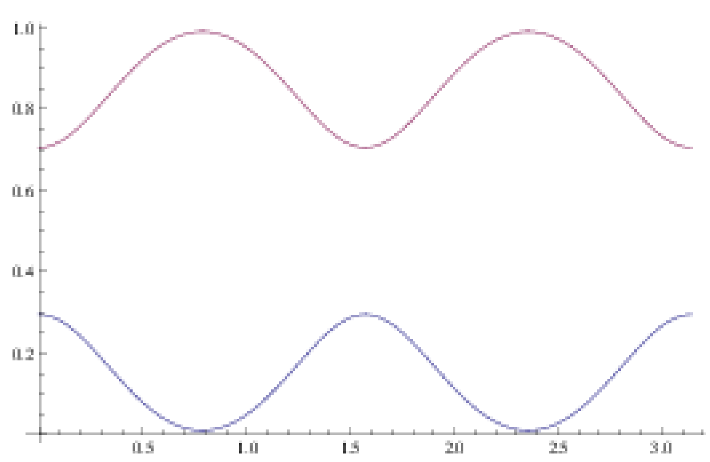
With n1 = 1, n2 = 2, n3 = 3

```
Plot[{e[1, 2, 3, 1, x], 1 - e[1, 2, 3, 1, x]}, {x, 0, Pi}]
```



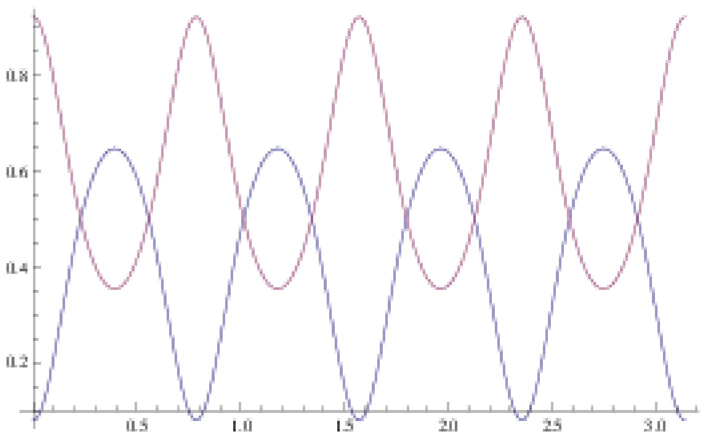
With n1 = 3, n2 = 2, n3 = 1

```
Plot[{e[3, 2, 1, 1, x], 1 - e[3, 2, 1, 1, x]}, {x, 0, Pi}]
```



With n1 = 2, n2 = 4, n3 = 1

```
Plot[{e[2, 4, 1, 1, x], 1 - e[2, 4, 1, 1, x]}, {x, 0, Pi}]
```



what happens if
$d = 0$? or
$n_1 = n_2 = n_3$ or
n_2 very very large
Does this simplify to known results?

Meaning you have to take extreme case what if n2 goes to infinity then R =1 or T=0. Does your result reproduce that?

this is a prelim tip. once you get a solution you are not sure whether is correct or not you need to filter or test it.

7.2 b) cont.

(imaginary part)

$$0 = +(n_2-n_1)(n_2+n_3)(n_1+n_2)\sin(2\omega n_2 d/c)$$

so $2\omega n_2 d/c = m\pi$ where $m \in \mathbb{Z}$

$$\boxed{d = \frac{m\pi c}{2\omega n_2}}$$

back to the real part:

note that we can now insert our solution for d,

to see that $\cos(2\omega n_2 d/c) = \cos(m\pi) = (-1)^m$

so our eqn becomes

$$0 = 4n_1n_2(n_2-n_3) - (n_2-n_1)(n_2+n_3)(n_1+n_2)(-1)^m + (n_2-n_1)^2(n_2-n_3)$$

$$= (n_2-n_3)\left(4n_1n_2 + (n_2-n_1)^2\right) + (n_1-n_2)(n_2+n_3)(n_1+n_2)(-1)^m$$

$$= (n_2-n_3)(4n_1n_2 + n_2^2 + n_1^2 - 2n_1n_2) + (n_1-n_2)(n_2+n_3)(n_1+n_2)(-1)^m$$

$$= (n_2-n_3)(n_1+n_2)^2 + (n_1+n_2)(n_1-n_2)(n_2+n_3)(-1)^m$$

$$= (n_2-n_3)(n_1+n_2) + (n_1-n_2)(n_2+n_3)(-1)^m$$

7.2 b)

to set $R=0$,

set $E_r = 0$

so $$\frac{E_i}{k_2+k_1}\left[\frac{4k_2k_1(k_2-k_3)}{(k_2+k_3)(k_1+k_2)e^{-2ik_2d}-(k_2-k_1)(k_2-k_3)} - (k_2-k_1)\right] = 0$$

$$0 = \frac{c}{(n_1+n_2)w}\left[\frac{4n_2n_1w^3(n_2-n_3)}{cw^2\left[(n_2+n_3)(n_1+n_2)e^{-2iwn_2d/c}-(n_2-n_1)(n_2-n_3)\right]} - \frac{w}{c}(n_2-n_1)\right]$$

$$0 = \left[\frac{4n_1n_2(n_2-n_3)}{(n_2+n_3)(n_1+n_2)e^{-2iwn_2d/c}-(n_2-n_1)(n_2-n_3)} - (n_2-n_1)\right]$$

$$0 = 4n_1n_2(n_2-n_3) - (n_2-n_1)\left[(n_2+n_3)(n_1+n_2)e^{-2iwn_2d/c} - (n_2-n_1)(n_2-n_3)\right]$$

Lets take real and imaginary parts

$$0 = 4n_1n_2(n_2-n_3) - (n_2-n_1)\left[(n_2+n_3)(n_1+n_2)\cos(2wn_2d/c) - (n_2-n_1)(n_2-n_3)\right]$$

(real part)

7.2 b) cont.

for M even:

$$0 = n_2 n_1 + n_2^2 - n_3 n_1 - n_3 n_2 + n_1 n_2 + n_1 n_3 - n_2^2 - n_2 n_3$$

$$0 = 2(n_2 n_1 - n_3 n_2)$$

$$0 = n_2(n_1 - n_3)$$

$n_2 = 0$ (unphysical)

for M odd

$$0 = n_2 n_1 + n_2^2 - n_3 n_1 - n_3 n_2 - n_1 n_2 - n_1 n_3 + n_2^2 + n_2 n_3$$

$$= n_2^2 - n_3 n_1$$

$$n_2 = \sqrt{n_1 n_3}$$

7.7 a)

in fourier space

$$\vec{A}(k, z, t) = \vec{\varepsilon} A(k) e^{ikz - i\omega t}$$

Thus in real space

$$\vec{E}(x, z, t) = \vec{\varepsilon} \int dk A(k) e^{ikx} e^{ikz - i\omega t}$$

$$= \vec{\varepsilon} \int dk A(k) e^{ikx + ikz - i\omega t}$$

$\frac{10}{50}$