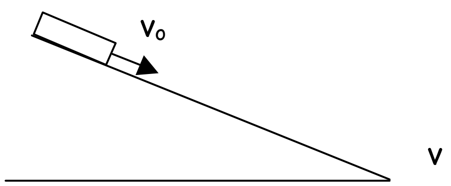
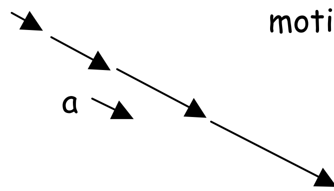
Lecture 7 Force and Motion

Practice with Free-body Diagrams and Newton's Laws

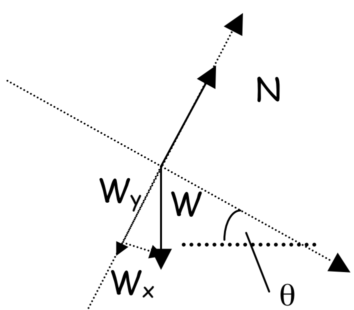
So now that we've practiced with Newton's Laws, we can go back to combine kinematics with Newton's Laws in this example.

Example 1:
Sammy Skier (75kg) starts down a 50m high, 10° slope on frictionless skis. What is his speed at the bottom?





motion diagram



What we need to do is solve for the acceleration. With the acceleration we can use one of the kinematic equations to solve for the velocity.

$(F_{net})_x = ma_x$

$= W_x = W\sin\theta = mg\sin\theta = ma_x$

$a_x = g\sin\theta = (9.8\text{m/s}^2)(\sin 10°) = 1.7\text{m/s}^2$

Now we can write down our known and unknowns:

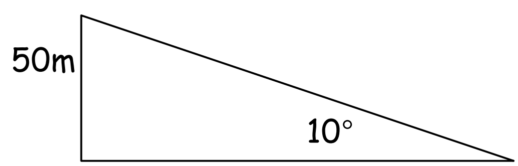
$ax = 1.7\text{m/s}^2$

$x_i = 0\text{m}$

$x_f = 50/\sin 10° = 287\text{m}$

$v_i = 0\text{m/s}$

$v_f = ?$



kinematic equations for constant acceleration

$$\bar{v} = \frac{v + v_o}{2}$$

$$v = v_o + a\Delta t$$

$$x = x_o + \frac{1}{2}(v + v_o)\Delta t$$

$$x = x_o + v_o\Delta t + \frac{1}{2}a\Delta t^2$$

$$v^2 = v_o^2 + 2a\Delta x$$

From this list the appropriate equation is:

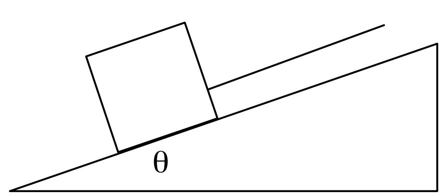
$$v^2 = v_o^2 + 2a\Delta x$$

$$v = \sqrt{v_o^2 + 2a\Delta x} = \sqrt{0 + 2(1.7m/s^2)(287m)} = 31m/s$$

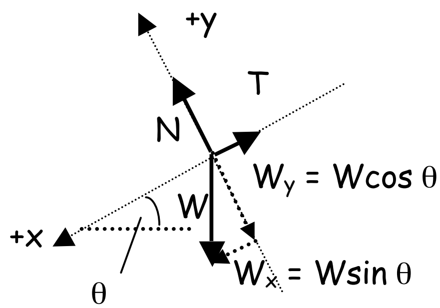
We could then ask if the answer is reasonable. The answer is yes. The units match. And it has the right order of magnitude. It is not unreasonable to go that fast as a skier.

What we've just worked is a classic inclined plane problem, which means that it is a force problem with a tilted coordinate system. This makes analyzing the forces a bit more challenging, but it's easy once you've practiced it a few times.

Example 2:
Burglars are trying to haul a 1000 kg safe up a frictionless ramp to their getaway truck. The ramp is tilted at angle θ. a) What is the tension in the rope if the safe is at rest? b) If the safe is moving up a ramp at a steady 1 m/s? c) If the safe is accelerating up the ramp at 1 m/s^2?



a) For the safe at rest, first we draw our force diagram:



Now we can make a statement of Newton's Second Law, keeping in mind that acceleration in each direction is zero → $a_x = a_y = 0$:

$$(F_{net})_y = N - W_y = N - W\cos\theta = 0$$

$$so,\ N = W\cos\theta = mg\cos\theta = (1000kg)(9.8m/s^2)(\cos 10^\circ) = 9850N$$

Now let's utilize Newton's Second Law in the x - direction :

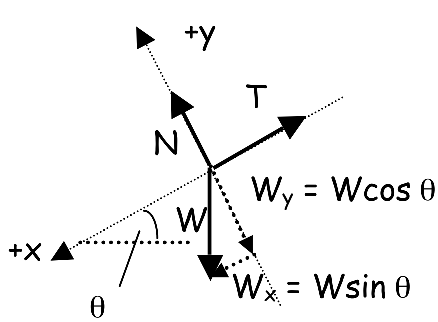
$$(F_{net})_x = W_x - T = W\sin\theta - T = 0$$

$$so,\ T = W\sin\theta = mg\sin\theta = (1000kg)(9.8m/s^2)(\sin 10^\circ) = 1700N$$

b) Now the safe is at constant speed of 1m/s. How does this change the problem?

Well, in the y-direction it is immediately clear that $a_y = 0$ because there's no motion in that direction. So that is the same as the first problem. What about a_x? Well if velocity isn't changing, then $\Delta v = 0$ and $a_x = 0$. This is the same as the first problem. The force diagram is the same, and also the statement of Newton's Second Law is the same – so T = 1700N in this problem too!

c) Now the safe is accelerating at 1m/s^2, what is the tension now?



$a_y = 0$ *still, so*:

$$(F_{net})_y = N - W_y = N - W\cos\theta = 0$$

$$so,\ N = W\cos\theta = mg\cos\theta = (1000kg)(9.8m/s^2)(\cos 10^\circ) = 9850N$$

Now let's utilize Newton's Second Law in the x - direction:

note: a_x *is a negative number because the safe is moving up the ramp, and we've designated down the ramp as positive*

$$(F_{net})_x = W_x - T = W\sin\theta - T = ma_x$$

$$so,\ T = W\sin\theta - ma_x = mg\sin\theta - ma_x = 1000kg(9.8m/s^2(\sin 10^\circ) + 1.0m/s^2) = 2700N$$

This number is reasonable considering it took 1700 N to just keep the safe on the ramp not accelerating. It makes sense that it takes more force to accelerate it up the ramp.

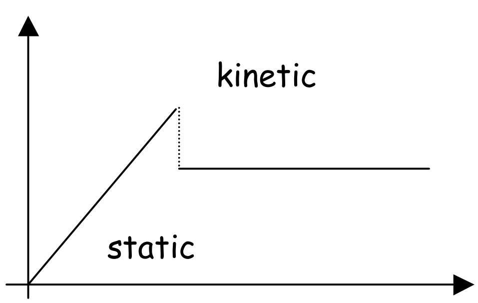
In that problem, we had a frictionless ramp, but what if we have to solve a problem that involves a ramp that is not frictionless? How can we calculate friction?

Let's assess what we already know about friction:

- it is a contact force, it requires contact between an agent of force and the object experiencing the force
- it acts opposite the direction of motion

It turns out that there are two types of friction: Static and Kinetic.

When something resists intended motion even though you are pushing on it, that is static friction. Static friction is a force that is variable. If I push on something and it does not move so I push harder and it still does not move, that is a function of static friction. Static friction can increase to a point, until the object moves and you have kinetic friction.



Notice the drop in friction from static to kinetic.
When you push something it doesn't move at first, until you push hard enough (static friction is resisting you). But when you get it going it requires less force (kinetic friction is resisting you).

How to calculate friction:
$f=\mu N$
μ is called the coefficient of friction.

The formula for kinetic friction is:
$f=\mu_k N$ where μ_k is the coefficient of kinetic friction.

The formula for kinetic friction is:
$f \le \mu_s N$ where μ_s is the coefficient of static friction.

You can see that the formula for kinetic friction gives you a constant value, where the coefficient of static friction has an upper limit and can be any value between 0 and $\mu_s N$ depending on how much it has to fight an applied force.

So to wrap up lecture let's try an example using friction:
A car traveling at 20 m/s stops in a distance of 50 m. Assume that the deceleration is constant. The coefficients of friction between a passenger and the seat are $\mu_k = 0.3$ and $\mu_s = 0.5$. Will a 70 kg passenger slide off the seat if not wearing a seat belt?

First we have to find the deceleration of the person which is the same as the deceleration of the car.

$$v_i = 20\ m/s$$

$$v_f = 0\ m/s$$

$$x_i = 0\ m$$

$$x_f = 50\ m$$

$$a = ?$$

Given what we know, the equation to use is:
$$v^2 = v_o^2 + 2a\Delta x$$

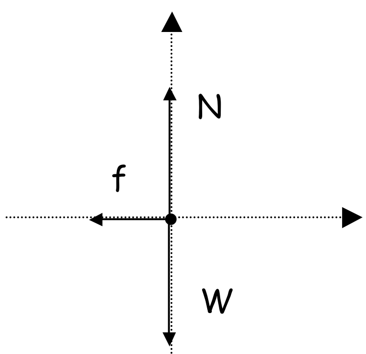
So now let's solve for a (which should be negative) since we're slowing down.

$$v^2 = v_o^2 + 2a\Delta x$$

$$v^2 - v_o^2 = 2a\Delta x$$

$$a = \frac{v^2 - v_o^2}{2\Delta x} = \frac{0 - (20\ m/s)^2}{2(50\ m)} = -4\ m/s^2$$

So we have a = -4 m/s^2. Now we can use Newton's Second Law to come up with the friction force on the passenger. What we need first is a free-body diagram:



$$(F_{net})_y = ma_y = 0$$

$$N - W = 0 \Rightarrow N = W$$

The maximum amount of friction available is from static friction.

$$f = \mu_s N = \mu_s mg = (0.5)(70kg)(9.8m/s^2) = 343N$$

Now we can calculate the amount of force to decelerate the person :

$$(F_{net})_x = ma_x = (70kg)(-4m/s^2) = -280N$$

Since the maximum amount of friction available is 343 N and the amount of force required to decelerate the person is 280 N, friction can easily keep the person from slipping off the seat.