Lecture 7 Force and Motion

Practice with Free-body Diagrams and Newton's Laws
Today we'll just work through as many examples as we can utilizing Newton's Laws and free-body diagrams.
Example 1:
An elevator is going up at a steady speed. Draw a free body diagram and write a statement of Newton's Second law. Is the tension force greater than equal to or less than the weight? Explain.


$$
\left(F_{\text {net }}\right)_{x}=m a_{x}=0
$$

since $v$ is constant $a_{x}=0$
$\mathrm{T}-\mathrm{W}=0$
$\Rightarrow T=W$

## Example 2:

Push a block across a table at steady speed. Since you're exerting a force on it, why isn't it accelerating? Identify all the forces and draw a free body diagram. How does the pushing force compare to the friction force? Is it greater than, less than, or equal to the friction?


## Example 3:

Push the same block quickly, then release it so it slides some distance before stopping. Draw a free body diagram, and determine the direction of acceleration based on Newton's Second Law.


So we're now ready to think about the problem of the ball tossed in the air. After it leaves your hand there is no contact force - so how does it move up? Well let's first ask the question why does it slow down? It slows down because the force of gravity is pulling it down. How does the acceleration point? How does the velocity point?


When acceleration and velocity point in opposite directions, we know an object has to be slowing down.
The idea of the ball in the air is the same as giving the box a push and letting it go. Did that require a constant contact force to make it go? No, it just required a one-time push. It would move forever in the absence of friction.
So finally we want to answer the question do we have to have a force up to make the ball travel up. The answer is no. Obviously there's nothing in contact with the ball, i.e. no contact force. But if we look at the motion just after the ball leaves the hand, we can invoke Newton's First Law. An object at rest tends to stay at rest, an object in motion tends to stay in motion at constant velocity, in the absence of an external force. Because the object has mass and you've given it a velocity by throwing it in the air, it will maintain that velocity in the absence of gravity.

## Example 4:

A 1000kg block hangs on a rope. Find the tension on the rope if the block is:
a) hanging on the rope
b) moving upward at a steady speed of $5 \mathrm{~m} / \mathrm{s}$
c) accelerating upward at $5 \mathrm{~m} / \mathrm{s}^{2}$
a) we know that it's just hanging, so there is no change in velocity, it's constantly zero, therefore, $\Delta v=0$, and acceleration is zero.

$\left(F_{\text {net }}\right)_{y}=m a_{y}$ since $v$ is not changing $a_{y}=0$
$\mathrm{T}-\mathrm{W}=\mathrm{ma} y=0$
$T=W=m g=(1000 \mathrm{~kg})\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)=9800 \mathrm{~N}$
b) moving upward with a speed of $5 \mathrm{~m} / \mathrm{s}$ :

In this case, it is moving so we could do a motion diagram:

From there we can draw a free-body diagram: If $a=0$ the free-body diagram should look exactly like the one before. In that problem it was moving with a constant velocity of $0 \mathrm{~m} / \mathrm{s}$. In this case it is moving with a constant velocity of $5 \mathrm{~m} / \mathrm{s}$.

$$
\left(F_{\text {net }}\right)_{y}=m a_{y}
$$


since $v$ is not changing $a_{y}=0$
$\mathrm{T}-\mathrm{W}=\mathrm{may}=0$
$T=W=m g=(1000 \mathrm{~kg})\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)=9800 \mathrm{~N}$
c) block is accelerating at $5 \mathrm{~m} / \mathrm{s}^{2}$

If we know that the block is accelerating upward, what can we say about
$F_{\text {net }}$ ? Since $a=F_{\text {net }} / m$, the direction of $a$ and $F_{\text {net }}$ must be the same.
Therefore $T$ must be greater than W.

$$
\begin{aligned}
& \left(F_{n e t}\right)_{y}=m a_{y} \\
& a_{y}=5 \mathrm{~m} / \mathrm{s}^{2} \\
& T-W=m a_{y} \\
& T=W+m a_{y}=m g+m a_{y} \\
& =1000 \mathrm{~kg}\left(9.8 \mathrm{~m} / \mathrm{s}^{2}+5 \mathrm{~m} / \mathrm{s}^{2}\right) \\
& =14800 \mathrm{~N}
\end{aligned}
$$

Example 5:
Find the components along the $x$ and $y$ axis of $W$ in each of these coordinate systems:




$W_{x}=W \sin \theta$
$W_{y}=W \cos \theta$


$$
\begin{aligned}
& W_{x}=0 \\
& W_{y}=W
\end{aligned}
$$

$$
W y=W \cos \theta
$$

$$
\begin{aligned}
& W_{y}=W \cos \theta \\
& W_{x}=W \sin \theta
\end{aligned}
$$



Example 6:
Sammy Skier ( 75 kg ) starts down a 50 m high, $10^{\circ}$ slope on frictionless skis. What is his speed at the bottom?



What we need to do is solve for the acceleration. With the acceleration we can use one of the kinematic equations to solve for the velocity.
$\left(F_{\text {net }}\right)_{x}=m a_{x}$
$=W_{x}=W \sin \theta=m g \sin \theta=m a_{x}$
$a_{x}=g \sin \theta=\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)\left(\sin 10^{\circ}\right)=1.7 \mathrm{~m} / \mathrm{s}^{2}$

Now we can write down our known and unknowns:
$a x=1.7 \mathrm{~m} / \mathrm{s} 2$
$x_{i}=0 \mathrm{~m}$
$x_{f}=50 / \sin 10^{\circ}=287 \mathrm{~m}$
$v_{i}=0 \mathrm{~m} / \mathrm{s}$
$\mathrm{v}_{\mathrm{f}}=$ ?

kinematic equations for constant acceleration
$\bar{v}=\frac{v+v_{o}}{2}$
$v=v_{o}+a \Delta t$
$x=x_{o}+\frac{1}{2}\left(v+v_{o}\right) \Delta t$
$x=x_{o}+v_{o} \Delta t+\frac{1}{2} a \Delta t^{2}$
$v^{2}=v_{o}^{2}+2 a \Delta x$

From this list the appropriate equation is:
$v^{2}=v_{o}^{2}+2 a \Delta x$
$v=\sqrt{v_{o}^{2}+2 a \Delta x}=\sqrt{0+2\left(1.7 \mathrm{~m} / \mathrm{s}^{2}\right)(287 \mathrm{~m})}=31 \mathrm{~m} / \mathrm{s}$
We could then ask if the answer is reasonable. The answer is yes. The units match. And it has the right order of magnitude. It is not unreasonable to go that fast as a skier.

