Lecture 5 Motion and Kinematics

Practice with Problem Solving / Using Dynamics Worksheets
Projectile Motion
Circular Motion

You are now well versed at drawing motion diagrams and kinematic graphs, the final piece you need to actually solve a problem is the mathematics.

A list of these equations is as follows (p.56):

*kinematic equations for constant acceleration*

$$\bar{v} = \frac{v + v_o}{2}$$

$$v = v_o + a\Delta t$$

$$x = x_o + \frac{1}{2}(v + v_o)\Delta t$$

$$x = x_o + v_o\Delta t + \frac{1}{2}a\Delta t^2$$

$$v^2 = v_o^2 + 2a\Delta x$$

Now we'll go through the actual process of solving a problem entirely.
A first example:
A train is moving at a steady 30 m/s. At t = 0s, the engine passes a light at x = 0. Determine the engine's position at t = 1s, also at t = 2s and t = 3s.

Take a moment to think about this to come up with the answer. We can then verify the answer using physics.
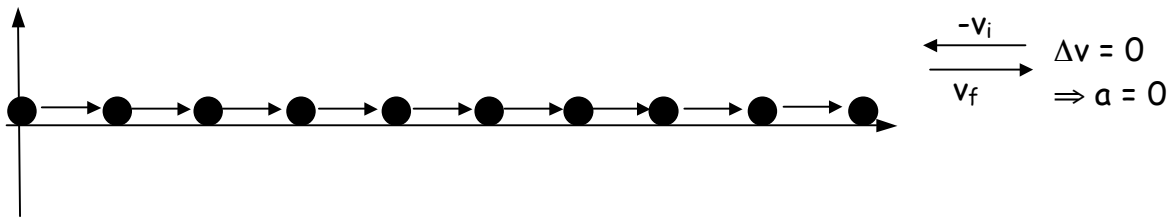
So the answer is easy right, if the train is moving at 30 m/s:
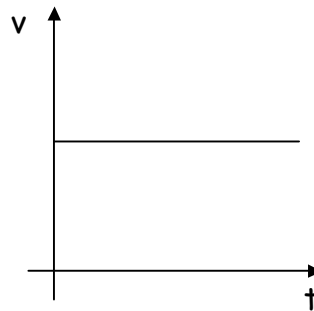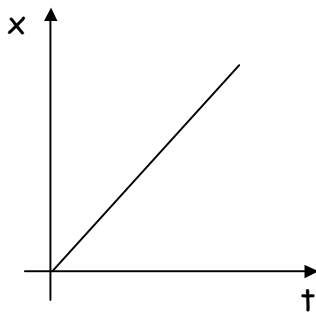after 1s it has moved 30 m
after 2s it has moved another 30 m for a total of 60 m
after 3s it has moved another 30 m for a total of 90 m

The point of this problem is to see if we can check this answer by doing the physics. The first step is to draw our motion diagram:



Easy so far. Now let's draw a displacement vs. time and velocity vs. time graph:



Now lets list the known information and the desired unknowns:

*kinematic equations for constant acceleration*

Known Information:
$v_i = v_f = 30$ m/s
$a = 0$
$x_o = 0$
$\Delta t = 1s, 2s, 3s$
Desired Unknowns:
$x_f$?

$$\bar{v} = \frac{v + v_o}{2}$$

$$v = v_o + a\Delta t$$

$$x = x_o + \frac{1}{2}(v + v_o)\Delta t$$

$$x = x_o + v_o\Delta t + \frac{1}{2}a\Delta t^2$$

$$v^2 = v_o^2 + 2a\Delta x$$

Finally, we list our assumptions and choose the right equation to use:
No special assumptions in this problem.

Now we can look at our list of equations and see which one involves all of our knowns and unknowns.

It looks like the one we need is:

$$x = x_o + \frac{1}{2}(v + v_o)\Delta t$$

Now solve for the desired quantity x.

*for* $\Delta t = 1s$

$$x = 0 + \frac{1}{2}(30m/s + 30m/s)(1s) = 30m$$

*for* $\Delta t = 2s$

$$x = 0 + \frac{1}{2}(30m/s + 30m/s)(2s) = 60m$$

*for* $\Delta t = 3s$

$$x = 0 + \frac{1}{2}(30m/s + 30m/s)(3s) = 90m$$

So we just went through the standard steps of solving a kinematics problem. You can use a dynamics worksheet to guide you through this process. [pass out dynamics worksheets]

So a special case of constant acceleration is when something is in freefall. When you release something and allow it to fall to the ground it is only under the influence of gravity. (assuming air resistance is negligible)

The acceleration due to gravity g = 9.8 m/s$^2$. This means that anything falling under the influence of gravity will increase its velocity by 9.8 m/s each second. No matter what the mass of the object it is subject to this acceleration. So now if this is true and you dropped a feather and a lead block, they should hit the floor at the same time! Unfortunately – I have neither of those things and you would have to suck the air out of the room so that the feather would hit the floor (air resistance)! But we'll do the next best thing.
[Drop penny vs. rubber ball to see that they hit the floor at the same time]

Let's try a problem involving an object in freefall.

Bob throws a ball straight up at 20 m/s, releasing the ball 1.5 m above the ground. What is the maximum height of the ball? What is the ball's impact speed as it hits the ground?

First, let's draw a picture and a motion diagram.

$y_f = ?$

$v_i = 20m/s$

$v_f = ?$
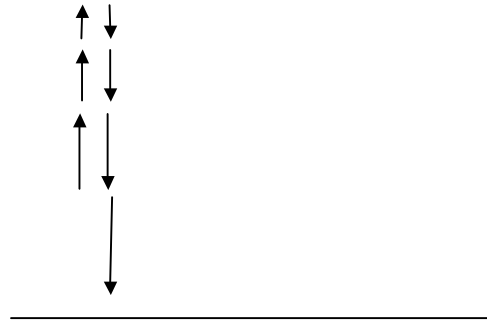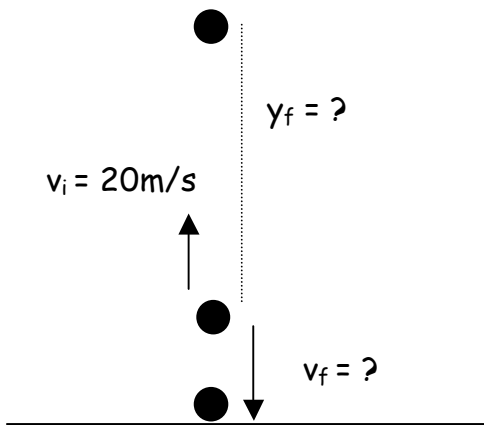
Now let's list knowns and desired unknowns:

Known Information:
$v_i$ = +20 m/s
a = -g
$y_o$ = +1.5
$v_f$ = 0
Desired Unknowns:
$y_f$?

$$\bar{v} = \frac{v + v_o}{2}$$

$$v = v_o + a\Delta t$$

$$x = x_o + \frac{1}{2}(v + v_o)\Delta t$$

$$x = x_o + v_o\Delta t + \frac{1}{2}a\Delta t^2$$

$$v^2 = v_o^2 + 2a\Delta x$$

Now we need to choose the equation that involves everything we know and the one thing we don't know:

$$v^2 = v_o^2 + 2a\Delta y$$

$$v^2 = v_o^2 + 2a(y - y_o)$$

$$v^2 - v_o^2 = 2a(y - y_o)$$

$$y = \frac{(v^2 - v_o^2)}{2a} + y_o = \frac{-(20m/s)^2}{2(-9.8m/s^2)} + 1.5m = 21.9m$$

Does this answer seem reasonable? Yes, it has the right units and 20m is definitely more reasonable than 200m. It has the right order of magnitude.

Now let's find its speed before it hits the ground. Let's put our initial point at the highest point, and the final point at the instant just before it hits the ground.

Known Information:
$v_i$ = +0 m/s
a = -g
$y_o$ = +21.9 m
$y_f$ = 0 m
Desired Unknowns:
$v_f$ = ?

$$v^2 = v_o^2 + 2a\Delta y$$

$$v^2 = v_o^2 + 2a(y - y_o)$$

$$v^2 = 0 + 2(-9.8 m/s^2)(0 - 21.9m)$$

$$v = \sqrt{429.24} = 20.7 m/s$$

Another special case in kinematics is projectile motion. You've already dealt with this in Tuesday's lab with the pennies. Projectile motion is a kinematics problem that you have to separate into 2 parts: x and y. This makes it a little scarier, but it's really just a matter of applying the same strategy we've been using all along. Let's start with a motion diagram:
All we need is an initial velocity and an acceleration.
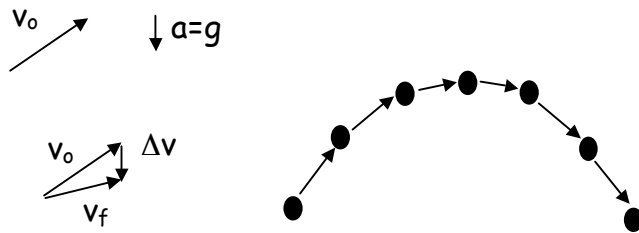


So now we've got our motion diagram, let's work on solving an actual problem:
A firehose sprays a stream of water at an angle of 33° with respect to the horizontal at a speed of 24 m/s. Find the highest point in the arc and determine at what point the stream hits the ground.

Since we already have a pretty good motion diagram, let's draw a quick picture to go with it:
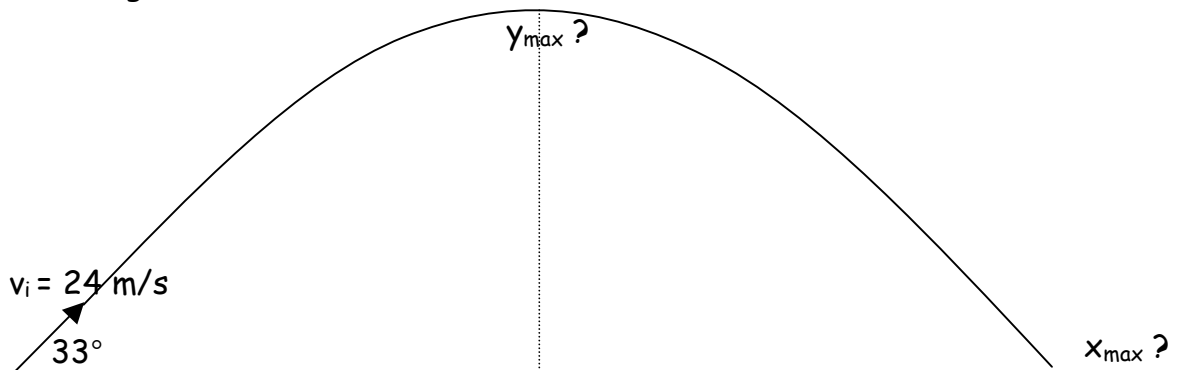


Now to identify knowns and unknowns:
We have to make two separate boxes to describe both the x and y direction.

We have to break the initial velocity into components:

$$v_{x_o} = 24\cos(33°) = 20.1 m/s$$

$$v_{x_o} = 24\sin(33°) = 13.1 m/s$$

*kinematic equations for constant acceleration*

$$\overline{v_x} = \frac{v_x + (v_o)_x}{2} \qquad\qquad \overline{v_y} = \frac{v_y + (v_o)_y}{2}$$

$$v_x = (v_o)_x + a_x\Delta t \qquad\qquad v_y = (v_o)_y + a_y\Delta t$$

$$x = x_o + \frac{1}{2}(v_x + (v_o)_x)\Delta t \quad y = y_o + \frac{1}{2}(v_y + (v_o)_y)\Delta t$$

$$x = x_o + (v_o)_x\Delta t + \frac{1}{2}a_x\Delta t^2 \quad y = y_o + (v_o)_y\Delta t + \frac{1}{2}a_y\Delta t^2$$

$$v_x^2 = (v_o^2)_x + 2a_x\Delta x \qquad\qquad v_y^2 = (v_o^2)_y + 2a_y\Delta y$$

So based on what we have what equation(s) can we use?
Anything with time is out.
Let's try

$$v_y^2 = (v_o^2)_y + 2a_y\Delta y$$

$$v_y^2 - (v_o^2)_y = 2a_y\Delta y$$

$$\frac{v_y^2 - (v_o^2)_y}{2a_y} = y_{max} - y_o$$

$$y_{max} = \frac{-(13.1 m/s)^2}{2(-9.8 m/s^2)} = 8.8m$$

Now we know $y_{max}$, but what about $x_{max}$?

If we look at our equations, we don't know enough stuff in the x direction yet.  But, maybe we can use the y information to help us out.  Is there anything common to both?

Time is common to both.  So we can find time in the  y-direction and since time is common to both we can apply it to the x-direction.