Lecture 3 / Day 2 Motion and Kinematics

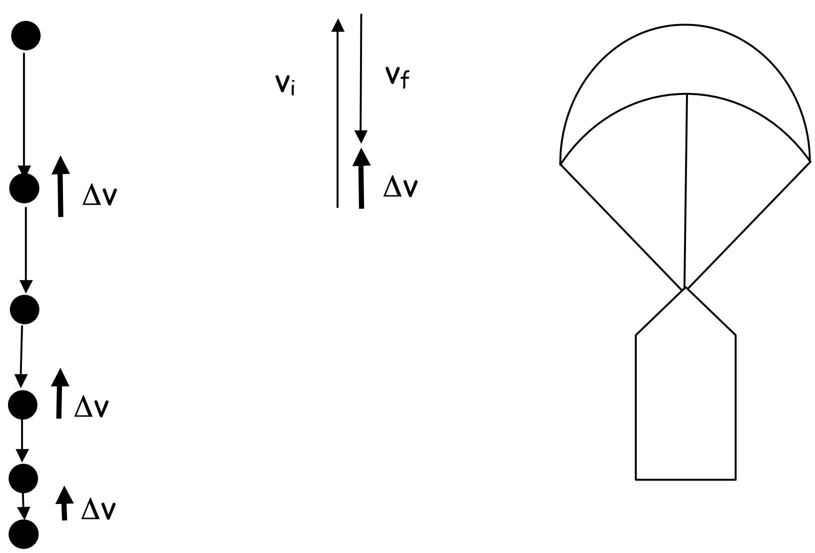
Acceleration
Complete Motion Diagrams
Turning Points

So we finished with an example talking about velocity and how velocity is what changes an initial position to a final position. But what changes an initial velocity to a final velocity? Acceleration is what changes an initial velocity to a final velocity.
The definition of velocity is:

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} \Rightarrow \Delta \vec{v} = \vec{a}\Delta t \quad \vec{v}_f = \vec{v}_i + \vec{a}\Delta t \quad or \quad \vec{v}_f = \vec{v}_i + \Delta \vec{v}$$

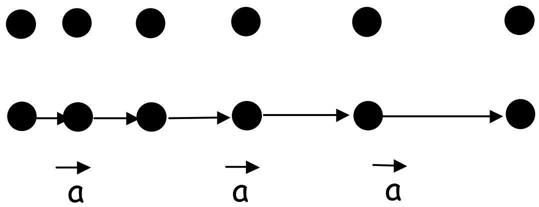
Since we know how to add and subtract vectors, let's do an example of a motion diagram and find acceleration.

Example:
A spacecraft lands on Mars with a parachute. Draw a motion diagram to describe the motion.



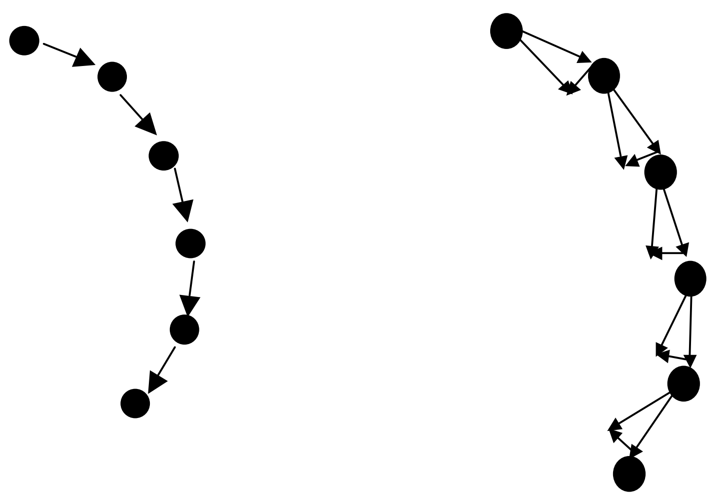
So we can see that the spacecraft is slowing down and that our acceleration vectors are pointing in the direction opposite to the velocity.

Let's do a few simple examples of a car on a track:
Draw the velocity and acceleration vectors for the car speeding up:

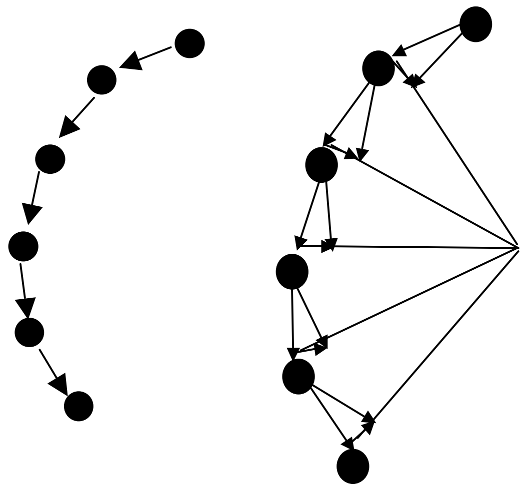


Notice for the car speeding up the acceleration points in the same direction as the velocity.

Now try the car going around a curve at constant velocity:



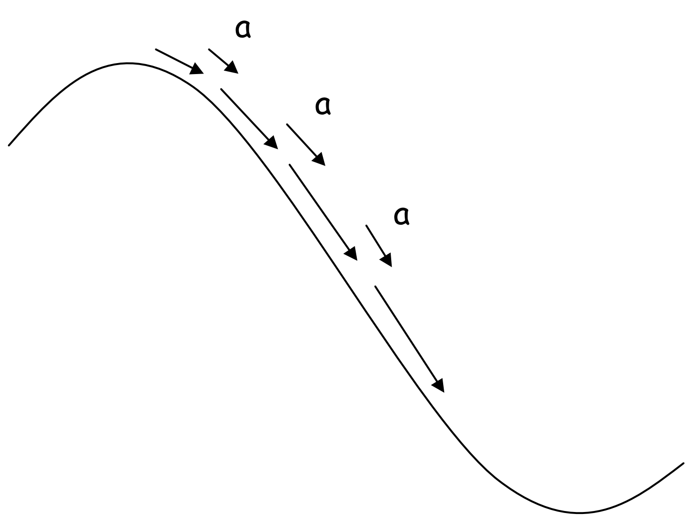
Notice that going around a curve at constant velocity there is an acceleration. But by definition, $\vec{a} = \frac{\overrightarrow{\Delta v}}{t}$, if $v_i = v_f$, $\Delta v = 0$ so $a = 0$. This would imply that going around a curve at <u>constant</u> velocity, acceleration should be zero! So why is $a \neq 0$? Think about this while trying your own diagram: a car going over the opposite curve at constant velocity.

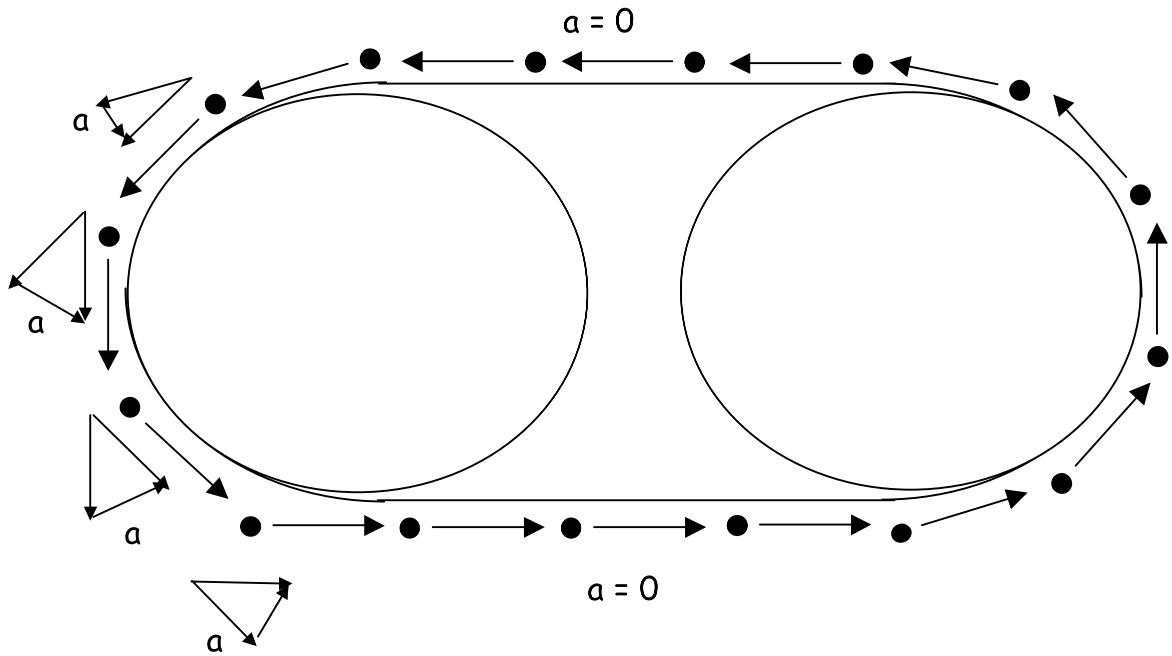


So let's answer the question, how can you have an acceleration at constant velocity? The truth is, the magnitude is constant, but the direction is changing. So there has to be an acceleration if the velocity is changing direction, because acceleration is what changes an initial velocity to a final velocity. Also notice that the acceleration points to a common point, the center of the circle. It makes sense if you think about it, to keep something going in a circle you have to have an acceleration toward the center. Think of a **ball on a string**, which you swing around. If you cut the string the ball will go off in a new direction – no longer in a circle. The string is what provides that acceleration to keep it in a circle.

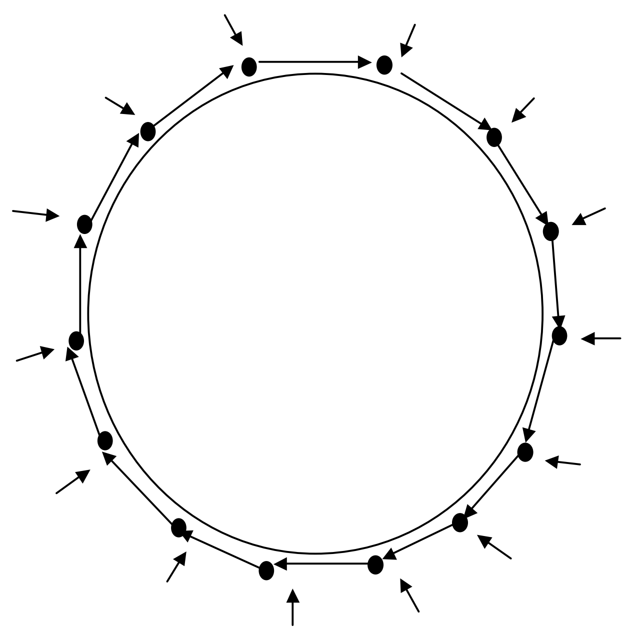
Now we'll have you do some examples:

- A car rolls down a hill after the parking brake fails
- Bob runs once around a track with straight sides and semicircular ends
- A ferris wheel goes around once at constant speed





No acceleration on the straight bits, but notice that acceleration points to the center of the semi-circles.

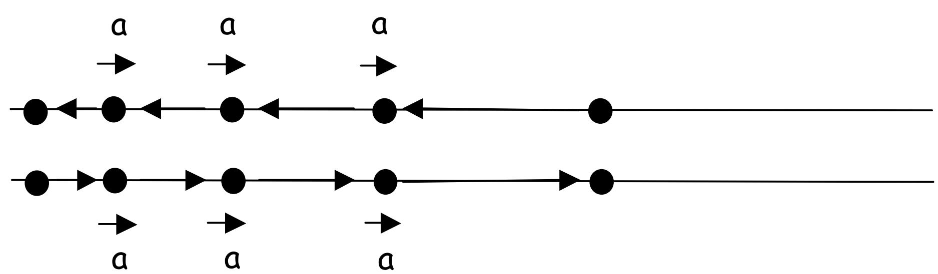


Acceleration for the ferris wheel should point toward the center.

Even though the speed of the ferris wheel doesn't change, there is an acceleration because the direction of the velocity is changing. So now let's try a quick quiz question.

A car drives over a hill at a steady 60 mph. Is it accelerating as it crosses the crest of a hill? Justify your answer using a complete motion diagram

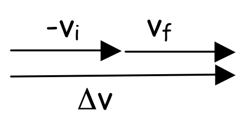
Now let's consider a new type of situation:
[cart attached to mass over pulley, push against force of mass and allow to return]
Now let's do a motion diagram including velocity and acceleration vectors.



Notice something very important, the object slows down when acceleration is opposite velocity, and the object speeds up when acceleration is in the same direction as velocity.

What about the turning point, what's the acceleration there?
We'll split the class into two groups, since there are two possibilities: $a = 0$, $a \neq 0$ each has to choose a spokesman and defend the position.

Ultimately, acceleration at the turning point cannot be zero by the definition of acceleration – acceleration is what changes initial velocity to final velocity.
If we look at the vectors:

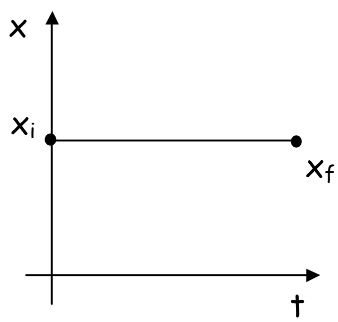


Δv is definitely not equal to zero. In fact it is pretty big compared to the other acceleration vectors. This is because you have to go from velocity to the left to a change that gives you velocity to the right. You have to

overcome the tendency to move to the left, stop the cart, then speed the cart up to the right.
Now split the class in two again (count off the class at random), have each group come to the board and put up one of these examples:

- A ball rolls up and down an incline
- A ball tossed up which comes down along the same path

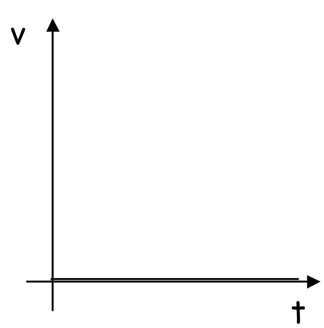
Now that we're clear on acceleration, we can expand our ideas about graphs. So far we've looked at velocity versus time graphs, but now we'll look at how position, velocity, and acceleration graphs relate.

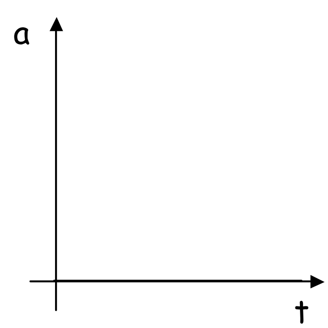


We've drawn the simplest graph possible here. This motion would represent an object with a constant position. Displacement $\Delta x = x_f - x_i$ is zero. Remember how the slope of a velocity graph is acceleration, it turns the slope of our position graph is velocity.

$slope = \frac{rise}{run} = \frac{\Delta x}{\Delta t}$ the units of slope in this case are m/s, units of velocity!

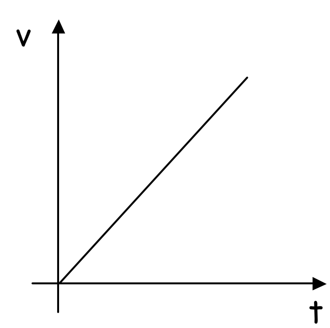
So let's consider the velocity of this graph. $\Delta x = 0$ in this case, so $v = 0$. Now let's draw the velocity vs. time graph even though it's trivial.





Acceleration graph is also easy since there's no change in velocity ($\Delta v = 0 \Rightarrow a=0$)

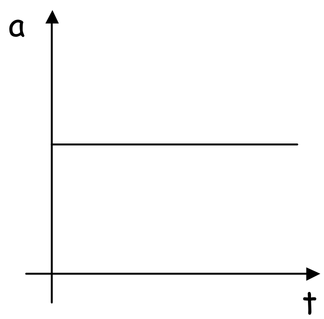
Now let's try something a little more challenging:



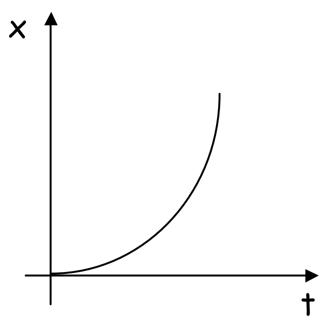
Let's draw a position and acceleration graph to go with this velocity graph.
The slope of a velocity graph gives you acceleration.

$$slope = \frac{rise}{run} = \frac{\Delta v}{\Delta t}$$

The slope here is constant and positive, so the acceleration graph should be constant and positive.



The position is a little more challenging. The slope of the position graph gives you the velocity. So we need to draw a position graph that gives a line with positive slope.



The slope of a parabola is a line.
The way to describe a parabola is:

$$x = at^2$$

$$slope = v = \frac{\Delta x}{\Delta t} = 2at$$

Notice that the slope of the position is a line with a constant positive slope.

So to recap:

$$position \xrightarrow{slope} velocity \xrightarrow{slope} acceleration$$

The area of a velocity graph gives you displacement.
The way to find displacement on a position graph is to take the difference in positions: $\Delta x = x_f - x_i$