Lecture 24 Pressure and Fluids

Recall that last time we talked about pressure in a liquid. We determined that the pressure in a fluid depended on two things, the pressure at the surface and the depth in the fluid.

$$pA = p_o A + mg$$

$$m = \rho V = \rho A d$$

$$pA = p_o A + \rho A d g$$

$$p = p_o + \rho d g$$

What we've found is called the hydrostatic pressure (at some depth d in a liquid), because our assumption is that the fluid is at rest. Notice that $p = p_o$ when d = 0. At the surface of the liquid the pressure should be the pressure at the surface. For a liquid open to the air, p_o = 101,300 Pa.

Now for a quick example:
A submarine cruises at a depth of 300m. What is the pressure at this depth? Give the answer both in pascals and and atmospheres.

The density of seawater is ρ = 1030 kg/m^3. The pressure at depth d = 300m is then found to be:

$$p = p_o + \rho g d = 1.013\times10^5 Pa + (1030 kg/m^3)(9.8 m/s^2)(300 m)$$

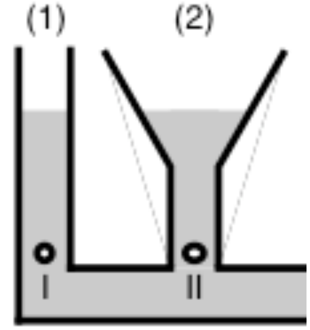
$$= 3.13\times10^6 Pa$$

Converting to atmospheres:

$$3.13\times10^6 Pa\left(\frac{1\,atm}{1.013\times10^5 Pa}\right) = 30.9\ atm$$

So to summarize our findings – the hydrostatic pressure in a liquid depends only on the depth and pressure at the surface. We can come to a couple of conclusions based on this premise:

1) A connected liquid in hydrostatic equilibrium rises to the same height in all open regions of a container.

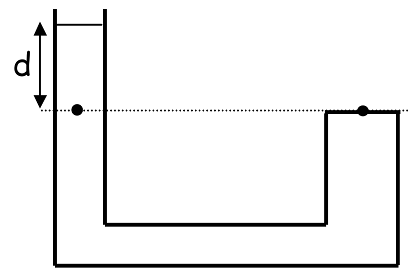


This is easily demonstrated in a picture. If the pressure depends on the pressure at the top of the liquid & the depth in the liquid, we can prove that $P_1 = P_2$.

$$p = p_o + \rho g h$$

since p_o and ρgh are the same for both points, we can draw the conclusion that the pressure must be the same for both points! If the pressure at point 2 was greater that that at point 1, we know that the fluid would flow until the pressure was the same at all points.

We can draw another conclusion which is the pressure is the same at all points on a horizontal line through a connected liquid in hydrostatic equilibrium.



Let's try an example:

Water fills a u-tube with an open and closed end. What is the pressure at the top of the closed tube?

We know that the pressure at the surface of the water on the left is at atmospheric pressure. We can then draw a horizontal line through both tubes. Since we know pressure depends on depth, we know the pressure along the line must be uniform. So if we find the pressure of a point along the line in the left tube, we then know the pressure along the same line in the right tube.
We can go through the equation for figuring out hydrostatic pressure:

$$p = p_o + \rho g h$$

$$p = p_o + \rho g d$$

$$p = p_{atm} + \rho g d$$

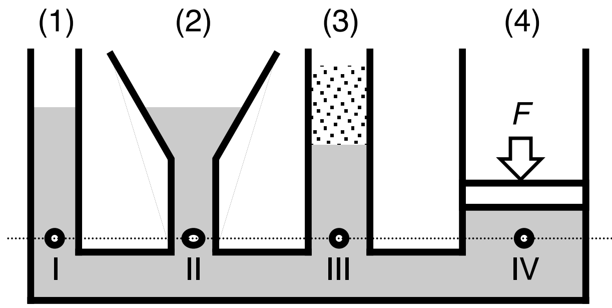
So we've figured out what the pressure must be at the top of the tube on the right. If the depth of the point is 60cm, we can figure out the pressure at this depth:

$$p = p_{atm} + \rho g d$$

$$p = 1.01 \times 10^5 Pa + (1000 \,{kg}/{m^3})(9.8 \,{kg \bullet m}/{s^2})(0.6m) = 1.07 \times 10^5 Pa$$

$$1.07 \times 10^5 Pa\left(\frac{1atm}{1.01 \times 10^5 Pa}\right) = 1.06atm$$

You can see that the increase in pressure is pretty minimal. Just as an aside, the rule of thumb for water is that you have to go about 10 meters to increase your pressure an atmosphere.



So now another question:
How does the pressure at each point (I,II,III,IV) compare?

The answer is that all the points are at the same height, so they have to be at the same pressure! We don't even have to worry about what's going on at the top of each tube. We know that if the pressure was not the same, the fluid would flow until there was no more difference.

Pressure is actually a very practical topic. As a doctor you measure blood pressure. As a cyclist you measure the pressure in your tires. Often you use what's called a gauge to measure pressure. This gets to the idea of gauge pressure versus absolute pressure. When you're taking a pressure reading of your tires, what you learn about is the gauge pressure.

So let's think about what gauge pressure might be. What do we know? If a tire is flat, the gauge will read zero. But does that mean there's absolutely no air in the tire? In essence, a vacuum? We also know that the atmosphere is constantly pushing in with 1 atm of pressure. To inflate something, we have to fill it with <u>at least</u> 1 atm of pressure, right? So with these facts in hand, come up with a formula involving three terms: p_{gauge}, $p_{absolute}$, p_{atm}.

$$p_{gauge} = p_{absolute} - p_{atm}$$

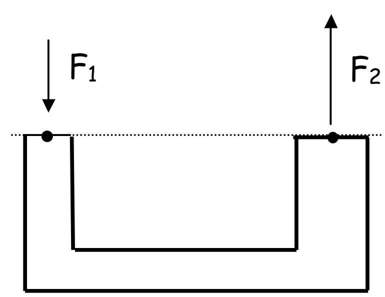
What a gauge does is to give the pressure above atmospheric pressure. This is valuable especially when you have to inflate something because before you can really begin to inflate it, you have to beat atmospheric pressure. If you've ever looked at a tire rating, it's a gauge pressure recommendation. It's telling you that you have to inflate your tires to 32 psi above atmospheric pressure. Now you ask, what is psi? Another unit of pressure is psi, pounds per square inch. The conversion from psi to Pa is 101.3 kPa = 14.7 psi.

Another important application of pressure is hydraulics. Hydraulics is when you use pressurized liquids to do useful work. One basic application is that you take a two-piston system (connected by fluid, one big and one small) and push on the smaller one. When you increase the pressure on the smaller piston, the pressure at all points in the liquid increases, and the bigger piston can then push out to do useful work.

The brake system in your car is a hydraulic system. When you step on the brake pedal, a piston is pushed into the master brake cylinder, which increases the pressure of the brake fluid. This pressure increase is then translated to your four wheels. Then your brake pads are pushed against the disk.

Going back to the original two-piston concept, let's analyze what's called a hydraulic lift (the thing they use to lift your car at a mechanic shop).

Assuming that the pressure along a horizontal line in a continuous fluid has to be the same, then $p_1 = p_2$.
If we go back to our original definition of pressure, then we can solve for the pressure at the larger piston:



$$p_1 = p_2$$

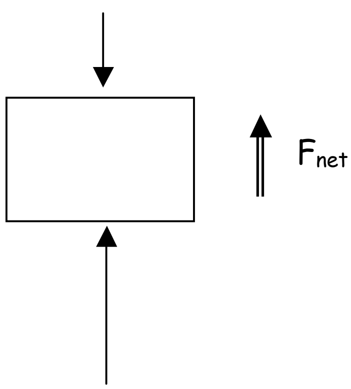
$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$F_2 = \frac{A_2}{A_1} F_1$$

We can see that the force at the larger piston must be greater than the force at the smaller piston because the ratio of areas is greater than 1. What we have here is called force multiplication. Using pressure as a lever all you have to do is worry about the areas of your pistons to get the force you want, enough to lift a car!

Another phenomenon that pressure can explain is buoyancy. Drop a rock into a lake. It sinks. A massive ship can float on the ocean, but a rock will sink. How do we explain this? The first concept to introduce is the buoyant force. When you put something in a fluid it experiences an upward force. Whether it sinks or floats depends on how much buoyant force it feels and how much downward force is fighting the buoyant (upward) force.

Understanding where the buoyant force comes from is actually pretty easy. We know that pressure increases with depth. So when you submerge something in liquid, the pressure pushing up from the bottom is greater (depth is greater) than the pressure pushing on the top. When you sum the forces, the net effect is a force pushing upward, i.e. the buoyant force.

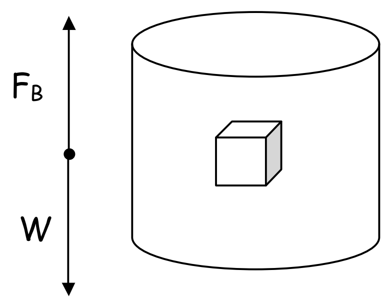


Understanding how to calculate the buoyant force is also straightforward. If we just imagine a bit of liquid in a container. There are two forces on it in the y-direction: the weight acts down and since it is in equilibrium there must be some upward force too.

$$F_{net} = ma = 0$$

$$F_{net} = F_B - W = 0$$

$$F_B = W$$



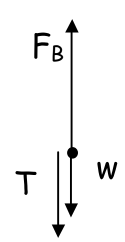
So there must be a force equal and opposite to the weight that keeps that liquid from dropping. Now, what if I stick a sugar cube with the exact same shape and volume in that spot displacing the liquid that was there? The weight of the sugar cube will be different, because that depends on the object that occupies that spot. But, there's no reason for the buoyant force to change. It is the force due to the pressure in the fluid. So it should stay the same if the fluid is staying the same. In this way we've just discovered how to calculate the buoyant force. The official name for this is Archimedes' principle: "A fluid exerts an upward buoyant force F_B on an object immersed in or floating on the fluid. The magnitude of the buoyant force equals the weight of the fluid displaced by the object."

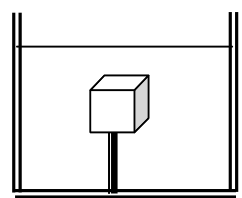
In equation form, Archimedes' Principle is:

$$F_B = \rho_f V_f g$$

where the subscripts refer to the fluid your object is floating/submersed in. So where does this equation come from? We call the volume of fluid displaced V_f. The density of the fluid ρ_f is easy to look up. So we can solve for the mass of the displaced fluid: $m = \rho_f V_f$. Now if we know $F_B = W$, $W = mg = \rho_f V_f g$. Super easy right! So easy, we should try an example.

A 10cm × 10cm × 10cm block of wood with a density of 700 kg/m^3 is held underwater by a string tied to the bottom of the container. What is the tension in the string?





$$F_{net} = F_B - T - w$$

$$T = F_B - w$$

$$T = F_B - w = m_f g - m_o g$$

$$T = \rho_f V_f g - \rho_o V_o g$$

$$T = \rho_f V_o g - \rho_o V_o g = (\rho_f - \rho_o) V_o g$$

$$T = (1000 \, {}^{kg}\!/_{m^3} - 700 \, {}^{kg}\!/_{m^3})(1.0 \times 10^{-3} m^3)(9.8 \, {}^{m}\!/_{s^2})$$

$$T = 2.94 N$$

Notice that the tension would disappear if the fluid of the object were the same as the density of the fluid. This is the case where the volume would just be filled with fluid instead of an object.

In discussing the buoyant force, there are three possible cases that can occur: the object sinks, floats, or has what's called neutral buoyancy.

We'll discuss the details of this next time and finish our discussion of pressure with what's called Bernoulli's equation.