

Lecture 22 Waves
Interference of Waves
Standing Waves
Sound

Last time we talked about how traveling waves can actually interfere with each other to bring about constructive and destructive interference. We even gave you an equation to work with to determine if you had constructive or destructive interference. Each term has significance:

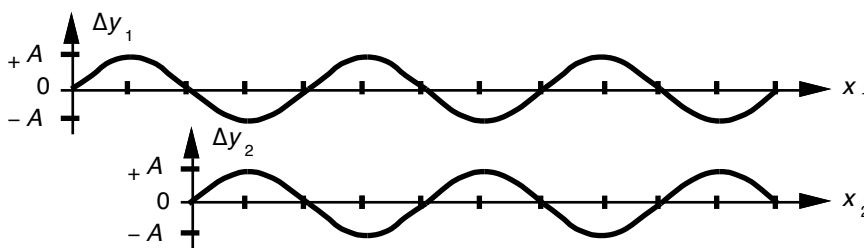
$$\Delta\Phi(x,t) = \left(\frac{2\pi\Delta x}{\lambda} + \Delta\phi \right)$$

↑
path length
difference

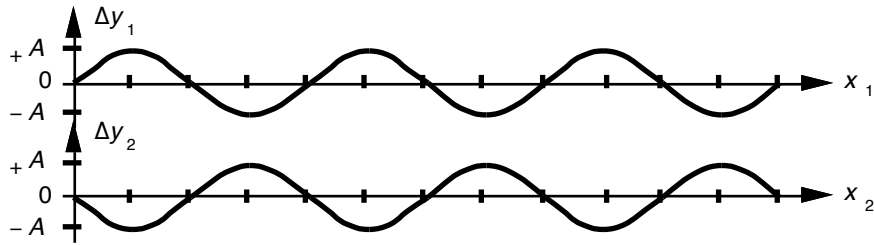
←
constant
phase
difference

So there are ultimately three ways to get constructive/destructive interference: a path length difference or a constant phase difference, or both.

Just to clear up what we mean by path length difference versus constant phase difference:



Here we have a path length difference. The second wave travels half a wavelength less than the first wave. If I calculated $\Delta\Phi$ for this I would come up with odd π . We would get destructive interference because we have troughs adding with crests.



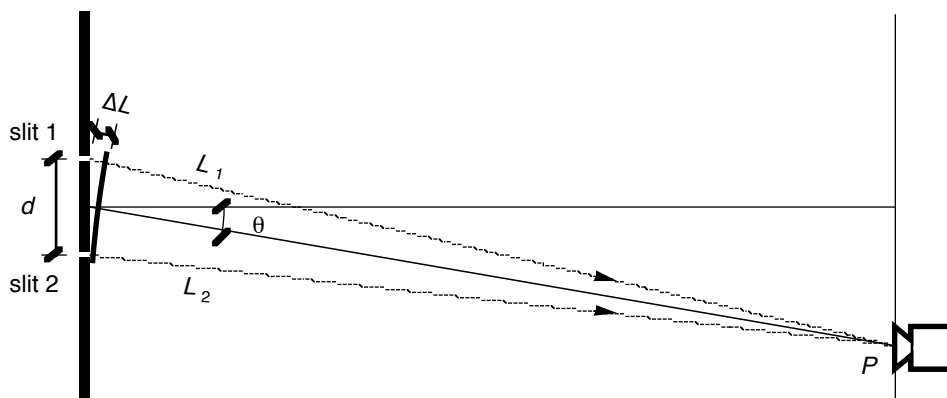
Here we have a constant phase difference. The second wave is shifted by a factor of π because it starts on a trough. So the constant phase difference $\Delta\phi$ would be π . The first wave has a constant phase of zero because it starts on a crest. It's not shifted at all. If we solved for $\Delta\Phi$ here it would be odd π because we're getting destructive interference, crests are adding to troughs.

An important application of this is light. A very famous experiment is the double slit experiment where you pass light through a couple of small slits. It sets up a pattern of constructive and destructive fringes. There's a cool applet that demonstrates this phenomena:

<http://www.colorado.edu/physics/2000/index.pl>

Each of the maxima (bright spots, constructive) and minima (dark spots, destructive) corresponds to interference. We can model the effect by simply looking at the addition of 2 light waves.

You start with one light source and two slits. This way the two waves start in phase. Then the only thing that matters is the path length difference.



We set up our experiment such that there is a known distance between the slits, d , and we pick a point along a screen opposite the double slits. Because we know that the waves are in phase the only thing we have to care about is

$\Delta x = x_2 - x_1$ - the path length difference. We know that if the path length difference is equal to a whole number of wavelengths we get constructive interference. If the path length difference is equal to a half number of wavelengths we get destructive interference. There is one more thing to consider: the geometry of the situation. It turns out that $\Delta x = d \sin \theta$. So now we have a whole new condition for interference:

$$\text{Constructive } n\lambda = d \sin \theta$$

$$\text{Destructive } (n + \frac{1}{2})\lambda = d \sin \theta$$

$$n = 0, 1, 2, 3, \dots$$

So let's try an example (we can check it with an applet when we're done)

<http://www.walter-fendt.de/ph14e/doubleslit.htm>

For a light source with $\lambda = 600 \text{ nm} = 600 \times 10^{-9} \text{ m}$

And a slit spacing of $d = 1000 \text{ nm} = 1000 \times 10^{-9}$

What is θ for the first maxima past the center and the first minima?

The center maxima ($n=0$) is easy, its angle is $\theta=0$. Each wave travels the same distance. So we want the $n=1$ maxima.

Our relationship is:

$$n\lambda = d \sin \theta, n=1$$

$$\sin \theta = \frac{\lambda}{d} = \frac{600 \times 10^{-9}}{1000 \times 10^{-9}} = .6$$

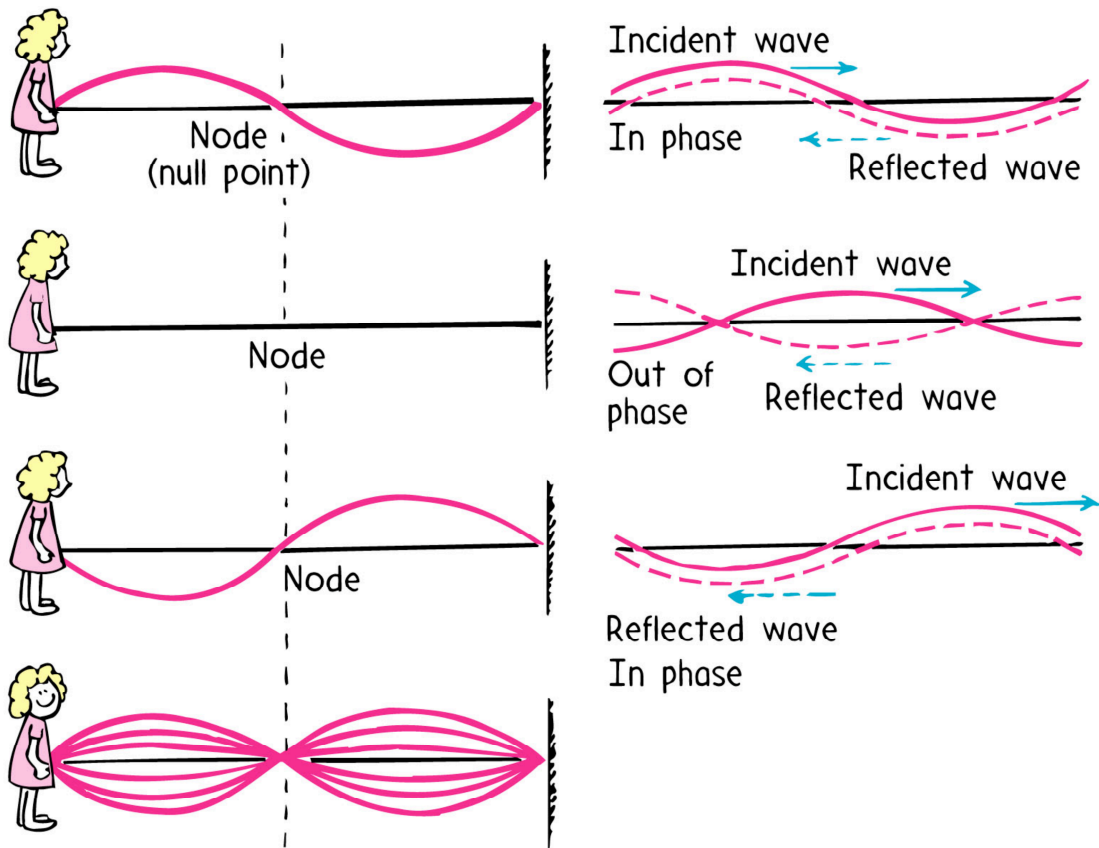
$$\theta = \sin^{-1}(.6) = 36.9^\circ$$

$$(n + \frac{1}{2})\lambda = d \sin \theta, n=0$$

$$\text{Now for the minima: } \sin \theta = \frac{\lambda}{2d} = \frac{600 \times 10^{-9}}{2(1000 \times 10^{-9})} = .3$$

$$\theta = \sin^{-1}(.3) = 17.5^\circ$$

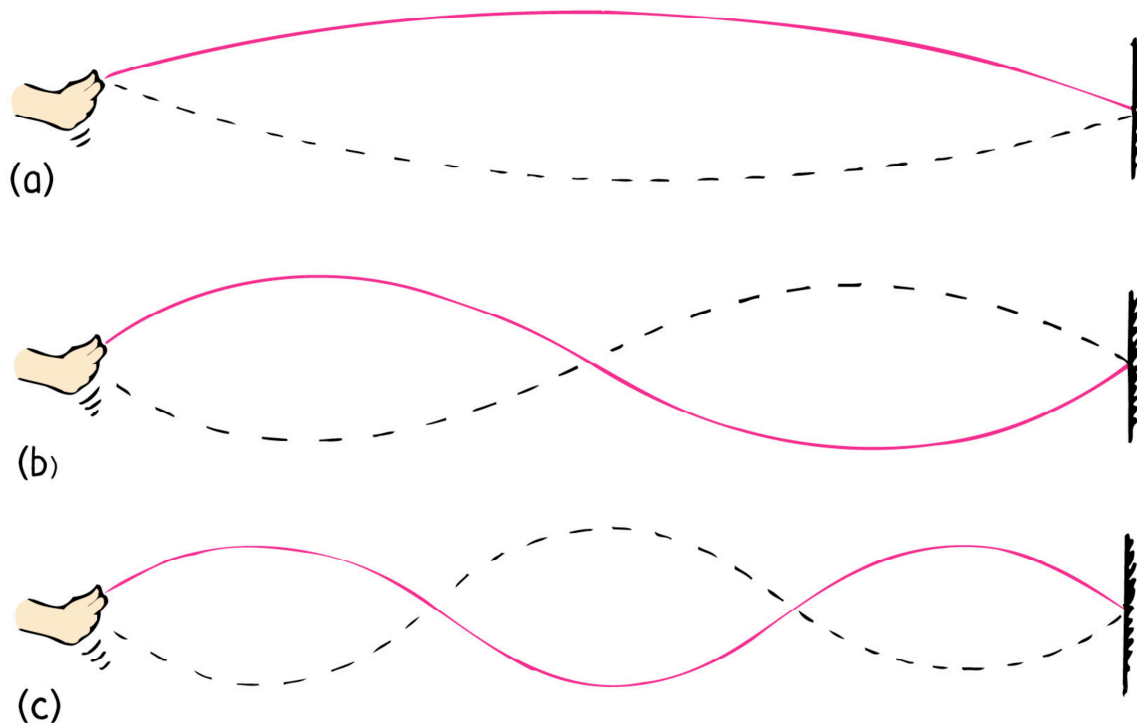
Another phenomena associated with wave interference is standing waves. We can create a standing wave in a system where we have reflected waves:



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A standing wave is just what it sounds like: a wave that doesn't really move, that is, the places of constructive and destructive interference are constant. Notice when the two waves are in phase, you get some sort of constructive interference (antinodes) and destructive interference (nodes) where the two waves add and cancel. What we mean by in phase is that the reflected wave goes in as a crest (trough) and leaves as a crest (trough). Clearly, we can see this in the picture. This produces a certain pattern where you have nodes at the ends and in the middle and antinodes in between.

We can actually set up various frequencies this way. The different frequencies are indicated by the different patterns (wavelengths).



Notice the lowest frequency has one antinode and two nodes. Let's try to compute the frequency of this standing wave:

$$v = \lambda f$$

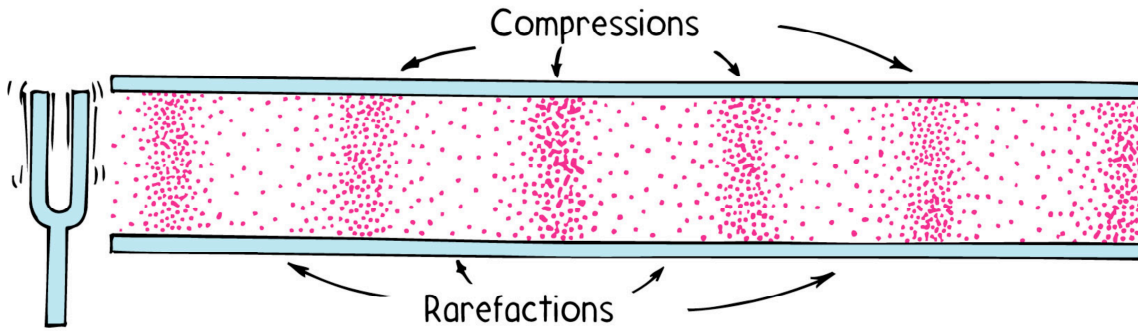
$$f = \frac{v}{\lambda}$$

$$\lambda = 2L$$

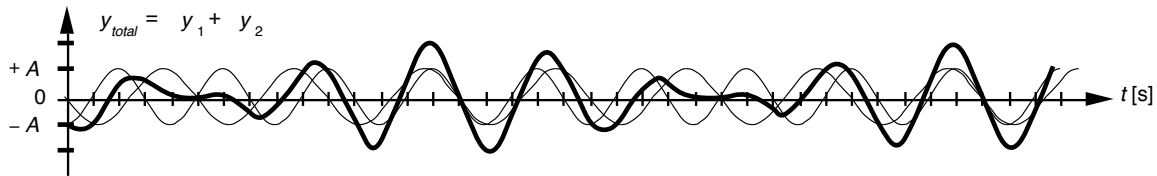
$$f_1 = \frac{v}{2L}, f_2 = \frac{v}{L}, f_3 = \frac{3v}{2L}$$

Don't forget v is a property of the medium, so it shouldn't change. We can also set up standing waves with different patterns if we change the boundary conditions. If we leave one end open and hold the other, we can set up a whole different set of frequencies. Now instead of a node at the end, we get an antinode-node pattern at the ends.

We can also talk about setting up standing waves in pipes, where instead of something like a rope - we're using air pressure differences to set up standing waves.



This is where we get musical notes from, setting up standing waves (musical notes). Multiple standing waves / frequencies can be set up in an instrument to form the sound we hear.



This combining of frequencies can also produce a phenomena called beats. If the two frequencies are close enough, we hear constructive and destructive interference over time. There's an applet that demonstrates this. To find the frequency of the beats we take the difference of the frequencies.

$$f_{beat} = f_1 - f_2$$

The beat frequency is the loud-soft-loud pattern we hear.

The other frequency we hear is the carrier frequency. It's the actual sound that gets loud, soft, and loud. The way we get this is to take the average of the two frequencies.

$$f_{carrier} = \left(\frac{f_1 + f_2}{2} \right)$$

If you think about combining two frequencies a higher one and a lower one, which will you hear - not the higher one, not the lower one. You hear the average of the two.

