Lecture 20 Oscillations (Chapter 11)
        Review of Simple Harmonic Motion Parameters
        Graphical Representation of SHM
   Review of mass-spring pendulum periods

Let's review Simple Harmonic Motion. Recall we used the example of a mass on a spring to introduce SHM. We see that it is smooth, not choppy, and it repeats itself over and over predictably (harmonic). This means it has a period. We can easily find the period of the motion by timing how long it takes to repeat its motion. We can also say other things about the motion.
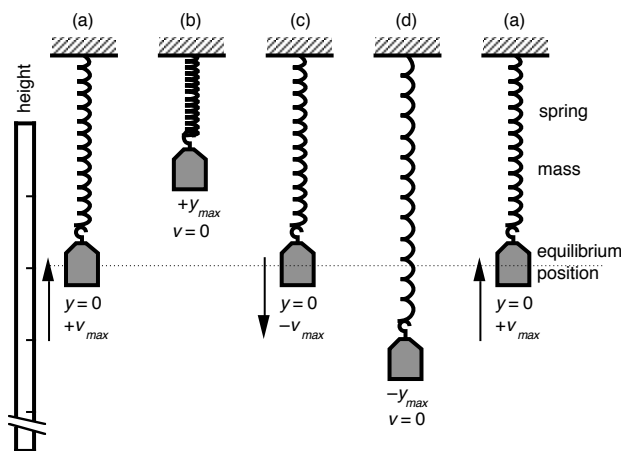
We know that net force / acceleration is zero at equilibrium and that the acceleration is maximum at the ends. We also know that velocity is a maximum at equilibrium and zero at the ends.

Again, we call the maximum displacement the Amplitude. It is determined by the initial conditions of the system (i.e. how far I decide to pull the spring down). The amplitude actually sets the total energy of the system. If we look at the formula for spring energy $E_s = \frac{1}{2} kx^2$, if we plug A in we get $E_s = \frac{1}{2} kA^2 \rightarrow$ this is the maximum energy. When you've stretched the spring this far, you have no kinetic and all potential energy. This means that you're setting the total energy of the system. We can use this to find the maximum velocity:
At equilibrium, we know $E_{Tot} = \frac{1}{2} kA^2$. We also know that the potential energy of the spring is zero, and so all we have is kinetic. So we have the maximum velocity at this point, since we have the maximum kinetic energy. We can draw a diagram reflecting this.
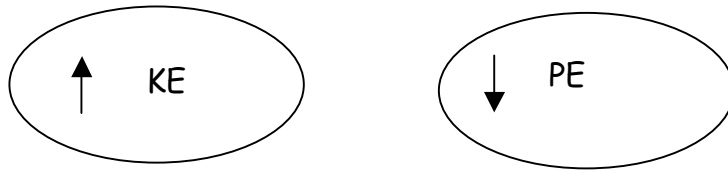
$$E_{Tot} = \frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2$$

$$v_{max} = \sqrt{\frac{kA^2}{m}}$$

So we can see here that the maximum velocity depends on the Amplitude. The bigger the amplitude, the bigger the maximum velocity (more total energy).

If we look at the motion of the mass on the spring over time, it has a distinctive motion. http://www.phy.ntnu.edu.tw/java/shm/shm.html (Figure 13.5, p.451) It is a sinusoidal (cosine or sine) curve. Notice that a sinusoidal curve has a period, i.e. it repeats its shape repeatedly. It also has an amplitude. Though your book uses a sine function, we're going to use a cosine function to describe the motion. The function tells us the position of the mass as a function of time.

$$y = A\cos\left(\frac{2\pi t}{T}\right) = A\cos(2\pi f t)$$

Notice that at t = 0s, y = A. Our system starts stretched at amplitude. When t = T/2, y = 0, our system is passing through equilibrium. So how would we draw this generic function? We know what it is generally going to look like. A cosine function starts at its maximum value.
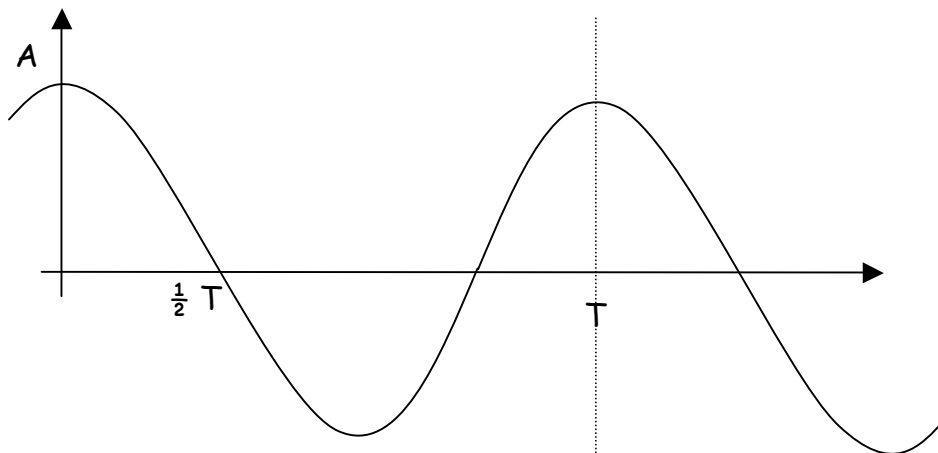
We also have functions to describe the velocity and acceleration of the mass: (surprise, they're sinusoidal too!)
for the velocity:

$$v = \frac{A}{T}\sin(\frac{2\pi t}{T}) = v_{max}\sin(\frac{2\pi t}{T})$$

The main difference between the position and velocity is that we have this sine function –

$y(t)$ for a certain position x

$T = \frac{1}{f}$
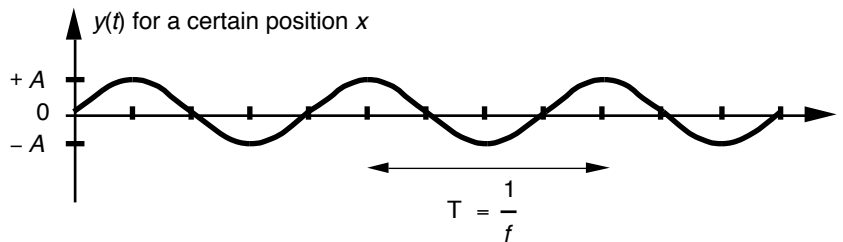
Notice we've started our function at zero instead of a maximum. When the position is at a maximum, the velocity is zero. When the velocity is at a max, the displacement is zero.

We can also talk about acceleration. The function is again a cosine function:

$$a = \frac{A}{T^2}\cos(\frac{2\pi t}{T}) = a_{max}\cos(\frac{2\pi t}{T})$$

The maximum acceleration of the system is A/T^2. Also notice that the acceleration is a maximum when the spring is at its maximum stretch. This makes sense right, if you consider that the maximum force is also at that position.

So now you're all familiar with the functions describing position, velocity, and acceleration. The one concept we haven't talked about is the idea of constant phase ϕ. What constant phase tells you is where your system starts:

$$y = A\cos\left(\frac{2\pi t}{T} + \phi\right) + B \rightarrow$$ this is the most general equation to start with.

$\phi = 0$

$\phi = \pi/2$
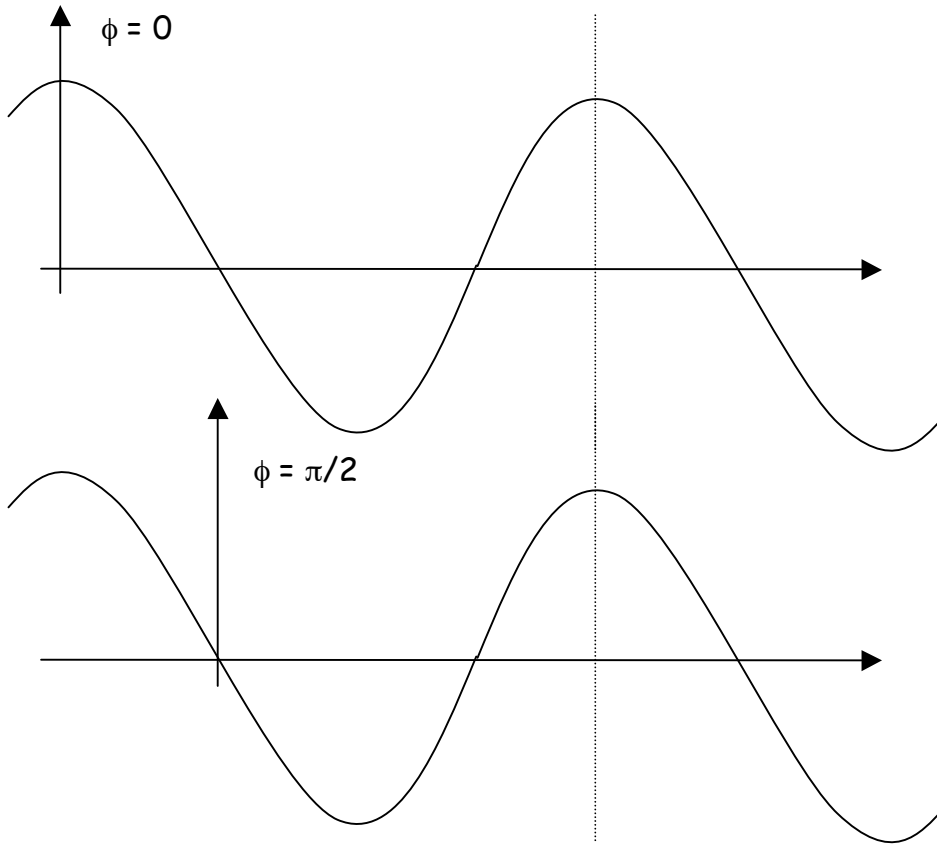
To include a phase shift, we simply move our starting point. All you have to understand is the connection between radians and how you can divide a cycle into radian measure.

$\phi = \pi$
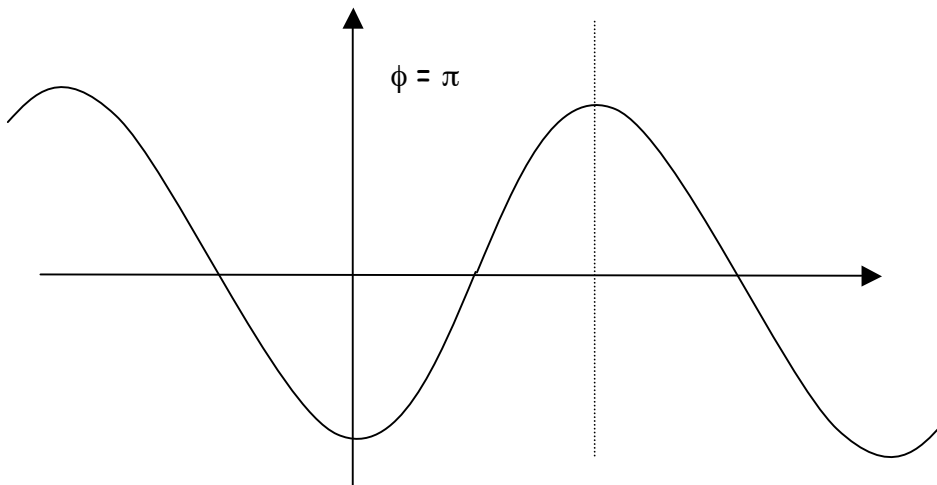
Now you're starting at the bottom of the motion. For a mass on a spring, it would be starting the motion where you've given it a pull down.

Now to explain B. B is the equilibrium value. If you move the point about which you oscillate, you change this value. The whole graph either shifts up or down.

Now we can go a little further into understanding oscillations by studying what happens when you put a bunch of oscillators together: http://www.cbu.edu/~jvarrian/applets/waves1/lontra_g.htm

When you put a bunch of them together you get a traveling wave. Let's get everybody to stand up and make a chain. We can demonstrate a traveling wave pretty easy this way. We can also investigate properties like speed and polarization.

A couple of important parameters are velocity and wavelength. The wavelength is the repeat distance of the wave (as opposed to period, which is the repeat time). Velocity of the wave is just the wavelength divided by the period.