

Lecture 18 Torque and Rotational Motion (Chapter 8)
Analyzing Angular Motion Problems
Torque
Newton's Second Law and Rotational Equilibrium

Last time we went through defining various rotational quantities like rotational velocity and rotational acceleration. I also mentioned quickly that we have a set of kinematic equations that describe rotational motion just like we have a set of kinematic equations that describe translational motion. So in the interest of figuring out how to use these equations, we should do an example:

A microwave oven has a rotating plate 30 cm in diameter for even cooking. The plate accelerates from rest at a uniform rate of 0.87 rad/s^2 for 0.50 s before reaching its constant final speed. (a) How many revolutions does the plate make before reaching its final speed? (b) What are the final angular speed of the plate and the tangential speed at its rim?

So, what are the equations we have available?

$$\theta = \omega \Delta t$$

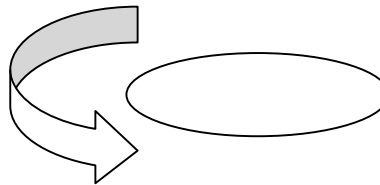
$$\bar{\omega} = \frac{\omega_o + \omega}{2}$$

$$\omega = \omega_o + \alpha \Delta t$$

$$\Delta \theta = \omega_o \Delta t + \frac{1}{2} \alpha \Delta t^2$$

$$\omega^2 = \omega_o^2 + 2\alpha \Delta \theta$$

view from above



Now we can list knowns and unknowns:

$$r = 15 \text{ cm} = 0.15 \text{ m}$$

$$\omega_o = 0$$

$$\alpha = 0.87 \text{ rad/s}^2$$

$$t = 0.50 \text{ s}$$

find $\Delta \theta$, ω & v

We can find the revolutions first (i.e. $\Delta\theta$):

$$\Delta\theta = \omega_o\Delta t + \frac{1}{2}\alpha\Delta t^2$$

$$\Delta\theta = 0 + \frac{1}{2}(0.87\text{ rad/s}^2)(0.50\text{ s})^2 = 0.11\text{ rad}$$

$$0.11\text{ rad} \left(\frac{1\text{ rev}}{2\pi\text{ rad}} \right) = 0.018\text{ rev}$$

So this means that the plate only goes through a tiny fraction of a revolution before it gets to its final speed. This makes sense since microwave plates don't move too quickly.

Now we can try to solve for ω :

$$\omega = \omega_o + \alpha\Delta t$$

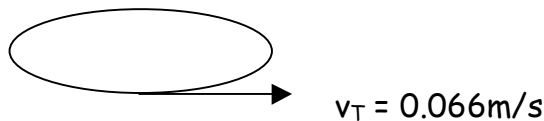
$$\omega = 0 + (0.87\text{ rad/s}^2)(0.50\text{ s}) = 0.435\text{ rad/s}$$

Now that we have the angular speed we can solve for the tangential speed at the rim, knowing the radius of the plate:

$$v = r\omega$$

$$v = (0.15\text{ m})(0.44\text{ rad/s}) = 0.066\text{ m/s}$$

So if you had something sitting at the edge of the plate, (an olive?) and it slipped as the plate accelerated, it would leave the plate with a velocity of 0.066 m/s.



We can now move on to talking more about torque. First, we should define some terms, namely translational versus rotational motion. (p. 261, Figure 8.1) has a nice figure of this. If we take something that is translating, i.e. moving in the x-direction, all of its particles are moving in that direction. If we look at something that is rotating, it has a fixed axis and the particles

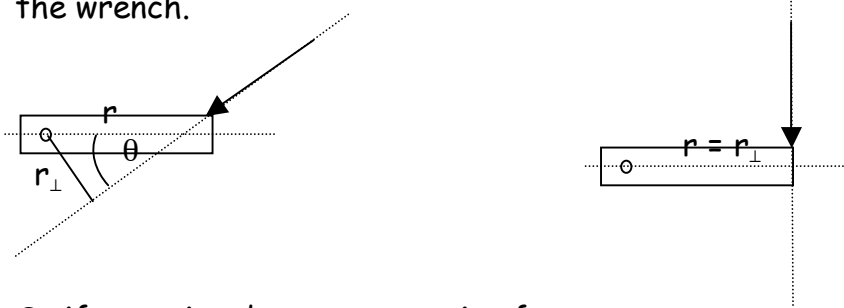
move with different velocities at any given time. Finally, to get something that is rolling, we add the translation and rotation vectors. Notice the object that is rolling has twice the velocity at the top than at the center, and is momentarily stationary at the point of contact. We can define the condition for rolling without slipping (if it slipped it wouldn't go anywhere) by looking at the diagram. This condition is defined as the velocity of the center of mass: $v_{CM} = r\omega$.

The way we get this is fairly straightforward (Figure 8.2, p.262):

$$v_{CM} = \frac{\text{displacement}}{\text{time}} = \frac{s}{t} = \frac{r\theta}{t} = r\omega$$

So recall last time we talked about how torque is rotational force. We defined torque as $\tau = rF \sin\theta$, where θ is the angle between r and F . We did some straightforward examples last time namely with $\theta = 90^\circ$. Now we should try some tougher examples. (Figure 8.4, p. 264)

The drain plug on a car's engine has been tightened to a torque of $25 \text{ m}\cdot\text{N}$. If a 0.15 m long wrench is used to change the oil, what is the minimum force need to loosen the plug? Assume the force makes a 30° with the length of the wrench.



So if we write down our equation for torque:

$$\tau = rF \sin\theta$$

$$\tau = 25 \text{ m}\cdot\text{N}$$

$$\tau = 0.15 \text{ m}(\sin 30^\circ)F = 25 \text{ m}\cdot\text{N}$$

$$F = 333 \text{ N}$$

Recall the lab from last time. You worked to try to balance the torques in a system such that $\tau_{\text{net}} = 0$. This is called rotational equilibrium. Though

torques are being exerted on a system, they sum to zero and we get no angular acceleration (rotational motion). When all the forces on a system add to zero we get translational equilibrium, and the (translational motion) acceleration of the system is zero.

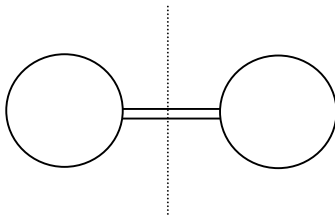
The meterstick scenario of your lab is a good example of a system in translational and rotational equilibrium. We can add up all the forces and set them equal to zero, just as we can add up all the torques and set them equal to zero.

Now that we've discussed equilibrium conditions we can discuss what happens when a system is not in equilibrium, when you have an angular acceleration. For a rigid body (something that's not a particle) we have the rotational analog to Newton's Second Law:

$$\tau_{net} = I\alpha$$

Where the net torque is proportional to angular acceleration. The I factor is called the moment of inertia. You can think of it like rotational mass. It not only matters how much mass is in a system, but also how it is distributed. (Demo: Disks with adjustable masses inside.)

The formula for Rotational Inertia is: Σmr^2 . This is if you have point masses in your system, like a barbell for instance. We'll do two different examples:

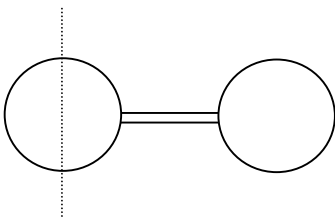


$$m_1 = m_2 = 30\text{kg}$$

$$x_1 = x_2 = 0.50\text{m}$$

$$\Sigma mr^2 = m_1x_1^2 + m_2x_2^2 = (30\text{kg})(0.5\text{m})^2 + (30\text{kg})(0.5\text{m})^2 = 15\text{kg} \cdot \text{m}^2$$

Now if we move the axis, it will change the calculation:



$$m_1 = m_2 = 30\text{kg}$$

$$x_1 = 0 \quad x_2 = 1\text{m}$$

$$\Sigma mr^2 = m_1 x_1^2 + m_2 x_2^2 = (30\text{kg})(0)^2 + (30\text{kg})(1.0\text{m})^2 = 30\text{kg} \cdot \text{m}^2$$

So if we look at Newton's Second Law, $\tau_{\text{net}} = I\alpha$ or $\alpha = \tau_{\text{net}}/I$, for a given torque the acceleration will be less for the second scenario because I is bigger. It would be tougher to get the second dumbbell to rotate because of where we've put the axis of rotation, though we haven't actually changed the masses!

But what if we don't have point masses to deal with? What if we have solid objects? It actually requires calculus to do this, so we've given you a table of objects to deal with (p.278). If you needed one of these on an exam, it would be provided to you. Notice that these all involve the mass and the radius in one form or another.

Next time we'll work on some torque problems involving Newton's Second Law. But what I want to talk about now is rotational kinetic energy. When something rotates it has kinetic energy associated with the rotation equal to $\frac{1}{2} I\omega^2$.

Let's go ahead and do a quick example:

A uniform, solid 1.0kg cylinder rolls without slipping at a speed of 1.8m/s on a flat surface. (a) What is the total kinetic energy of the cylinder? (b) What percentage of this total is rotational kinetic energy? We can list our knowns and unknowns:

$$M = 1.0\text{kg}$$

$$v_{CM} = 1.8\text{m/s}$$

$$I_{CM} = \frac{1}{2} MR^2$$

$$v_{CM} = r\omega$$

$$K = K_{\text{translational}} + K_{\text{rotational}}$$

$$K = \frac{1}{2} Mv^2 + \frac{1}{2} I\omega^2$$

$$K = \frac{1}{2} Mv^2 + \frac{1}{2} \left(\frac{1}{2} MR^2 \right) \left(\frac{v_{CM}}{R} \right)^2 = \frac{1}{2} Mv^2 + \frac{1}{4} Mv^2 = \frac{3}{4} Mv^2 = (0.75)(1.0\text{kg})(1.8\text{m/s})^2 = 2.43\text{J}$$