

Lecture 17 Circular Motion (Chapter 7)

Angular Measure

Angular Speed and Velocity

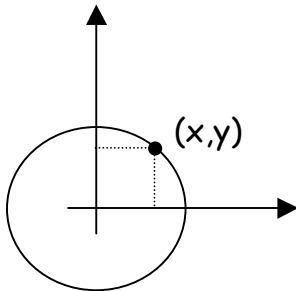
Angular Acceleration

We've already dealt with circular motion somewhat. Recall we learned about centripetal acceleration: when you swing something around in a circle the net force points toward the center of the circle. The centripetal acceleration can be calculated using the velocity of the object and the radius of the circle:

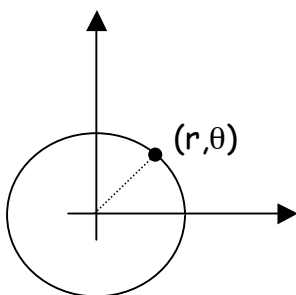
$$a_c = \frac{v^2}{r}$$

We jumped into centripetal acceleration because we were talking about how forces and acceleration were related. But now we're ready to fill out our understanding of circular motion.

So we'll start with Angular Measure. Consider a particle moving in a circular path. (p. 220, Figure 7.1)



We have our traditional Cartesian (x-y) coordinate system. But this isn't optimal for measuring something going in a circle.



There is another coordinate system, polar coordinates, that is better for circular motion. For something moving in a circle, r is constant. So while x and y constantly change for a particle moving in a circle, only θ changes in polar coordinates.

We can use r and θ to find x and y :

$$x = r \cos \theta$$

$$y = r \sin \theta$$

Recall that we defined linear displacement $\Delta x = x_f - x_i$. For circular motion, we can define angular displacement $\Delta\theta = \theta_f - \theta_i$.

Another variable important to circular motion is arc length (s). Arc length is the distance between two points on a circle. (Figure 7.2, p. 220)

The formula is:

$$s = r\theta$$

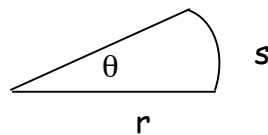
The units of arc length are meters, assuming that r is measured in meters and θ is measured in radians. There are 2π radians in 360° . There are π radians in 180° .

If we solve for θ , we get:

$\theta = \frac{s}{r}$ which is the ratio of two lengths. This makes a radian a pure number, dimensionless quantity.

Let's try an example:

You measure the length of a distant car to be subtended by an angular distance of 2.0° . If the car is actually 5.0m long, approximately how far away is the car?



$$s = r\theta$$

$$r = \frac{s}{\theta}$$

$$2^\circ \left(\frac{\pi}{180^\circ} \right) = 0.035 \text{ rad}$$

$$r = \frac{5.0m}{0.035rad} = 143m$$

Now that we understand angular measure, we can move onto angular speed and velocity.

At this point, we've gone over speed and velocity a great deal. We understand that velocity is $\frac{\Delta x}{\Delta t}$. Now for circular motion we have to define

the angular analog:

$$\omega = \frac{\Delta \theta}{\Delta t}$$

The units are typically radians/second, but revolutions/minute (rpm) are also common units. The direction is defined based on whether the speed is clockwise or counter-clockwise. The rule to follow is to use your right hand and wrap it in the direction of motion. If the angular speed is counter-clockwise, your thumb points up (positive direction). This is the direction of the vector associated with counter-clockwise motion. If the angular speed is clockwise, your thumb points down (negative direction). This is the direction of the vector associated with clockwise motion. This is just a convention that has been defined arbitrarily because it's easier to define vectors up and down than it is to define them clockwise and counter-clockwise.

Let's try an example:

A gymnast on a high bar swings through two (clockwise) revolutions in a time of 1.90s. Find the angular velocity of the gymnast.

The angular displacement of the gymnast is

$$\Delta \theta = -2 \text{ rev} \left(\frac{2\pi \text{ rad}}{1 \text{ rev}} \right) = -12.6 \text{ rad}$$

$$\omega = \frac{\Delta \theta}{\Delta t} = \frac{-12.6 \text{ rad}}{1.90 \text{ s}} = -6.63 \text{ rad/s}$$

We can talk about the rate of angular change, but what if we want to know the linear velocity of the circular motion? (Figure 7.6, pp.224)

There is a very easy relationship:

$$v = r\omega$$

ω has to be in radians per second. This means v will be in meters per second.

Let's try an example:

The tangential speed of a particle on a rotating wheel is 3.0m/s. If the particle is 0.20 m from the axis of rotation, how long will it take for the particle to go through one revolution?

So what do we need to do?

go from tangential velocity to angular velocity, then use the fact that we know there are 2π radians in one revolution to get the amount of time it takes to complete one revolution.

$$\omega = \frac{v}{r} = \frac{3m/s}{0.2m} = 15 \text{ rad/s}$$

$$\frac{2\pi \text{ rad}}{15 \text{ rad/s}} = 0.42 \text{ s}$$

Of course now that we've covered angular velocity the next logical thing to cover is angular acceleration.

It is defined in terms of the change of angular velocity over time:

$$\alpha = \frac{\Delta\omega}{\Delta t} \quad \text{the units of angular acceleration are commonly radians/s}^2.$$

We can do a quick example to demonstrate how to use angular acceleration:

During an acceleration, the angular speed of an engine increases from 700 rpm to 3000 rpm in 3.0s. What is the average angular acceleration of the engine?

$$\omega_o = 700 \text{ rev/min} \left(\frac{2\pi \text{ rad}}{1 \text{ rev}} \right) \left(\frac{1 \text{ min}}{60 \text{ s}} \right) = 73 \text{ rad/s}$$

$$\omega = 3000 \text{ rev/min} \left(\frac{2\pi \text{ rad}}{1 \text{ rev}} \right) \left(\frac{1 \text{ min}}{60 \text{ s}} \right) = 314 \text{ rad/s}$$

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{314 \text{ rad/s} - 73 \text{ rad/s}}{3 \text{ s}} = 80 \frac{\text{rad}}{\text{s}^2}$$

Each second the angular velocity of the engine is increasing by 80 rad/s.

Something to point out now is that just as we had a set of equations for linear displacement, velocity, acceleration \Rightarrow we have a set of kinematic equations for angular displacement, velocity, and acceleration. (Table 7.2, p.236)

We'll come back during the next lecture to work a problem utilizing these equations, but for now we're going to segue into ideas of rotational force.

Recall that we had Newton's second law which told us $F_{\text{net}} = ma$. This implies that force and acceleration are connected. We've just defined angular acceleration, therefore there must be some rotational force connected to it. In physics we do define a rotational force. We call it torque. The basis of torque is the idea that to produce a change in rotational motion, we need a rotational force.

Let's think about what factors might be involved in a rotational force. If you think about trying to loosen a tight bolt, how do you maximize this effort?

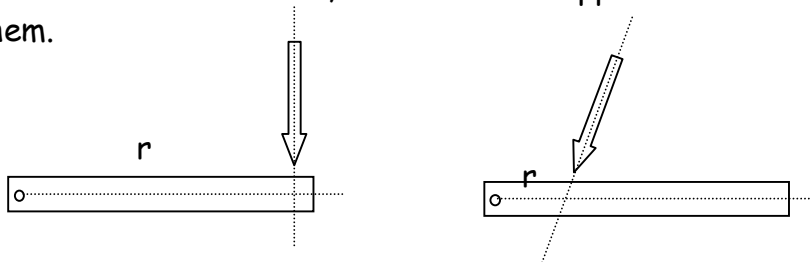
First, do you hold the wrench close to the head or toward the end of the handle?

Does it matter how hard you push?

These questions make sense when you consider the definition of torque:

$$\tau = rF \sin \theta$$

r is called the lever arm, F is the force applied and θ is the angle between them.



Obviously, the first case is going to have the maximal effect. The lever arm is longer and the angle between the force and lever arm is 90° . The biggest that the sine function can be is 1. So by exerting the force at 90° , we're maximizing the torque. Also, by making the lever arm (r) as long as possible, we're also maximizing the torque.

Let's try a problem utilizing balanced torques:

We have three masses suspended from a meterstick. The question is how much mass must be suspended from the right side of the meterstick to be in equilibrium (no angular acceleration).

We have four forces to deal with here: Force of gravity on the meterstick, weight of m_1 , weight of m_2 , weight of m_3 (unknown).

So what we can do is to draw a diagram showing all the potential torques:

