Lecture 16 Energy

Conservation of Energy (including W_{nc} and W_{ext})
Elastic vs. Inelastic Collisions

Another issue we haven't dealt with yet is what to do when a force like friction does work. Work is a transfer of energy, so if we're dealing with conservation of energy we have to include it somehow. I'm going to write down a more complete statement of conservation of energy now:

$$\Delta K + \Delta U + W_{nc} = W_{ext}$$

This formula now includes all the possible energy transformations and transfers that can occur. So far we've only analyzed systems where conservative forces and no external forces have been present. Now we'll introduce non-conservative forces and external forces.

Conservative and non-conservative forces are defined with respect to the work they do. A conservative force can do work to transfer energy from one system to another (gravity can do work to transfer energy from potential to kinetic, spring forces are also conservative) so that mechanical energy can be conserved. A non-conservative force can do work to transfer energy from the system to the environment (friction can do work to transfer energy to heat the environment) so that mechanical energy cannot be conserved. External forces (like a push or pull) transfer energy into the system, again mechanical energy cannot be conserved under an external force.

Let's go through some scenarios and see if we can list the basic elements:

- which objects are included in a system
- conservative interaction forces, non-conservative interaction forces, external forces
- Specify the value of ΔK, ΔU, W_{nc}, W_{ext}.
- Energy transformations
- Energy transfers

Pushing a block across a table (with friction) at constant speed.

objects in system: block

external force: push, non-conservative force: friction

$\Delta K = 0$, $\Delta U = 0$, $W_{nc} < 0$, $W_{ext} > 0$

The work done by the push is positive (energy going into system), the work done by friction is negative (energy leaving the system). These contributions are equal and opposite.

The block sliding to a halt after you release it.

objects in system: block

non-conservative force: friction

$\Delta K < 0$, $\Delta U = 0$, $W_{nc} < 0$, $W_{ext} = 0$

Energy is transferred from the kinetic energy system out to the environment through the work done by friction.

Picking up the block and placing it on a high ledge.

objects in system: block

external force: lifting force,

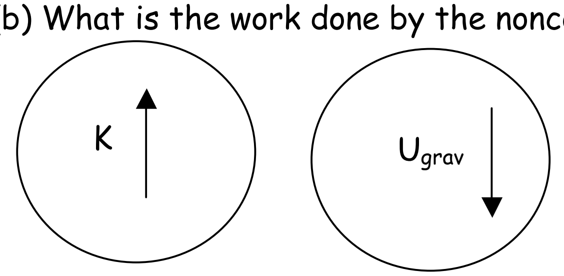
$\Delta K = 0$, $\Delta U > 0$, $W_c > 0$, $W_{nc} = 0$, $W_{ext} > 0$

There is no change in kinetic because initial and final velocity are zero. The positive work done by the lifting force is transferred into the gravitational potential energy system by the conservative force of gravity.

Let's try a problem including the full statement of conservation of energy:

A 50 kg student on a sled starts from rest at a vertical height of 20 m above the horizontal base of a hill and slides down. (a). If the sled and the student have a speed of 10 m/s at the bottom of the hill, this system is: conservative, nonconservative, or neither. Why?

(b) What is the work done by the nonconservative force?



$$\Delta K + \Delta U = 0$$

Let's assume that we have a conservative system, that there are no non-conservative forces taking energy out of the system and putting it into the environment. We'll put our reference level at the bottom of the hill so that U=0 there.
If mechanical energy is conserved:

$$(K+U)_i = (K+U)_f$$

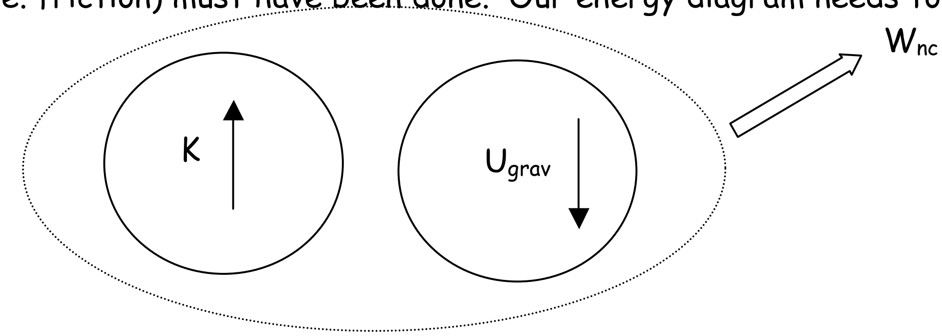
$$K_i = 0 \qquad U_f = 0$$

$$U_i = K_f$$

$$U_i = mgy_i = (50kg)(9.8m/s^2)(20m) = 9800J$$

$$K_f = \frac{1}{2}mv_f^2 = (0.5)(50kg)(10m/s)^2 = 2500J$$

Somehow, we've lost energy. In the final state we only have 2500J of energy when we started with 9800J. So work by non-conservative forces (i.e. friction) must have been done. Our energy diagram needs to change:



We can even find the work done by the non-conservative force. It's just the difference between the initial and final energy.

$E_i - E_f$ = 9800J – 2500J = 7300J

So, this system is a non-conservative system because friction did 7300 J of negative work and gave that energy to the environment.

Recall that we discussed collisions briefly with conservation of momentum. But we never discussed the difference between elastic and inelastic collisions. But now that we know about energy, we can understand the difference.

In an elastic collision, the total <u>kinetic</u> energy of a system is conserved. For instance, two billiard balls colliding represents an elastic collision. Kinetic energy may be exchanged between the two balls, but $K_i = K_f$. (Figure 6.13 pp.197)

In an inelastic collision, the total <u>kinetic</u> energy of a system is not conserved. Energy is lost through work done by non-conservative forces. Heat, sound, or light may be generated. You may permanently deform one of the objects involved in the collision. You can think of a clay ball colliding with a billiard ball. In an inelastic collision $K_f < K_i$. (Figure 6.12 pp.195)

Let's do an example of an elastic collision:

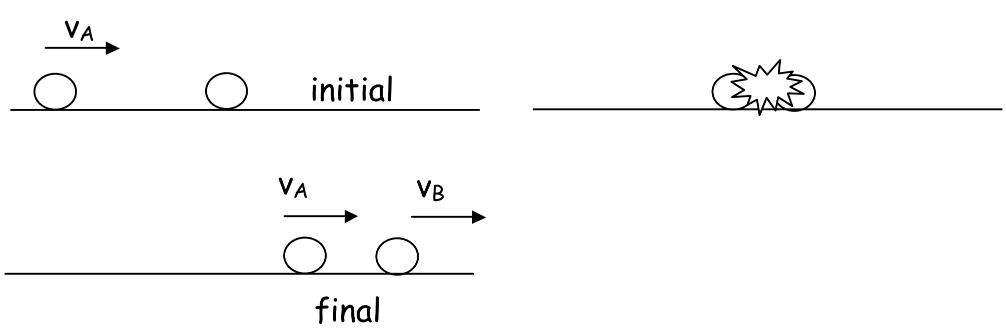
A 4.0 kg ball with a velocity of 4 m/s in the +x-direction collides head on elastically with a stationary 2.0 kg ball. What are the velocities of the balls after the collision?

In an elastic collision we can say:

$$K_i = K_f$$

$$P_i = P_f$$

So let's utilize these relationships along with a diagram to solve the problem.



$$(K_A + K_B)_i = (K_A + K_B)_f$$

$$(K_A)_i = (K_A + K_B)_f$$

$$\frac{1}{2} m_A (v_A^2)_i = \frac{1}{2} m_A (v_A^2)_f + \frac{1}{2} m_B (v_B^2)_f$$

$$m_A[(v_A^2)_i - (v_A^2)_f] = m_B (v_B^2)_f \qquad (1)$$

$$P_i = P_f$$

$$(p_A + p_B)_i = (p_A + p_B)_f$$

$$(m_A v_A)_i = (m_A v_A + m_B v_B)_f$$

$$m_A[(v_A)_i - (v_A)_f] = m_B (v_B)_f \qquad (2)$$

Using equations 1&2 we can go through some algebra (p.197) to come up with a third equation we can use:

$$(v_A)_i + (v_A)_f = (v_B)_i$$

From there you can do some more algebra to come up with equations for $(v_A)_f$ and $(v_B)_f$:

$$(v_A)_f = \left(\frac{m_A - m_B}{m_A + m_B} \right) (v_A)_i$$

$$(v_B)_f = \left(\frac{2m_A}{m_A + m_B} \right) (v_A)_i$$

Notice that the final velocity of ball A and ball B depends on the initial velocity of ball A. This makes sense since ball A is the only ball carrying momentum in the initial state.

If we look at three special limiting cases, these equations make sense. (p. 199)

If $m_A = m_B$, ball A stops and ball B shoots off.

If $m_A >> m_B$, ball A maintains most of its initial velocity and ball B shoots off with lots of velocity. (Think bowling ball hitting golf ball.)

If $m_A \ll m_B$, ball A recoils off ball B and ball B basically doesn't move. (Think golf ball hitting bowling ball.)

So now let's refer back to our original problem.

$$(v_A)_f = \left(\frac{m_A - m_B}{m_A + m_B}\right)(v_A)_i = \left(\frac{4kg - 2kg}{4kg + 2kg}\right)(4m/s) = 1.3m/s$$

$$(v_B)_f = \left(\frac{2m_A}{m_A + m_B}\right)(v_A)_i = \left(\frac{2(4kg)}{4kg + 2kg}\right)(4m/s) = 5.3\,m/s$$

The small ball shoots off with a quick velocity while the larger ball slows down slightly.