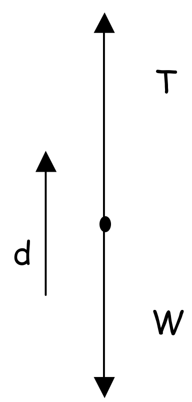
Lecture 14 Energy

Problem from last time
Spring Potential Energy
Gravitational Potential Energy

Recall last time a caffeinated drink was promised to the person who could solve this problem in terms of Work and Energy.

A 200g ball is lifted upward on a string. It goes from rest to a speed of 2m/s in a distance of 1m. What is the tension (assumed to stay constant) in the string?



We can solve this first with Newton's Second Law:

$F_{net} = ma$
$T - W = ma$, but what is a?

First, we need to solve for the amount of time it takes:
$x = x_0 + \frac{1}{2}(v + v_0)\Delta t$
$\Delta t = 2(x - x_0)/(v + v_0) = 2\,(1m)/(2m/s) = 1s$

Now we can find $a = \Delta v/\Delta t = (2m/s)/(1s) = 2m/s^2$
$T = W + ma = (.2kg)(9.8m/s^2) + (.2kg)(2m/s^2) = 2.36N$

Now let's use the Work/Energy approach:
$W_T = Td\cos\theta = Td$
$W_w = Wd\cos\theta = -Wd$

$W_{net} = Td - Wd$
At this point we can ask, "what else do we know about energy in this problem?" We know that the net work has to be positive because the ball is speeding up, i.e. there is energy being put into the system. We know also that the energy that is increasing is kinetic, so we can equate the two expressions:

$\Delta K = W_{net}$
$K_f - K_i = W_{net}$

$\frac{1}{2}mv^2 = Td - Wd$

$T = (\frac{1}{2}mv^2 + Wd)/d = [\frac{1}{2}(.2kg(2m/s)^2) + (.2kg)(9.8m/s^2)(1m))]/(1m) = 2.36N$

Both approaches are equivalent – often times energy will yield the answer faster.

We know all about kinetic energy now, but we've mentioned potential energy a couple of times and haven't done anything with it yet. Today we'll do some work to try and understand the potential energy of a spring. A spring is just one example of a system than can have potential energy, but it's a good one to start with.

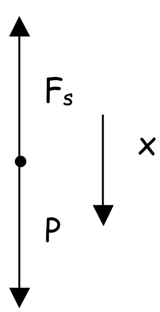
Le't recap what we talked about last time with respect to the springs. Recall the spring constant tells us about the springiness or stiffness of the spring. The stiffer the spring, the higher the spring constant.
The units of the spring constant are force/length (Newtons/meter).

We get this from looking at Hooke's law: $F_S = -k\Delta x$, if we solve for k, we get $k = F_S/\Delta x$

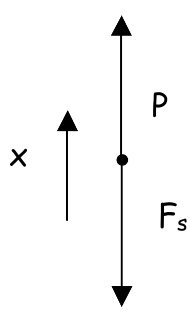
So the spring constant tells you for every meter you stretch (compress) the spring the number of Newtons of force that it will pull (push) back with.

Let's look at a real spring to study this relationship:

If I pull the spring down, it pulls up.



Notice that if I pull the spring down, the force of the spring acts up. This is the negative sign in Hooke's law. The force is opposite the displacement. If I compress a stiff spring, the force is then down.



Now that we have Hooke's Law we are ready to try a physics problem involving a spring.

A 20cm long spring is attached to a wall. The spring stretches to a length of 22cm when pulled horizontally with a force of 100 N. What is the value of the spring constant?
$F_s = -k\Delta x$, so $k = F/\Delta x = (100\text{ N})/(.22\text{ m} - .20\text{ m}) = 5000\text{ N/m}$ (Notice that I didn't use the negative sign. This is because the spring constant is a positive quantity. It is just a constant that tells is how stiff/squishy our spring is.

The same spring is now used in a tug-of-war. Two people pull on the ends, each with a force of 100N. How long is the spring while it is being pulled?

We know the spring resting is 20 cm long. Also we can use Hooke's law to find the amount it has been stretched on each end.

Assume we take the +x direction: (if we pull in the positive direction, $F_s < 0$)
$\Delta x = -F_s/k = -(-100\text{N})/(5000\text{N/m}) = 0.02\text{ m} = -2\text{ cm}$

So, since each end is being pulled, we have a total stretch of 4 cm and now including the resting length (20 cm)…the total length of the spring is 24 cm.

Just to remind you, the whole discussion of springs came about because I was trying to segue into potential energy. Now that we know about Hooke's Law, we know enough to talk about potential energy. So if I do work on a spring to compress it for instance, I put energy into it. If I hold it, where has that energy gone? It has gone into potential energy. The compressed spring has potential energy.

One argument we can make in favor of this statement is that if I release the spring it will gain kinetic energy (speed up). Where does the kinetic energy come from? Potential energy! Potential energy is converted to kinetic when the spring is released.

It turns out the potential energy of a spring is related to the quantities that force depends on (Recall that $F_s = -k\Delta x$):
$E_s = \frac{1}{2} k\Delta x^2$

By unit analysis, we can figure out that this is right.
N•m = N/m • m^2 = N•m = Joule, so it has units of energy, yay!

We can try a new problem:
A 0.30 kg block sliding on a horizontal frictionless surface with a speed of 2.5 m/s strikes a light spring that has a spring constant of 3.0•10^3 N/m.

(a) What is the total mechanical energy of the system? (0.94 J)

$$E_{mech} = K + U$$

$$K = \frac{1}{2}mv^2 = (0.5)(0.30kg)(2.5m/s)^2 = 0.94J$$

$$U = 0 \text{ since block is sliding and spring is not compressed yet}$$

(b) What is the kinetic energy K_1 of the block when the spring is compresed a distance x_1 = 1.0cm? (0.79J)

As the block hits the spring, it slows down.

Kinetic energy is decreasing, but spring potential

is increasing.

We can calculate the amount of energy lost

lost to spring potential:

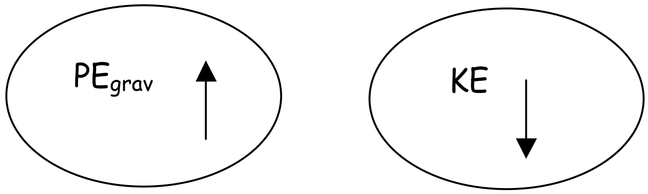
$$U = \frac{1}{2}k\Delta x^2 = (0.5)(3000N/m)(0.01m)^2 = 0.15J$$

Now we can take the initial Kinetic and subtract this loss from it:

$$K_{final} = 0.94J - 0.15J = 0.79J$$

We've alluded to conservation of energy in this problem. Kinetic energy was transformed into spring potential energy, i.e. the total energy of the system in the initial and final states was the same.

If we toss a ball up into the air, it starts out with a lot of kinetic energy, right. As it moves, it slows down though. So the kinetic energy went somewhere. Where did it go? It went into potential energy. Gravity did work → work is just a transfer of energy → energy was transferred into a potential energy system. Here's where we introduce the energy interaction model.



ΔPE + ΔKE = 0

This is the equation that describes the conservation. Any increase in potential energy is equal and opposite to the decrease in kinetic energy. But how to calculate Potential Energy?

The formula depends on changes in height: $\Delta PE = mg\Delta y$. It makes sense that if gravity is doing work here, it should depend on the force of gravity (mg) and the displacement (distance traveled) Δy.

Now let's try to solve a problem using this model and our knowledge of energy. If the initial velocity of the thrown ball is 5m/s, what is its maximum height?

$$\Delta PE_g + \Delta KE = mg\Delta y + \frac{1}{2}m\Delta v^2 = 0$$

$$mg\Delta y = -\frac{1}{2}m\Delta v^2$$

$$\Delta y = -\frac{\Delta v^2}{2g} = -\frac{v_f^2 - v_i^2}{2g} = -\frac{0-25}{2(9.8)} = 1.3m$$

We did this whole problem without even thinking of Newton's Laws. Many times the energy picture will be a faster, easier way of solving problems. No messy vectors.