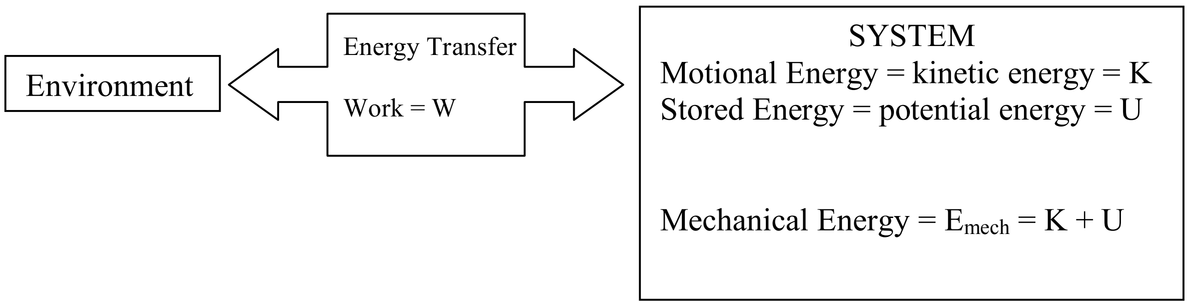
Lecture 13 Energy

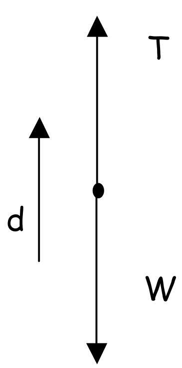
Last time we talked defined energy in various ways. Recall we talked about energy by making analogies to money. We talked about energy by developing a model:




Finally, we talked about the details of how work and energy are connected. Let's try a problem to review those concepts:

Compare two scenarios in terms of work and energy:
1) A ball is lifted at constant speed from the ground to a height of 1m.
2) A ball is lifted at increasing speed from the ground to a height of 1m.

First, we can draw a force diagram:



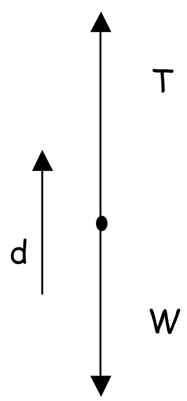

For something moving with constant speed, a = 0
so $F_{net} = 0 \Rightarrow T = W$

If we look at the work done:
$W_w = Wd\cos\theta = -Wd$ (since we have $\cos 180° = -1$)
$W_T = Td\cos\theta = Td$ (since we have $\cos 0° = 1$)

$W_{net} = W_T + W_w = Td - Wd = 0$ since $W = T$
So there is no net work done on this system.

From an energy standpoint this makes sense. We said that if work is positive, energy is entering the system and if work is negative, work is leaving the system. So what we're observing here is that the positive work done by the tension force in lifting the object is balanced by the negative work done by gravity. There is no net energy input or output from the system.

Now let's look at the second scenario, the ball speeding up:




If the ball is speeding up, there must be an acceleration since there is a change in velocity. Since the velocity is in the positive direction, the acceleration must be in the same direction.

$F_{net} \neq 0$, $T - W = ma \Rightarrow T = W + ma$

Now for our work picture:

$W_w = Wd\cos\theta = -Wd$ (since we have $\cos 180° = -1$)

$W_T = Td\cos\theta = Td$ (since we have $\cos 0° = 1$)

$W_{net} = W_T + W_w = Td - Wd$

$= (W + ma)d - Wd = Wd + mad - Wd = mad$

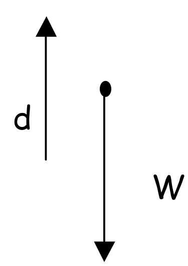
So we have net positive work!

This should make sense. We have net positive work:

- There is more energy going into the system than leaving
- We see the system is speeding up, i.e. kinetic energy is increasing

Let's introduce a third scenario: What if the net work of the system was negative. Work with a partner to draw a force diagram for this situation and come up with an equation for $W_{net.}$

One way to do this is to throw the ball up and let it slow down. The force diagram would then look like this:




$F_{net} = ma$

$- W = ma \Rightarrow a = - g$

From a work standpoint:

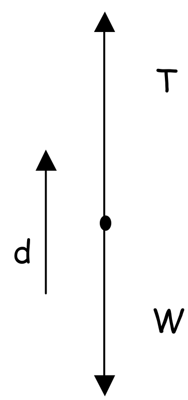
$W = mgd\cos\theta = -mgd$

$W_{net} = -mgd$

Clearly, we have negative net work in this situation. This means we have a net loss of energy to the environment. This makes sense, as the object is slowing down kinetic energy is decreasing, therefore energy is being lost.

We can now try an example with some numbers.

A 200g ball is lifted upward on a string. It goes from rest to a speed of 2m/s in a distance of 1m. What is the tension (assumed to stay constant) in the string?




We can solve this first with Newton's Second Law:

$F_{net} = ma$
$T - W = ma$, but what is a?

First, we need to solve for the amount of time it takes:
$x = x_0 + \frac{1}{2}(v + v_0)\Delta t$
$\Delta t = 2(x - x_0)/(v + v_0) = 2\,(1m)/(2m/s) = 1s$

Now we can find $a = \Delta v/\Delta t = (2m/s)/(1s) = 2m/s^2$
$T = W + ma = (.2kg)(9.8m/s^2) + (.2kg)(2m/s^2) = 2.36N$

Now let's use the Work/Energy approach:
$W_T = Td\cos\theta = Td$
$W_w = Wd\cos\theta = -Wd$

$W_{net} = Td - Wd$
At this point we can ask, "what else do we know about energy in this problem?" We know that the net work has to be positive because the ball is speeding up, i.e. there is energy being put into the system. We know also that the energy that is increasing is kinetic, so we can equate the two expressions:

$\Delta K = W_{net}$
$K_f - K_i = W_{net}$
$\frac{1}{2}mv^2 = Td - Wd$
$T = (\frac{1}{2}mv^2 + Wd)/d = [\frac{1}{2}(.2kg(2m/s)^2) + (.2kg)(9.8m/s^2)(1m))]/(1m) = 2.36N$

Both approaches are equivalent – often times energy will yield the answer faster.

We know all about kinetic energy now, but we've mentioned potential energy a couple of times and haven't done anything with it yet. Today we'll do some work to try and understand the potential energy of a spring. A spring is just one example of a system than can have potential energy, but it's a good one to start with.

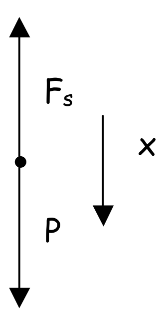
First let's talk about some intrinsic properties of a spring. One important property to physicists is the stiffness/springiness of a spring. We can quantify this using what's called a spring constant. A very stiff spring has a large spring constant and a spongy spring has a low spring constant. The real importance of defining a spring constant is that it gives you a sense of how much force a spring will pull or push with when it is displaced from its resting position.

The units of the spring constant are force/length (Newtons/meter)

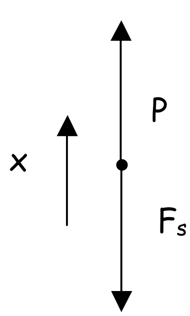
So the spring constant tells you for every meter you stretch (compress) the spring the number of Newtons of force that it will pull (push) back with.

The formula that this comes from is called Hooke's Law:
$F_s = -k\Delta x$
What this formula says is that the force of a spring is proportional to how much you stretch it ($\Delta x$) and the stiffness of the spring (k). Let's look at a real spring to study this relationship:

If I pull the spring down, it pulls up.




Notice that if I pull the spring down, the force of the spring acts up. This is the negative sign in Hooke's law. The force is opposite the displacement. If I compress a stiff spring, the force is then down.



Now that we have Hooke's Law we are ready to try a physics problem involving a spring.

A 20cm long spring is attached to a wall. The spring stretches to a length of 22cm when pulled horizontally with a force of 100 N. What is the value of the spring constant?
$F_s = -k\Delta x$, so $k = F/\Delta x = (100\ N)/(.22\ m - .20\ m) = 5000\ N/m$ (Notice that I didn't use the negative sign. This is because the spring constant is a positive quantity. It is just a constant that tells is how stiff/squishy our spring is.

The same spring is now used in a tug-of-war. Two people pull on the ends, each with a force of 100N. How long is the spring while it is being pulled?

We know the spring resting is 20 cm long. Also we can use Hooke's law to find the amount it has been stretched on each end.

Assume we take the +x direction: (if we pull in the positive direction, $F_s < 0$)
$\Delta x = -F_s/k = -(-100N)/(5000N/m) = 0.02\ m = -2\ cm$

So, since each end is being pulled, we have a total stretch of 4 cm and now including the resting length (20 cm)…the total length of the spring is 24 cm.

Just to remind you, the whole discussion of springs came about because I was trying to segue into potential energy. Now that we know about Hooke's Law, we know enough to talk about potential energy. So if I do work on a spring to compress it for instance, I put energy into it. If I hold it, where has that energy gone? It has gone into potential energy. The compressed spring has potential energy.

One argument we can make in favor of this statement is that if I release the spring it will gain kinetic energy (speed up). Where does the kinetic energy come from? Potential energy! Potential energy is converted to kinetic when the spring is released.

It turns out the potential energy of a spring is related to the quantities that force depends on (Recall that $F_s = -k\Delta x$):
$E_s = \frac{1}{2} k\Delta x^2$

By unit analysis, we can figure out that this is right.
N•m = N/m • $m^2$ = N•m = Joule, so it has units of energy, yay!

We can try a new problem:
A 0.30 kg block sliding on a horizontal frictionless surface with a speed of 2.5 m/s strikes a light spring that has a spring constant of 3.0•103 N/m. (a) what is the total mechanical energy of the system? (0.94 J) (b) What is the kinetic energy $K_1$ of the block when the spring is compressed a distance $x_1$ = 1.0cm? (0.79J)