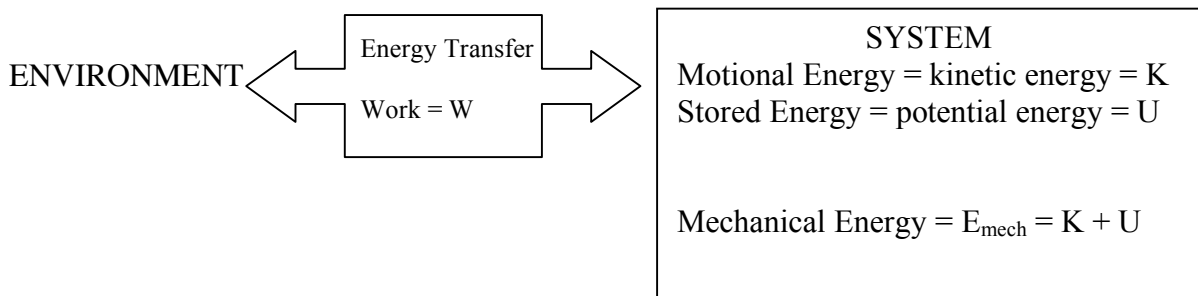Lecture 12 Energy

We are now at the point where we can talk about one of the most powerful tools in physics, energy. Energy is really an abstract concept. We have indicators of energy (temperature, velocity for example) – things that imply the existence of energy – but we can't reach out and grab energy. Something that we understand well is money. We can make a direct analogy between energy and money.

- money, like energy, has no single, unique definition
- one vague definition of energy is "the ability to do work"
- one vague definition of money is "the ability to purchase goods"
- both can be transformed and transferred in various ways
- kinetic energy = cash
- potential energy = savings account
- work = getting a paycheck or paying bills

So now that we've mentioned these terms : kinetic, potential, work we should try to explain them more. Let's develop a model:

ENVIRONMENT

Energy Transfer

Work = W

SYSTEM
Motional Energy = kinetic energy = K
Stored Energy = potential energy = U

Mechanical Energy = E_{mech} = K + U

Work in the physics definition is a mechanical transfer of energy to or from a system by the pushes and pulls of external forces. When energy is added to a system, work is positive, and when energy is removed from a system, work is negative.

Here we can get into thinking of two different concepts: a property vs. a process. Energy is a property of a system (the system has a certain amount of energy) versus a process like work (work changes the energy of the system, it is not something the system contains). Another way of seeing this is that we can define changes in a property (ΔE, ΔK, ΔU), but we would never define changes in a process ΔW. We can't define $W_{initial}$ and W_{final} the way

we can define $E_{initial}$ and E_{final}. Work is an energy transfer – by definition a system can't "contain" a transfer.

Now we can start filling in the details of our model. We've mentioned kinetic energy as an energy of motion, but we can be even more specific: $K = \frac{1}{2}mv^2$ you see a direct dependence on velocity. The faster an object moves, the more kinetic energy it has. The more mass it has, the more kinetic energy it has. Now we can ask the question, what causes kinetic energy to change? What causes changes in velocity (i.e. acceleration)? Looking at Newton's Second Law, we can see that force changes velocity, and in turn causes changes in kinetic energy. How can we then calculate those changes? What we're really calculating is a transfer of energy (work) due to a force exerted on the system. Let's stop for a moment and consider a specific example.

If a cart is moving on a track, the way we would slow it down would be to exert a force on it.

You do negative work on the system because the cart slows down, i.e. energy is leaving the system.

So we can infer W∝F, but if we look at units we are missing something: Force has units of force (Newtons) while work should have units of energy. But what are units of energy? Let's go back to our definition of Kinetic Energy, $K = \frac{1}{2}mv^2 = kg \bullet \left(\frac{m}{s}\right)^2 = kg \bullet \frac{m}{s^2} \bullet m = N \bullet m = Joule(J)$

So the unit of energy is a Newton times a meter, otherwise defined as a Joule. Back to our work relationship W∝F, but there's something missing. If work has to be defined in Joules (Newtons × meters) it follows that we should include a distance in the formula. The actual formula is W = Fdcosθ . where does the angle come in? If we go back to our example of the cart on the track

In this picture the angle between F and d (displacement of the cart) is 180°. This gives us W = Fdcos180° = -Fd. What if we exerted a force at another angle?
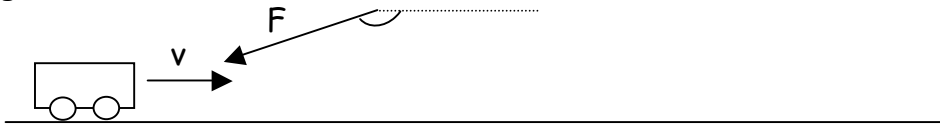
Now the angle is < 180°, estimate 150°. W = Fdcos150° = -0.86Fd. We can see that this is less than when the two were 180° apart. We can see that stopping the cart would be more efficient if we pushed directly against the cart, as opposed to pushing at an angle, i.e. we can transfer more energy out of the system: Fd > 0.86Fd. The cosine term accounts for this difference in efficiency.

Let's try an example using work and energy. Take a cart attached to a mass/pulley system.
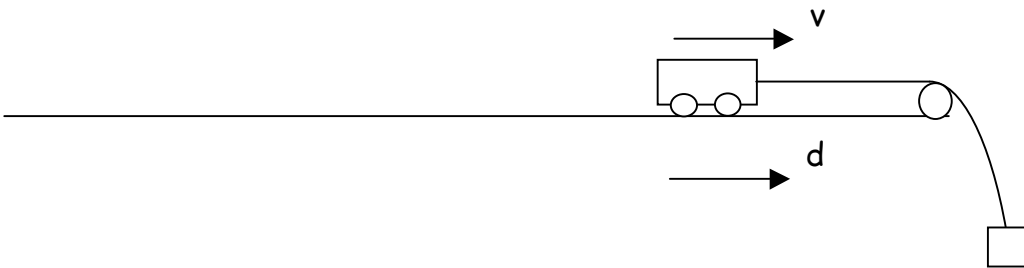
If we allow the mass to pull the cart, we can then analyze the forces involved and see which ones do work:
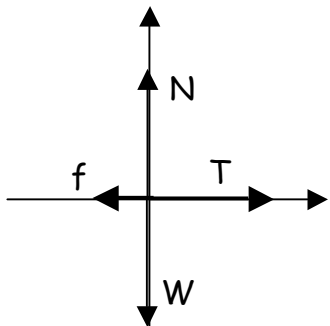

First, let's look at the normal force and weight. If Work = Fdcosθ, where θ is the angle between F and d, the normal force and weight are at an angle of 90° with respect to the displacement. Therefore they do no work. They do not affect the (kinetic) energy of the system.

Now for friction and Tension. First tension – we can predict whether it does positive or negative work. We know the cart is speeding up. Therefore there must be some positive work being done on the system. Now let's look at the work done by the tension force: W = Tdcos0° = Td. This is definitely

positive, and we observe that our system is speeding up. So these two facts support each other. Now for friction: W = fdcos180° = -fd. Friction is doing negative work, which would imply that the system is losing energy i.e. it should be slowing down. We can make two observations. First, if we removed the tension, the cart would slow down, friction would transfer the kinetic energy out of the system. With the tension in the system, we can imply that since the system is speeding up, the positive work must be winning out over the negative work, that is:

Td > fd or Td – fd > 0 so there is a net transfer of energy into the system and we can see this by virtue of the fact that the cart is speeding up. Velocity is an indicator of Kinetic Energy. If velocity increases so does Kinetic Energy (K = ½ mv^2).

Now let's look at the reverse situation: if I push the cart up the track against the force of tension in the string.
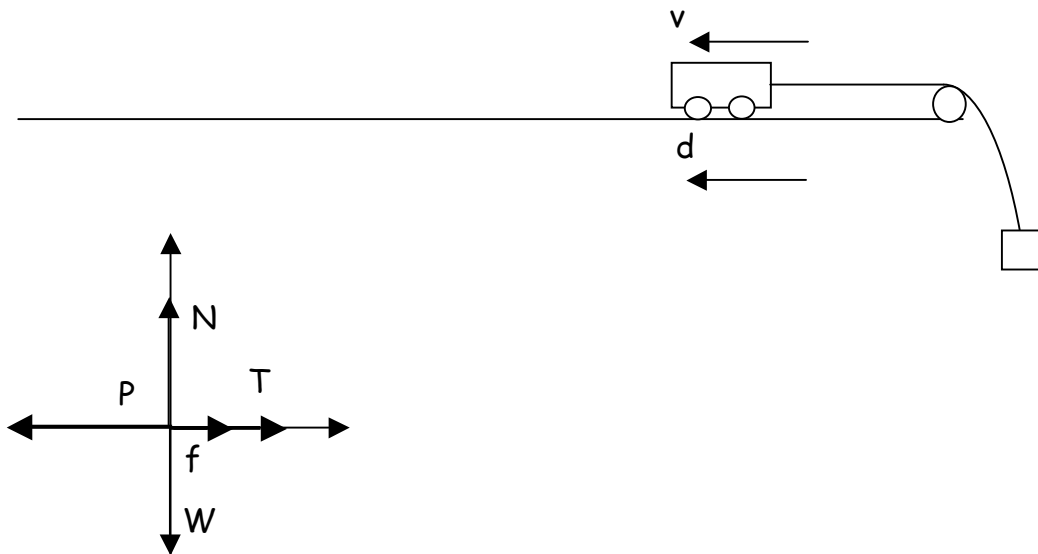
Now, if we assume that there is no acceleration in the system, I'm pushing the cart with a constant velocity, then there should be no net work in the system. W_{net} = 0 since Δv = 0 which implies ΔE = 0. The energy leaving the system (negative work) balances the energy entering the system (positive work). So let's break the work down force by force. The normal force and weight do no work because they are perpendicular to the displacement. The work done by my pushing is positive W = Pdcos0° = Pd. I am putting energy into the system. The tension and friction are doing negative work, W_T =

Tdcos180° = -Td, W_f = fdcos180° = -fd. We can come up with an equation from all these expressions:
Wnet = Pd – Td – fd = 0 (assuming a = 0)

Let's summarize our findings:
- work is a transfer of energy into (W > 0) or out of (W < 0) a system
- friction does negative work (always acts opposite displacement)
- normal force does no work (always acts perpendicular to the displacement)

Now let's analyze a new scenario. If I swing a ball over my head (in a horizontal circle) at a constant speed, what is the work done in this problem?

F_c
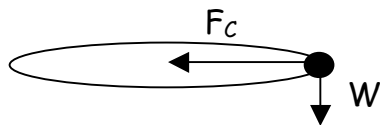
First, we label the forces.

W

The displacement vectors point tangent to the circle (top view) :
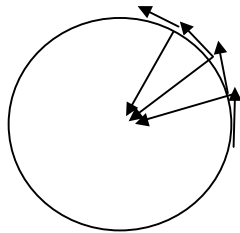
The force is always perpendicular to the displacement in this case. There is no work done by this force, nor is there work done by the weight (which is also perpendicular to the displacement).