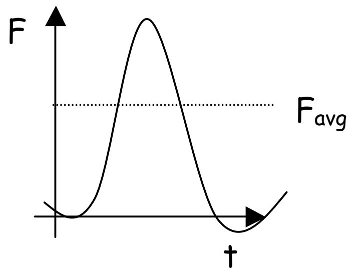
Lecture 11 Impulse and Momentum

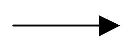
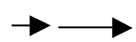
We last left off by talking about how the area under a force vs. time curve is impulse.

Recall that for our golf ball we had a strongly peaked force curve:



You have to have a changing force because as you initiate contact, you have a rapid increase and as you take the club away, you lose that contact. If we find the impulse (Δp), what we've found is the area under the F_{avg} curve since impulse = $\Delta p = F_{avg}\Delta t$.

So now that we know all about impulse, we can talk about conservation of momentum. Remember that conservation means initial = final. Recall that we did a specific example where mass was conserved.

Let's look at a situation where we have a collision. Collisions are the archetypal situation for studying momentum. We have two balls of equal mass, one moving toward the other at rest. The initial momentum of this system is easy. It's the momentum of ball 1. 
Now consider the final momentum. Ball 1 strikes ball 2 and it shoots off with a lot of momentum while ball 1 moves with just a little momentum. So the final momentum is the addition of these two. 
In this way we can see the initial and final momentum match. It is just a matter of how the components of the system share the momentum.
So how do we justify this conservation. It's not exactly an obvious conclusion. We can start from Newton's Second Law:

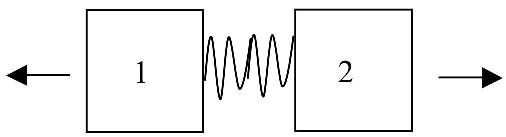
$$F_{net} = ma = m\frac{\Delta v}{\Delta t} = \frac{mv_f - mv_i}{\Delta t} = \frac{p_f - p_i}{\Delta t} = \frac{\Delta p}{\Delta t}$$

$$F_{net} = \frac{\Delta p}{\Delta t}$$

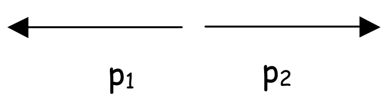
From here we can ask the question, what happens when $F_{net} = 0$?

$$F_{net} = \frac{\Delta p}{\Delta t} = 0 \Rightarrow \Delta p = 0, so\ p_f = p_i$$

The significance of this is that when there is no net external force in a system, momentum is conserved. Now let's do an example to clarify some of the details. We take a system of two blocks separated by a spring.



If we allow the spring to expand and push the blocks away from one another we can use the conservation of momentum to analyze the system. The spring force is an internal force to the system (no external forces, like friction), so we can say that $F_{net} = 0$, and hence, $\Delta p = 0$. We then define our initial and final conditions. Initially, the system is at rest. In the final state you have two separate contributions to the momentum, p_1 and p_2.



Now, let's assign some quantities to the problem. Let m_1 = 1.0kg and m_2 = 2.0kg. If m_1 has a velocity of 1.8 m/s to the left, what is the velocity of m_2?

Because the only force acting in this problem is the spring which is internal to the system we can employ conservation of momentum. Recall that we can use the conservation of momentum since there are no external forces (i.e. friction).

$$\Delta P = 0$$

$$P_i = P_f$$

$$0 = p_2 - p_1$$

$$p_2 = p_1$$

$$m_2 v_2 = m_1 v_1$$

$$v_2 = \frac{m_1 v_1}{m_2} = \frac{(1.0kg)(1.8m/s)}{(2.0kg)} = 0.9m/s, \textit{to the right}$$

Another good example to try:

A 2000 kg car is traveling north at 15 m/s when it overtakes and crashes into a 5000 kg truck also traveling north and moving with a speed of 10 m/s. Find the velocity of the combined wreckage the instant after the collision.

Initially we have the momentum of the car and truck separately. In the final state we have the collective velocity of the car and truck stuck to each other.

$P_i = p_{car} + p_{truck}$
$P_f = p_{car+truck}$
Since there are no external forces (the forces of the car crashing into the truck are internal) → $\Delta P = 0$, so $P_i = P_f$
$Pi = m_{car}v_{car} + m_{truck}v_{truck}$
$Pf = (m_{car}+m_{truck})V$
$m_{car}v_{car} + m_{truck}v_{truck} = (m_{car}+m_{truck})V$
$V = (m_{car}v_{car} + m_{truck}v_{truck})/(m_{car}+m_{truck}) = (30000+50000)/(7000) = 11.4$ m/s
This has the right units, and it also falls between 10m/s and 15m/s.

A 10g bullet is fired into a 1kg wood block, where it lodges. Subsequently, the block slides 4.00m across a wood floor ($\mu_k = 0.2$ for wood on wood). What was the bullet's speed?

First we need to define our initial and final conditions:

Initially we have the bullet moving and the block at rest:

$P_i = p_{bullet}$ $P_f = P_{block+bullet}$

$\Delta P = 0$ since there are no external forces.

$P_i = P_f$

$p_{bullet} = p_{block+bullet}$

$mv = (M + m)V$

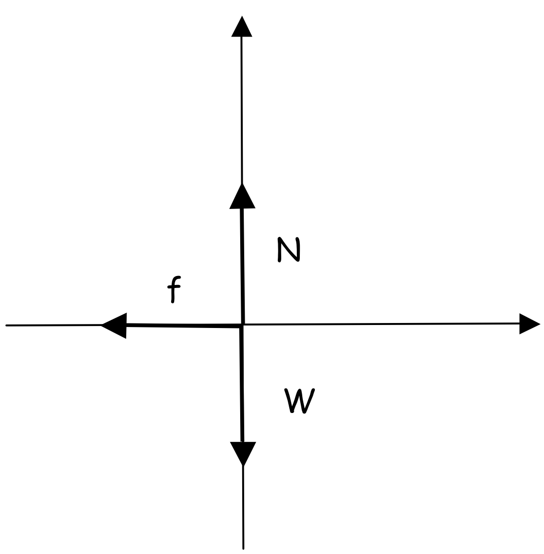
$v = \frac{(M + m)V}{m}$

We have all the masses, but we don't know the velocity of the mass+block system. So we have to go back to our kinematics equations.

We can list our knowns and unknowns:
v_o = ?
v_f = 0
Δx = 4.0m
a = ?

But we can find a using Newton's Second Law!
$F_{net} = ma$

We should go ahead and make a free body diagram:



$(F_{net})_x = -f = ma_x$

$f = \mu N$

$\Rightarrow -\mu N = ma_x$

$(F_{net})_y = N - W = ma_y = 0$

$N = W = mg$

$a_x = \frac{-f}{m} = -\frac{\mu mg}{m} = -\mu g = -0.2(9.8) = 1.96 = -2.0 m/s^2$

Now we can go back to find the velocity using kinematic equations:

$$v^2 = v_o^2 + 2a\Delta x$$

$$v_o = \sqrt{v^2 - 2a\Delta x}$$

$$v_o = \sqrt{0 - (2)(-2m/s^2)(4.0m)}$$

$$v_o = 4m/s$$

Now recall the original solution we came up with for the bullet:

$$v = \frac{(M+m)V}{m}$$

$$v = \frac{(1kg + .01kg)(4m/s)}{(.01kg)} = 404m/s$$