This may be translated in terms of the partial wave analysis by the following equations:

$$\text{For } \eta_1 = -1: \quad p = \sqrt{\tfrac{6}{5}}\, T_{\frac{1}{2}} + \sqrt{\tfrac{4}{5}}\, T_{\frac{3}{2}} \qquad (10)$$

$$f = -\sqrt{\tfrac{4}{5}}\, T_{\frac{1}{2}} + \sqrt{\tfrac{6}{5}}\, T_{\frac{3}{2}} \qquad (11)$$

$$\text{For } \eta_1 = +1: \quad d = -\sqrt{\tfrac{2}{7}}\, T_{\frac{1}{2}} - \sqrt{\tfrac{12}{7}}\, T_{\frac{3}{2}} \qquad (12)$$

$$g = \sqrt{\tfrac{12}{7}}\, T_{\frac{1}{2}} - \sqrt{\tfrac{2}{7}}\, T_{\frac{3}{2}}\, , \qquad (13)$$

where p, f, d and g are the amplitudes of the partial waves. In this way we obtain for the ratio's:

$$\text{For } \eta_1 = -1: \quad |p/f| = 1.8 \begin{smallmatrix} +0.9 \\ -0.5 \end{smallmatrix} \qquad (14)$$

$$\text{For } \eta_1 = +1: \quad |d/g| = 0.65 \pm 0.17 \qquad (15)$$

So the data of table 1 imply that f and g can not be neglected compared with p and d respectively. This is not surprising although in the partial wave analysis for (spin 0 + spin $\frac{1}{2}$) systems higher waves are usually suppressed. This effect need not necessarily be expected here, since one deals with a (spin $\frac{3}{2}$ + spin 0) system. In the (0, $\frac{1}{2}$) system only one partial wave is defined for total angular momentum $\frac{5}{2}$ and given parity, in the (0, $\frac{3}{2}$) system these quantum numbers allow for two partial waves.

In this analysis it has been assumed, following Armenteros et al., that interference effects with the background can be neglected. The smallness of the odd momenta in table 1 supports this approach. Then one can use these data to say something about the parity of the $Y_1^*(1765)$. For this purpose one needs some model of the interaction.

If the matrix element is assumed to be the same for decay into a wave l or $l+2$, the last one will still be suppressed by the centrifugal barrier, certainly for low values of the momenta of the outgoing particles. In reaction (1) the value of the momentum is 1.3, expressed in the pion rest-mass, which is already a relativistic value for the outgoing pion. So a potential model for the interaction is not expected to be good. If we adopt an empirical formula as given by Lichtenberg [3] and many others:

$$\Gamma \approx k \left\{ \frac{(kR)^2}{1 + (kR)^2} \right\}^l , \qquad (16)$$

in which R is of the order of an inverse pion mass, we get for the ratios (14) and (15) a value 2 if $kR = 1$ and a value 1.25 if $kR = 2$. This favours the minus sign of η_1.

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ON THE APPARENT SHIFT OF THE RHO MESON MASS IN PHOTOPRODUCTION

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For two-pion photoproduction, the interference between ρ^0 production and the "Drell mechanism" is shown to shift the apparent rho meson mass by about 25 MeV. This can explain the mass shift experimentally observed for ρ^0 production.

Recently several experiments have been done on photoproduction

$$\gamma + p \rightarrow \rho^0 + p \qquad (1)$$

of the neutral rho meson [1-4] at photon energies of a few GeV. In all of these, the mass value at which the rho meson peak appeared was found to be 25 to 35 MeV lower than the usually accepted value [5] of 765 MeV. It is the purpose of this note to show that there exists a natural explanation for this mass shift, as being due to an interference effect.

Our basic assumption is that the amplitude of reaction (1) is mainly absorptive. This may be justified by the observation that the characteristic features of reaction (1) resemble those of elastic diffraction scattering, namely a nearly energy-independent total and forward differential cross section $d\sigma/dt$, and a momentum transfer distribution similar to that of diffraction scattering [1-3]. We do not have to specify the detailed mechanism of the reaction * but, in addition to the rho production proper (fig. 1a), we consider "Drell-type" processes [6, 7] in which a virtual pion is diffraction-scattered on the proton (fig. 1b, c), and which give a non-resonant background. The interference between these contributions will be shown to produce an apparent shift in the mass of the rho meson of about 25 MeV. The sign of the shift cannot be predicted; it depends, in particular, on the sign of the $\rho\pi\pi$ coupling constant.

According to our assumption, we make the following phenomenological Ansatz for the rho production amplitude:

$$T_\rho(s,t) = \pm\, \mathrm{i}cF(s)\exp(\tfrac{1}{2}at)\ ,$$

where $s=(k+p)^2$, $t=(p'-p)^2$ (see fig. 1 for notation), and $F(s)=\frac{1}{2}(s-m_N^2)$ is the flux factor. The exponential dependence on the four-momentum transfer is, according to the experiments [1-3], similar to that found in pion-nucleon diffraction scattering, with $a\approx 8\ \mathrm{GeV}^{-2}$. The normalization factor c is related to the total rho production cross section by

$$c \approx \left[\frac{a}{4\pi}\,\sigma_\rho(s)\exp\left(a\,\mathrm{Min}\,|t|\right)\right]^{\frac{1}{2}}$$

and is approximately independent of s.

One can now write down a matrix element for production of a $(\pi^+\pi^-)$ pair via an intermediate ρ^0 meson state:

$$M_1 = -\sqrt{2}(2\pi)^{-\frac{5}{2}}(k_0p_0q_{10}q_{20}p_0')^{-\frac{1}{2}}\, g_{\rho\pi\pi}\, T_\rho(s,t)\times$$
$$\times\,(m_\rho^2-\omega-\mathrm{i}\gamma)^{-1}\, e_\mu^{(\gamma)}(k)\left[\delta_{\mu\nu}-\frac{q_\mu k_\nu}{(q\cdot k)}\right](q_{1\nu}-q_{2\nu})\ ,$$

where $\omega=q^2=(q_1+q_2)^2$ and $\gamma=m_\rho\Gamma(\omega)$. The $\rho\pi\pi$ coupling constant $g_{\rho\pi\pi}$ is related to the width $\Gamma(m_\rho^2)$ of the rho meson. We also take [e.g.8]

$$\Gamma(\omega)=\frac{m_\rho}{\omega^{\frac{1}{2}}}\left[\frac{\frac{1}{4}\omega-m_\pi^2}{\frac{1}{4}m_\rho^2-m_\pi^2}\right]^{\frac{3}{2}}\Gamma(m_\rho^2)\ .$$

For the Drell diagrams (figs. 1b, c), the matrix element is

$$M_2+M_3 = -2^{\frac{3}{2}}(2\pi)^{-\frac{5}{2}}\, e(k_0p_0q_{10}q_{20}p_0')^{-\frac{1}{2}}\, e_\mu^{(\gamma)}(k)\times$$
$$\times\left[\frac{T_-(s_2,t)}{m_\pi^2-t_1}\,q_{1\mu}-\frac{T_+(s_1,t)}{m_\pi^2-t_2}\,q_{2\mu}\right]$$

with $s_1=(p'+q_1)^2$, $s_2=(p'+q_2)^2$, $t_1=(q_1-k)^2$ and $t_2=(q_2-k)^2$. Here, the amplitudes $T_\pm(s,t)$ for elastic $\pi^\pm p$ diffraction scattering enter. Using the optical theorem, they may be written

$$T_\pm(s,t)=\mathrm{i}\,\frac{\sigma_\pm(s)}{8\pi}\,F_{\pi N}(s)\exp(\tfrac{1}{2}at)\ ,$$

where $\sigma_\pm(s)$ are the total $\pi^\pm p$ cross sections, and

$$F_{\pi N}(s)=\tfrac{1}{2}[s-(m_N+m_\pi)^2]^{\frac{1}{2}}\,[s-(m_N-m_\pi)^2]^{\frac{1}{2}}\ .$$

If now the value of the total rho production cross section $\sigma_\rho(s)$ is taken from experiment, then the matrix element $M_1+M_2+M_3$ corresponding to fig. 1 is completely known, apart from the sign of $g_{\rho\pi\pi}$ and the sign of Im $T_\rho(s,t)$. After squaring and performing the polarization average, one arrives at a differential cross section which is a sum of six terms, namely the three quadratic terms and three interference terms. By integrating over the appropriate variables one gets the cross section as a function of $\omega^{\frac{1}{2}}$, the mass of the $(\pi^+\pi^-)$ system.

Fig. 2 shows the result for a photon energy of $k_0^{\mathrm{LAB}}=4$ GeV. In this example, the parameters used are $m_\rho=765$ MeV, $\Gamma(m_\rho^2)=124$ MeV, $\sigma_\rho=13\ \mu$b and $\sigma_+\sigma_-=35$ mb. The Drell graphs were calculated in the Coulomb gauge in the center-of-momentum system.

The first diagram, M_1, alone gives an approximate Breit-Wigner cross section with a maximum at 765 MeV. The Drell diagrams M_2 and M_3 produce a smooth background, representing a total cross section of about 6 μb. They also interfere with the imaginary part of M_1, yielding a rapidly varying contribution in the vicinity of the rho mass. This results in a skewing and shifting of the peak. The sign of the interference terms is

* The features mentioned suggest a diffraction production mechanism. A model along these lines has been discussed by Berman and Drell [10]. It is at least in qualitative agreement with the experimental observations on reaction (1).

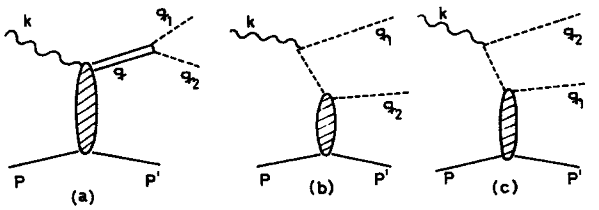

Fig. 1. Diagrams corresponding to the three matrix elements M_1, M_2 and M_3 for the process $\gamma + p \rightarrow \pi^+ + \pi^- + p$.

such that a downward shift results if Im $T_\rho(s,t) > 0$ and if $g_{\rho\pi\pi}$ has the same sign as the charge of the pion. The maximum is then shifted to 750 MeV, and the mean value of the points at half maximum becomes 740 MeV, so that an effective shift of the peak of about 25 MeV will be observed.

These reaults do not depend critically on the photon energy, since the three amplitudes considered here have a similar s dependence. We do not worry about possible modifications to make the amplitude gauge-invariant [9] because the essential result will remain unchanged.

An obvious further contribution to reaction (1) is given by the one-pion-exchange matrix element [10, 11] *. Its size cannot be calculated because the $\rho\pi\gamma$ coupling constant is unknown. The experiments [1-3] indicate, however, that its contribution is comparatively small. Furthermore, after summing over polarizations there are no interference terms between the amplitudes considered here, and the one-pion-exchange amplitude. It is therefore not likely to affect our conclusions.

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* According to Soloviev [12] also in photoproduction $\gamma + \pi \rightarrow \pi + \pi$ on pions the rho meson peak should appear shifted towards lower mass values. Extending the argument to virtual pions, one expects a similar shift in case of a one-pion-exchange mechanism for reaction (1).

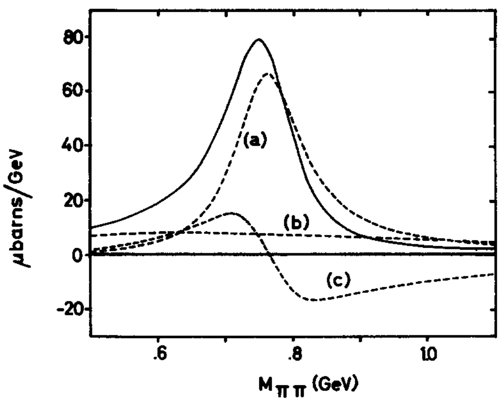

Fig. 2. Cross section as a function of the $(\pi^+\pi^-)$ mass. Curve (a) gives the contribution of M_1, while (b) shows the contribution of M_2 and M_3 and their mutual interference term, and (c) that of the interference term between M_1 and M_2, and between M_1 and M_3. The full curve is the sum of all six terms.

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