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Theory of Strong Interactions*

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"There is a large experimental program on production of K particles by nuclear collisions and by photons, scattering, and interactions of those mesons with nuclei, etc. But just between us theoretical physicists: What do we do with all these data? We can't do anything. We are facing a very serious problem. . . . Perhaps the results of all experiments will produce some idiotic surprises, and some dope will be able to calculate everything from some simple rule. What we are doing can be compared with those complicated models invented to explain the hydrogen spectra which turned out to satisfy very simple regularities."

R. P. FEYNMAN

All the symmetry models of strong interactions which have been proposed up to the present are devoid of deep physical foundations. It is suggested that, instead of postulating artificial "higher" symmetries which must be broken anyway within the realm of strong interactions, we take the *existing exact* symmetries of strong interactions more seriously than before and exploit them to the utmost limit. A new theory of strong interactions is proposed on this basis.

Following Yang and Mills we require that the gauge transformations that are associated with the three "internal" conservation laws—baryon conservation, hypercharge conservation, and isospin conservation—be "consistent with the local field concept that underlies the usual physical theories." In analogy with electromagnetism there emerge three kinds of couplings such that in each case a massive vector field is coupled linearly to the conserved current in question. Each of the three fundamental couplings is characterized by a single universal constant. Since, as Pais has shown, there are no other internal symmetries that are exact, and since any successful theory must be simple, there are no other fundamental strong couplings. Parity conservation in strong interactions follows as the direct consequence of parity conservation of the three fundamental vector couplings. The three vector couplings give rise to corresponding current-current interactions. Yukawa-type couplings of pions and K particles to baryons are "phenomenological," and may arise, for instance, out of four-baryon current-current interactions along the lines suggested by Fermi and Yang. All the successful features of Chew-Low type

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meson theories and of relativistic dispersion relations can, in principle, be in accordance with the theory whereas none of the predictions based on relativistic Yukawa-type Lagrangians are meaningful unless ω/M is considerably less than unity.

Simple and direct experimental tests of the theory should be looked for in those phenomena in which phenomenological Yukawa-type couplings are likely to play unimportant roles. The fundamental isospin current coupling in the static limit gives rise to a short-range repulsion (attraction) between two particles whenever the isospins are parallel (antiparallel). Thus the low-energy s-wave πN interaction should be repulsive in the $T = \frac{3}{2}$ state and attractive in the $T = \frac{1}{2}$ state in agreement with observation. In $\pi\Sigma$ s-wave scattering the $T = 0$ state is strongly attractive, and there definitely exists the possibility of an s-wave resonance at energies of the order of the K^-p threshold, while the $T = 1$ $\pi\Sigma$ phase shift is likely to remain small; using the K matrix formalism of Dalitz and Tuan, we might be able to compare the "ideal" phase shifts derived in this manner with the "actual" phase shifts deduced from K^-p reactions. It is expected that the two-pion system exhibits a resonant behavior in the $T = 1$ (p-wave) state in agreement with the conjecture of Frazer and Fulco based on the electromagnetic structure of the nucleon. The three pion system is expected to exhibit two $T = 0$, $J = 1$ resonances. It is conjectured that the two $T = \frac{1}{2}$ and one $T = \frac{3}{2}$ "higher rsonances" in the πN interactions may be due to the two $T = 0$ 3π resonances and the one $T = 1$ 2π resonance predicted by the theory. Multiple pion production is expected at all energies to be more frequent than that predicted on the basis of statistical considerations. The fundamental hypercharge current coupling gives rise to a short-range repulsion (attraction) between two charge-doublet particles when their hypercharges are like (opposite). If the isospin current coupling is effectively weaker than the hypercharge current coupling, the KN "potential" should be repulsive and the $\bar{K}N$ "potential" should be attractive, and the charge exchange scattering of K^+ and K^- should be relatively rare, at least in s states. All these features seem to be in agreement with current experiments. Conditions for the validity of Pais' doublet approximation are discussed. The theory offers a possible explanation for the long-standing problem as to why associated production cross sections are small and K^- cross sections are large. The empirical fact that the ratio of $(K\bar{K}2N)$ to $(K\Lambda N) + (K\Sigma N)$ in NN collisions seems to be about twenty to thirty times larger than simple statistical considerations indicate is not surprising. The fundamental baryonic current coupling gives rise to a short-range repulsion for baryon-baryon interactions and an attraction for baryon-antibaryon interactions. There should be effects similar to those expected from "repulsive cores" for all angular momentum and parity states in both the $T = 1$ and $T = 0$ NN interactions at short distances though the $T = 1$ state may be more repulsive. A simple Thomas-type calculation gives rise to a spin-orbit force of the right sign with not unreasonable order of magnitude. The ΛN and ΣN interactions at short distances should be somewhat less repulsive than the NN interactions. Annihilation cross sections in $N\bar{N}$ collisions are expected to be large even in Bev regions in contrast to the predictions of Ball and Chew. The observed large pion multiplicity in $N\bar{N}$ annihilations is not mysterious. It is possible to invent a reasonable mechanism which makes the reaction $p + \bar{p} \rightarrow \pi^+ + \pi^-$ very rare, as

recently observed. Fermi–Landau–Heisenberg type theories of high energy collisions are not expected to hold in relativistic NN collisions; instead the theory offers a theoretical justification for the "two-fire-ball model" of high-energy jets previously proposed on purely phenomenological grounds.

Because of the strong short-range attraction between a baryon and antibaryon there exists a mechanism for a baryon-antibaryon pair to form a meson. The dynamical basis of the Fermi–Yang–Sakata–Okun model as well as that of the Goldhaber–Christy model follows naturally from the theory; all the *ad hoc* assumptions that must be made in order that the compound models work at all can be explained from first principles. It is suggested that one should not ask which elementary particles are "more elementary than others," and which compound model is right, but rather characterize each particle only by its internal properties such as total hypercharge and mean-square baryonic radius. Although the fundamental couplings of the theory are highly symmetric and universal, it is possible for the three couplings *alone* to account for the observed mass spectrum. The theory can explain, in a trivial manner, why there are no "elementary" particles with baryon number greater than unity provided that the baryonic current coupling is sufficiently strong. The question of whether or not an $|S| = 2$ meson exists is a dynamical one (not a group-theoretic one) that depends on the strength of the hypercharge current coupling. A possible reason for the nonexistence of a $\pi^{0'}$ (charge-singlet, non-strange boson) is given. The theory realizes Pais' principles of economy of constants and of a hierarchy of interactions in a natural and elegant manner.

It is conjectured that there exists a deep connection between the law of conservation of fermions and the universal V-A weak coupling. In the absence of strong and electromagnetic interactions, baryonic charge, hypercharge, and electric charge all disappear, and only the sign of γ_5 can distinguish a fermion from an antifermion, the fermionic charge being diagonalized by γ_5; hence $1 + \gamma_5$ appears naturally in weak interactions. Parity conservation in strong interactions, parity conservation in electromagnetic interactions, parity non-conservation in weak interactions can all be understood from the *single common* principle of generalized gauge invariance. It appears that in the future ultimate theory of elementary particles all elementary particle interactions will be manifestations of the five fundamental vector-type couplings corresponding to the five conservation laws of "internal attributes"—baryonic charge, hypercharge, isospin, electric charge, and fermionic charge. Gravity and cosmology are briefly discussed; it is estimated that the Compton wavelength of the graviton is of the order of 10^8 light years.

It is suggested that every conceivable experimental attempt be made to detect directly quantum manifestations of the vector fields introduced in the theory, especially by studying Q values of pions in various combinations in $N\bar{N}$ annihilations and in multiple pion production.

I

Although in recent years there have been rapid advances in our empirical knowledge of strong interactions of strange particles, virtually no progress has been made in our basic theoretical understanding of those interactions beyond the selection rule proposed by Gell-Mann (*1*) and by Nakano and Nishijima (*2*).

The various symmetry models proposed and worked out by a number of physicists all over the globe have not led to a single fruitful prediction; if there is any significant achievement along these lines at all, it is precisely the following negative result obtained by Pais (*3*, *4*): *There are no internal symmetries stronger than those implied by charge independence that work to all orders and that are not contradicted by experiments.*

There are some theoretical reasons to believe that all those symmetry models which have been proposed up to the present are devoid of deep physical foundations. Take the "global symmetry" (universal π coupling) model of Gell-Mann (*5*) and Schwinger (*6*), for instance. It had its genesis in earlier works of Wigner (*7*, *8*) in which a possible connection between the conservation of baryons and the universality of π couplings is discussed in analogy with electromagnetism. Unfortunately the pion field is pseudoscalar not vector, and the baryon density to which the pion field is coupled is a pseudoscalar density (or pseudovector density) not a conserved-vector density. Hence the tight interrelation that exists in electrodynamics among the conservation of electric charge, the very existence of the electromagnetic field and the universality of the coupling, the interrelation that can be so elegantly formulated in terms of invariance under *space-time dependent* gauge transformations, has no analog in mesodynamics.

A similar criticism applies to the now obsolete model of Schwinger (*9*) in which the pion field is regarded as a dynamical manifestation of hypercharge (= the sum of strangeness and baryon number); if there is any dynamical manifestation of hypercharge at all, it must be a *vector* field coupled universally to the conserved current constructed out of fields having nonvanishing hypercharge, a point to which we shall come back later. Yet, despite the failure of Schwinger's earlier model, his original idea that one should associate *dynamical* features to internal attributes such as hypercharge is truly profound, and should not be dismissed casually.

The "cosmic symmetry" (universal K coupling) model has had a somewhat different *raison-d'être* (*10*, *11*). There the starting point of the investigations was the question of how to guarantee parity conservation in the K couplings. The disappointing result that we must rely so heavily on the structure of Yukawa-type Lagrangians makes the author (who is incidentally one of the originators of the model) feel that this model too should not be taken seriously (even though we started by asking the right question).[1] Other models such as the one based on the coupling constant relation $G(K\Sigma N) = G(K\Xi\Lambda) \neq 0$, $G(K\Lambda N) = G(K\Xi\Sigma) = 0$ seem even more difficult to justify on theoretical grounds.

No matter to which model we may subscribe, the proposed symmetry must be immediately broken by subsequent interactions which are unfortunately also

[1] The author is indebted to constructive criticisms of Professor T. D. Lee on this point.

"strong." It looks as though we created a symmtery in order that it might soon be broken.[2]

These considerations, together with Pais' result mentioned earlier, seem to indicate that for more than two years we have hunted fruitlessly for "higher" symmetries which do not even exist. After having spent considerable time (and energy) on various symmetry models, the present author is convinced that there are no simple patterns in the Yukawa-type Lagrangians in which pions and K particles are coupled linearly to baryons, and that all those symmetry models proposed up to now are mere *mental* exercises devoid of any *physical* significance whatsoever.

Yet we would like to believe in Gell-Mann's remark (if not in Gell-Mann's model) that nature is simple if you know how to look at her (*12*). So we are led to the view that we should look for simple and elegant patterns not in Yukawa-type Lagrangians but *elsewhere*. Perhaps the Yukawa-type couplings of π and K are phenomenological manifestations of some other couplings which possess more aesthetically appealing features.

Even apart from strange particle physics, there is reason to believe that the Yukawa coupling of the pseudoscalar field to the nucleon field is not so well founded as the electromagnetic coupling on *a priori* theoretical grounds. In the case of electromagnetism the very structure of the coupling and, in a certain sense, the very existence of it are determined and necessitated by the requirement that the gauge transformation, the invariance under which leads to the conservation law of electric charge, be *local* in character. No analogous argument is known for Yukawa-type couplings of spin-zero fields. Therefore, as long as we regard the *ps-ps* coupling (or *ps-pv* coupoing) as fundamental, there remains the following question, which has troubled the present author ever since his first contact with field theory: Why is it that the Creator was so supremely imaginative when he declared, "Let there be light," while he did not use any imagination whatsoever when he switched on the γ_5 coupling of the pion field to the nucleon field?[3] It appears that only by forsaking the idea that the Yukawa interaction is fundamental can we restore the depth, simplicity, beauty, and elegance that are so characteristic of true physical theories.

Now we must start from the very beginning as though we did not know any

[2] In this connection the reader may be interested in the following remark made by Professor A Salam, which, in many ways, stimulated the present investigations. "Classical physical theories are profound. Take the second law of thermodynamics, for instance: Heat cannot flow spontaneously from a colder to a hotter body. Compare this to what you have been doing. You propose some symmetry, and ten seconds later you are already trying to figure out how to break it."

[3] Some people may prefer to argue that the γ_5 coupling is as well-founded as the electromagnetic coupling because they are both renormalizable in the Dyson sense.

meson theory of the Yukawa form. What should be the guiding principles in constructing a new theory of strong interactions? First of all, the theory should be deeply rooted in symmetry laws that hold *exactly* in the absence of the electromagnetic and weak couplings. Instead of looking for artificial higher symmetries, we should take the *existing* symmetries more seriously than ever before and *exploit them to the utmost limit.* Apart from this, simplicity should be the only other guiding principle. The present paper is an attempt to construct a new theory of strong interactions based on these two principles.

In Section II three fundamental strong couplings of the theory are discussed. In Section III pion phenomena are treated with special emphasis on s-wave π-baryon scattering. Section IV is concerned with K-particle phenomena. In Section V baryon-baryon and baryon-antibaryon phenomena are discussed. Section VI is concerned with more general and speculative problems in strong interactions. Discussions on the origin of weak interactions and brief cosmological speculations appear in Section VII. Section VIII points out further experimental and theoretical directions to be explored. The reader who does not wish to be bothered with details may read the Abstract and Sections II, VI, and VIII without loss of continuity.

II

What are the exact conservation laws of strong interactions? First of all, the number of baryons minus the number of antibaryons is conserved. Historically, although the conservation law of baryons (or of heavy particles) is usually attributed to Wigner (*7, 8*), the first clear statement of this conservation law can be traced back to a now almost forgotten work of Stueckelberg (*13*) in which "Erhaltungssatz der schweren Ladung" is treated on a par with "Erhaltungssatz der electrischen Ladung."[4] In analogy with electromagnetism one unit of "baryonic charge" is carried by each baryon. Baryonic charge is one (and probably the only) characteristic that distinguishes those fermions which can interact strongly from those fermions which can interact *only* electrogmagnetically and/or weakly. Therefore, although the law of conservation of baryons holds to an amazing degree of accuracy for any interaction (*14*), we are inclined to the view that baryonic charge is a *dynamical* attribute having something to do with strong interactions.

Secondly there is the conservation law of isospin, which formally expresses symmetries implied by charge independence. Historically this law was first clearly formulated by Wigner (*15*) in connection with nucleon-nucleon scattering, even though something like the validity of this law seemed to have been implicitly assumed in Heisenberg's work (*16*) in which the notion of isospin was first intro-

[4] The author is indebted to Professor G. Wentzel for calling attention to the work of Stueckelberg.

duced. The law of isospin conservation seems to hold well for low-energy nuclear physics and pion-nucleon physics (*17*, *18*) and has been tested in some of the strange particle interactions (*19*, *20*). Further evidence is urgently needed, but we assume that this law holds exactly in the absence of the electromagnetic and weak couplings.

In addition to baryon number B, isospin T, and the third component of isospin T_3, at least one more internal quantum number is needed to specify a particle. For instance, when we say that there is a particle with $B = 0$, $T = 1/2$ and $T_3 = 1/2$, we cannot tell whether we are referring to K^+ or $\bar{K}^0$. We can choose one out of the following three, electric charge Q, strangeness $S = 2(Q - T_3) - B$, and hypercharge $Y = S + B$. Of these electric charge seems to have nothing to do with strong interactions because leptons which cannot interact strongly can bear electric charge, and also because the very coupling that is intimately related to the conservation of electric charge destroys one of the exact internal symmetries of strong interactions. Now a unit of Y is associated with a half unit of isospin. Between S and Y we choose Y as the third fundamental internal attribute because systems with integral isospins can be made up of systems with half-integral isospins but not vice versa. Note that we have implicitly assumed that Q is always integral whereas T can be half-integral as well as integral. That the electromagnetic coupling destroys the isotropy of isospin space in such a way that Q is *sometimes displaced* with respect to T_3 in a peculiar manner is one of the deepest mysteries of elementary particle physics. Once this mystery is resolved, we shall be able to be more confident about our choice. We can give more formal arguments for choosing Y, but such arguments do not seem to throw any light on the fundamental issue.[5]

Now comes the question: How do we formulate conservation laws of internal attributes? Take the conservation of baryonic charge, for instance. If baryon conservation holds there must not exist any Hermitian matrix connecting a state with $B = 1$ to a state with $B = 0$. This means that the relative phase between the $B = 0$ state and the $B = 1$ state is arbitrary, nonmeasurable, and devoid of physical significance. We can apply

$$|A> \rightarrow \exp(iB\lambda)\,|A>, \tag{2}$$

[5] Consider the eigenvalues of G^2 where G stands for the G conjugation operator introduced by Michel (*21*) and by Lee and Yang (*22*), which amounts to the inversion operator in isospin space. We can readily show $G^2 = (-1)^Y$. If we follow an argument given by Wick *et al.* (*23*), the above relation implies that it is meaningless to compare the phase of a system with $Y = \pm 1$ with that of a system with $Y = 0$ so that Y conservation becomes a "superselection rule" as far as strong interactions are concerned provided that G is "good." We note a striking analogy between Y conservation and fermion conservation by recalling $(UK)^2 = (-1)^n$ where UK is the antiunitary time reversal operator of Wigner and n stands for fermion number. It is worth recalling that the significance of $Y = S + B$ was first pointed out by d'Espagnat and Prentki (*24*) who identified it as "isofermion number."

where λ is an arbitrary real constant and $|A>$ is a state with definite baryon number, and no physical change results. (2) implies the corresponding change in field operators

$$\psi \rightarrow \exp(iB\lambda)\psi, \tag{3}$$

where ψ stands for any field, B being zero for a meson field and unity for a baryon field. It is well known that in the usual Lagrangian formalism the requirement that the Lagrangian be invariant under (3) leads to the conservation law of total baryonic charge. Hypercharge conservation can be treated in an identical manner. In case of isospin we have $\psi \rightarrow \exp(i\mathbf{T}\cdot\boldsymbol{\lambda})\psi$ where $\boldsymbol{\lambda}$ is a constant real vector in isospin space.

So far everything may seem straightforward. However, if we think deeply about this conventional formalism, there is something rather unsatisfactory about what we have been doing. We are told that a change in the phase factor for a baryon field does not lead to any new physical situation. Yet λ in (2) is a real constant independent of space-time. Isn't the relative phase factor of states at two different space-time points separated by a space-like distance also arbitrary? Why are we not allowed to choose independent phases at different space-time points? In other words, why can't we let λ and (2) and (3) be a function of space-time? Why are we forced to apply the same $\exp(i\lambda)$ to all baryon states simultaneously everywhere in the universe? It appears that we are *almost* forced to believe in the idea of action-at-a-distance if we are always required to choose the phase factor in such a way that it is constant at all space-time points.[6]

In case of isospin conservation essentially identical questions have been asked and to a certain extent answered by Yang and Mills (*25*) in whose work the most fundamental idea in the present investigations has originated. The principle of isospin conservation implies that the orientation of isospin is of no physical significance. The differentiation between a proton and a neutron is purely arbitrary in the absence of the electromagnetic coupling. Yet, in the usual isospin formalism, once we decide what we call a proton at one space-time point, for instance here in Chicago, what we should call a proton at some other space-time point, for instance in Dubna or in a distant galaxy, is no longer arbitrary. Yang and Mills then remark that "this is not consistent with the localized field concept that underlies the usual physical theories," and go on to explore the possibility of requiring all interactions to be invariant under *independent* isospin rotations at all space-time points.

In quantum electrodynamics it has been known for a long time that we are free to make an independent change of phase of an electrically charged field at every space-time point (*26*). Suppose we did not know anything about the existence of A_μ. Even then, if we demand that the so-called gauge transformation,

[6] Various experts do not seem to agree on this difficult point.

the invariance under which leads to the conservation of electric charge, be *local* in character, we are forced to introduce a new field, which is to be identified with the electromagnetic field A_μ, coupled universally to the conserved current constructed out of electrically charged fields. This is because the transformation

$$\psi \rightarrow \exp(ie\Lambda(x))\psi \tag{4}$$

alone will not maintain invariance unless it is counteracted by

$$A_\mu \rightarrow A_\mu + \partial\Lambda/\partial X_\mu . \tag{5}$$

Similarly Yang and Mills have shown that if we require that the gauge transformation associated with isospin conservation be *local* in character, we are forced to introduce a vector field with isospin unity coupled universally to the isospin current constructed out of all fields having nonvanishing isospins.

This is a very profound idea—perhaps the most profound idea in theoretical physics since the invention of the Dirac theory. It essentially states that, if we have a conservation law of some internal attribute, there must necessarily exist a vector-type interaction corresponding to it in order that the conservation law in question be consistent with the concept of localized fields. To borrow Schwinger's words, internal attributes should have "dynamical manifestations." To put this idea more succinctly, internal symmetry *ergo* dynamics. It puzzles the author that the idea so profoundly physical has received so little attention in the past five years.

We can immediately generalize Yang and Mill's idea to hypercharge conservation and baryon conservation. There emerge three fundamental vector couplings corresponding to the three internal conservation laws of strong interactions. At this point, let us recall Pais' result that there exist no other internal symmetries that are exact. In addition, we would like to believe that any successful theory must be simple. Thus we are led to the view that these three couplings, which are the only couplings deeply rooted in the exact internal symmetries of strong interactions, are the *only* "fundamental" couplings of strong interactions. After all, there is no compelling reason why other fundamental strong couplings should exist.

We now write down the three fundamental interaction Lagrangians of strong interactions[7]:

$$\mathfrak{L}_T = -f_T \mathbf{B}_\mu^{(T)} \cdot \mathbf{J}_\mu^{(T)}, \tag{6}$$

$$\mathfrak{L}_Y = -f_Y B_\mu^{(Y)} J_\mu^{(Y)}, \tag{7}$$

$$\mathfrak{L}_B = -f_B B_\mu^{(B)} J_\mu^{(B)}, \tag{8}$$

[7] Throughout the paper we use the metric $x_\mu y_\mu = \sum_k^3 x_k y_k + x_4 y_4$ with $x_4 = ix_0$. The γ matrices are the ones defined by Pauli (*27*) with $\gamma_\mu = \gamma_\mu^+$.

which we call, respectively, the fundamental isospin current coupling (the original Yang–Mills coupling), the fundamental hypercharge current coupling, and the baryonic current coupling. $\mathbf{B}_\mu^{(T)}$, $B_\mu^{(Y)}$, and $B_\mu^{(B)}$ are the vector fields analogous to the electromagnetic field A_μ ; $\mathbf{J}_\mu^{(T)}$, $J_\mu^{(Y)}$, and $J_\mu^{(B)}$ are the current densities constructed out of fields having isospin, hypercharge, and baryonic charge, respectively. It is to be noted that the baryonic current coupling (8) has been previously considered by Lee and Yang (*28*) and by Fujii (*29*). If the fields were bare, we would have

$$\begin{aligned}\mathbf{J}_\mu^{(T)} &= i\bar{\psi}_N \frac{\boldsymbol{\tau}}{2} \gamma_\mu \psi_N - \bar{\boldsymbol{\psi}}_\Sigma \times \gamma_\mu \boldsymbol{\psi}_\Sigma + i\bar{\psi}_\Xi \frac{\boldsymbol{\tau}}{2} \gamma_\mu \psi_\Xi + \boldsymbol{\phi}_\pi \times \frac{\partial \boldsymbol{\phi}_\pi}{\partial x_\mu} \\ &+ i\left(\frac{\partial \phi_K^{+}}{\partial x_\mu} \frac{\boldsymbol{\tau}}{2} \phi_K - \phi_K^{+} \frac{\boldsymbol{\tau}}{2} \frac{\partial \phi_K}{\partial x_\mu}\right) + \mathbf{f}_{\mu\nu}^{(T)} \times \mathbf{B}_\nu^{(T)},\end{aligned} \tag{9}$$

$$J_\mu^{(Y)} = i\bar{\psi}_N \gamma_\mu \psi_N - i\bar{\psi}_\Xi \gamma_\mu \psi_\Xi + i\left(\frac{\partial \phi_K^{+}}{\partial x_\mu} \phi_K - \phi_K^{+} \frac{\partial \phi_K}{\partial x_\mu}\right), \tag{10}$$

$$J_\mu^{(B)} = i\bar{\psi}_N \gamma_\mu \psi_N + i\bar{\psi}_\Lambda \gamma_\mu \psi_\Lambda + i\bar{\boldsymbol{\psi}}_\Sigma \gamma_\mu \boldsymbol{\psi}_\Sigma + i\bar{\psi}_\Xi \gamma_\mu \psi_\Xi, \tag{11}$$

$$\mathbf{f}_{\mu\nu} \equiv \frac{\partial \mathbf{B}_\mu^{(T)}}{\partial x} - \frac{\partial \mathbf{B}_\nu^{(T)}}{\partial x_\mu} - f_T \mathbf{B}_\mu \times \mathbf{B}_\nu . \tag{12}$$

All the notations are conventional, for instance, $\boldsymbol{\psi}_\Sigma$ is the direct product of a four-component Dirac spinor in Lorentz space and a three-component isovector in isospin space. The last term in (9), the presence of which was pointed out in Yang and Mill's original paper, arises from the fact that the $\mathbf{B}_\mu^{(T)}$ field possesses isospin so that it can interact with itself. If an $|S| = 2$ meson ($D^{\pm}$ meson ?) exists, we may add the corresponding hypercharge current to (10) with coefficient two since this meson would correspond to a doubly hypercharged particle. In any case the idea of introducing a new field whenever a new particle is discovered seems distasteful. Some people may prefer to argue that not all elementary particles are really "elementary." If you want to regard a pion as a bound state of N and $\bar{N}$, you may omit the $\boldsymbol{\phi}_\pi \times (\partial \boldsymbol{\phi}_\pi / \partial x_\mu)$ term in (9). The important point is that our fundamental Lagrangians (6), (7), and (8) make sense regardless of whether you believe in a theory in which all particles are elementary or in a theory in which "some elementary particles are more elementary than others" *as long as whatever you believe in conserves isospin, hypercharge, and baryonic charge at every space-time point.*

In reality particles are never "bare"; so we may ask in what sense (6)–(8) and (9)–(11), where field operators are now "clothed," still approximate reality. One of the most important features of our theory is that the universality of each of the fundamental coupling still holds in low-energy limits with "clothed" operators, i.e., the coupling constants are not renormalized by the process of being "clothed." Take the proton, for instance. It may virtually disintegrate into a neutron and a π^+, but the coupling of $\mathbf{B}_\mu^{(T)}$ to the isospin current of the physical proton in *low-energy limits* is the same as the coupling between $\mathbf{B}_\mu^{(T)}$ and the bare proton since $\mathbf{B}_\mu^{(T)}$ can interact with the isospin of π^+ as well as with the isospin of n. The universality of coupling in our theory is very much like the universality that holds in the conserved-vector theory of weak interactions proposed by Gershtein and Zel'dovich (*30*) and by Feynman and Gell-Mann (*31*).

Note also that the universality of any one of the three couplings, for instance of the baryonic current coupling, is not destroyed as we "switch on" the other two couplings, the hypercharge current coupling and the isospin current coupling. This feature is not shared with any of the so-called symmetry models of strong interactions proposed up to now. Take the global symmetry model, for instance. We are told that all G_π's are equal in the absence of the K couplings. However, it is easy to convince ourselves that, as soon as we switch on the asymmetric K couplings, the new G_π's which are now renormalized by the K couplings are no longer equal. It is precisely for this reason that the author feels that we should be rather skeptical about Gell-Mann's conjecture that global symmetry has something to do with baryon conservation (*5*). How can global symmetry which is so readily destroyed by almost as strong "subsequent" couplings have anything to do with the sacred conservation law to which we all owe our very existence? We are led to believe that *the only "universal" scheme of strong interactions that makes any sense at all is the one that exploits the notion of conserved-vector currents.*

We have already remarked that all strong interactions are manifestations of the three fundamental couplings (6)–(8). Parity is necessarily conserved in strong interactions because our fundamental couplings conserve parity. Time reversal invariance also holds because the reality of f_T, f_Y, and f_B follows from the Hermiticity requirement. By the CPT theorem charge conjugation invariance is also valid.

A few remarks about the properties of the three kinds of B fields. Because $\mathbf{J}_\mu^{(T)}$ is even under the G conjugation operation whereas $J_\mu^{(Y)}$ and $J_\mu^{(B)}$ are odd under G, we have

$$G\mathbf{B}_\mu^{(T)}G^{-1} = \mathbf{B}_\mu^{(T)}, \tag{13}$$

$$GB_\mu^{(Y)}G^{-1} = -B_\mu^{(Y)}, \tag{14}$$

$$GB_\mu^{(B)}G^{-1} = -B_\mu^{(B)}. \tag{15}$$

This means that if the mass of the B_T quantum (corresponding to the $B_\mu^{(T)}$ field) is greater than $2\mu_\pi$, the B_T quantum *decays strongly* into 2π (and, if energetically possible, into 4π), and if the masses of the B_Y and B_B quanta are greater than $3\mu_\pi$, they *decay strongly* into 3π (5π, etc.). If the B_Y or B_B mass is less than $3\mu_\pi$, its decay modes are identical to those of the ρ^0 meson introduced by Nambu (*32*), the major decay modes being $\pi^0 + \gamma$ and $2\pi + \gamma$. Possible experiments to detect the various B quanta whose lifetimes are of the order of 10^{-21}–10^{-22} sec are briefly discussed in Section VIII.

By this time an intelligent reader must have made the following objection: The B fields cannot be massive because the mass term $\mu^2 B_\mu^{\ 2}$ in the Lagrangian certainly does not satisfy your gauge principle. This is a valid objection, perhaps the most serious objection to our theory. We would like to believe that the mass terms do vanish for the bare Lagrangian, and that the empirical mass terms (which are there in spite of the fact that the original Lagrangian does not contain any fundamental length) reflect, in a certain sense, a failure of our present-day field theory which demands that we have to have a bare mass to produce a selfmass.[8] So we look for possible mechanisms that are responsible for the masses of the various B quanta. For $\mathbf{B}_\mu^{(T)}$ Yang and Mills have already pointed out that the fact that the $\mathbf{B}_\mu^{(T)}$ field can interact with itself implies that the $\mathbf{B}_\mu^{(T)}$ field *can* be massive. In case of $B_\mu^{(Y)}$ and $B_\mu^{(B)}$ there are no such self-interactions; hence one may be tempted to argue that the corresponding quanta have to be massless for exactly the same reason as the photon is massless (*34*). This argument *might* break down, however, if there exists an *effective* mutual interaction between $B_\mu^{(Y)}$ and $B_\mu^{(B)}$. For instance, since both the B_Y quantum and the B_B quantum can decay strongly into three pions, a B_Y may convert itself into three pions which subsqeuently form a B_B. (Our situation here is somewhat reminiscent of a beam of "pure" K^0 particles which acquires a $\bar{K}^0$ component after a long time.) Such a mechanism may well make both B_Y and B_B massive. It might not be entirely rediculous to entertain the hope that an *effective* mass term which *seems* to violate our gauge principle may arise out of the fundamental Lagrangians which strictly satisfy our gauge principle.[9]

In this connection we should recall an interesting work of Lee and Yang (*28*). They have examined the experimental consequences of the possible existence of the *massless* $B_\mu^{(B)}$ field. In analogy with Coulomb's law there would be a repulsive force between two nucleons which falls off as $1/r^2$. Hence the observed gravitational attraction would be given by

$$F = -\frac{GM_1M_2}{r^2} + \frac{f_B^{\ 2}}{4\pi}\frac{N_1N_2}{r^2}, \tag{16}$$

where M_1 and M_2 are the rest masses of the two objects in question, and N_1 and

[8] An alternative approach would be to assume the mass of the B field to start with, and introduce an auxiliary scalar field to save the gauge principle, using techniques developed by Stueckelberg (*33*). This is essentially what Fujii (*29*) has done. Such an approach, however, does not answer the basic question of why the photon is massless while the B quanta are massive.

[9] This point has been criticized rather severely by Dr. R. E. Behrends, Professor R. Oppenheimer, and Professor A. Pais.

N_2 stand for the total numbers of nucleons in the object 1 and the object 2, respectively. Because the rest mass of a nucleus depends on the binding energy whereas for the f_B^2 term only the atomic number is relevant, the ratio of the observed gravitational mass to the inertial mass would vary from object to object as the packing fraction varies. From the experimental result of Eötvös Lee and Yang concluded that

$$\frac{f_B^2}{4\pi} \Big/ GM_N^2 < 10^{-5}. \tag{17}$$

Thus if the mass of the B_B quantum were zero, the coupling (8) could not possibly have anything to do with strong interactions. Using the same argument we can convince ourselves that if the B_Y quantum were massless, the coupling (7) would have nothing to do with strong interactions.

We admit that we lack satisfactory answers to the questions of the masses of the various B quanta. We must assume that they are all massive lest the whole edifice of our theory should crumble down. One of the reasons why the present work is submitted for publication in spite of the B mass problem is that the author hopes that the publication may prompt some clever ideas along this line.[10]

It is noteworthy that in classical electrostatics where the question of the photon mass does not enter we can invent an argument which illustrates the connection between the arbitrariness in the absolute scale of the electrostatic potential and the conservation of electric charge. In 1949, using an elementary but penetrating argument, Wigner (7) showed that nonconservation of electric charge together with the arbitrariness in the absolute scale of the electrostatic potential leads to a contradiction with energy conservation. It is evident from his writings that a great deal of effort has been made by him to invent an analogous argument in the case of baryon conservation.[11] The tragic error made by Wigner (and unfortunately inherited by Gell-Mann and by Schwinger) is that he identified "baryonic charge" with "mesonic charge," which identification is the starting point of what has later become known as global symmetry; we now know that this road leads to a dead end. Our proposal is that the analog of the electrostatic potential should be the longitudinal component of our $B_\mu^{(B)}$ field. Following Wigner, at least in classical "baryostatics" we can establish a connection between the conservation of baryons and the arbitrariness in the absolute scale of the longitudinal com-

[10] Several critics of our theory have suggested that, since we are not likely to succeed in solving the mass problem, we might as well take (6)–(8) with massive B fields as the starting point of the theory, forgetting about the possible connection with the gauge principle. This attitude is satisfactory for all practical purposes. However, the author believes that in any theory every effort should be made to justify the fundamental couplings on *a priori* theoretical grounds.

[11] See especially footnote 9 of Ref. 7.

ponent of the $B_\mu^{(B)}$ field independently of the question of the B_B mass.[12] Perhaps our difficulty arises from the fact that in the quantum theory of fields the field quantities play significant roles in the fundamental formalism while in classical theory only the derivatives of the fields are relevant physical quantities, a point of view recently emphasized by Aharonov and Bohm (*35*).

Since our ideas are rather novel, it is not too surprising that there are difficulties associated with our theory. It would be a pity to give up our theory on account of the B mass problem just as it would have been a pity to give up Bohr's atomic model on account of the difficulties associated with the notion of "quantum jumps." In the following we pursue our investigations with the assumption that the masses of the various B quanta are of the order of $3\mu_\pi$ to $6\mu_\pi$ because, if a fundamental length exists, it is likely to be in the neighborhood of the nucleon Compton wavelength, and also because there do not seem to be such "particles" with masses less than $3\mu_\pi$.

If the coupling constants $f_T^2/4\pi$, $f_Y^2/4\pi$, and $f_B^2/4\pi$ were small, an exchange of a single B quantum between two currents in each case would lead to an effective Hamiltonian of the form

$$H_T = -\frac{f_T^2}{4\pi\mu_T^2} \mathbf{J}_\mu^{(T)} \cdot \mathbf{J}_\mu^{(T)}, \tag{18}$$

$$H_Y = -\frac{f_Y^2}{4\pi\mu_Y^2} J_\mu^{(Y)} J_\mu^{(Y)}, \tag{19}$$

$$H_B = -\frac{f_B^2}{4\pi\mu_B^2} J_\mu^{(B)} J_\mu^{(B)}, \tag{20}$$

provided that the square of the invariant momentum transfer were considerably smaller than μ^2 in question. These are current-current interactions reminiscent of the V-A weak coupling.

[12] The argument goes roughly as follows. We assume that baryon conservation is violated but the energy necessary to create or destroy a baryon is independent of the absolute scale of the "baryostatic" potential. We create a proton at a distance of about 0.4×10^{-13} cm from another proton. This requires some energy. As we shall show in Section V, the pp interaction is repulsive at such a distance because of our baryonic current interaction. But by our assumption the energy needed to create the proton is independent of this repulsive "baryostatic" potential characteristic of the pp interaction at short distances, and is equal to the energy E necessary to create a proton at any point. Let the system "go." The two protons fly apart, releasing the energy we associate with the short-range pp repulsion. Now after the two protons are well separated, we destroy one of the protons. According to our assumption that no physics shall depend on the absolute scale of the baryostatic potential, we regain exactly the energy E. Thus the net effect is that we have obtained the energy we associate with the short-range pp repulsion out of nothing. Since this is a contradiction, we have shown that the arbitrariness of the absolute scale of the baryostatic potential implies baryon conservation.

It is to be emphasized here that the connection between such current-current interactions and Yang–Mill type theories was previously discussed by Feinberg and Gürsey (*11*). One of them (Gürsey) remarked in a seminar at the Institute for Advanced Study in December, 1958 that if all strong interactions arise from Yang–Mills type arguments, parity conservation in strong interactions follows immediately. In their paper, however, they seem to have had the opinion that (18) and possibly (19) should be introduced as perturbations to break the very high symmetry (Tiomno's six-dimensional symmetry (*36*)) of the Yukawa-type couplings of π and K.[13]

As we shall show later, the numerical values of the coupling constants lie between unity and 20. Hence to regard (18), (19), and (20) as effective Hamiltonians is a poor approximation. Yet it turns out that in the static limit a "potential" due to the longitudinal component of the B field is correctly given by that expected from Born approximation calculations even if the coupling is strong. This has a familiar analog in quantum electrodynamics; Coulomb's law holds regardless of whether $e^2/4\pi$ is as small as 1/137 or as large as 137. For instance, the static potential between two nucleons arising from the fundamental baryonic coupling has the familiar Yukawa form

$$V = \frac{f_B^2}{4\pi}\frac{\exp(-\mu_B r)}{r} \tag{21}$$

both in the strong coupling limit and in the weak coupling limit (or in any other case). The corresponding potential in the $N\bar{N}$ case is just the negative of (21).

In our theory the couplings (6)–(8) are the only "fundamental" couplings of strong interactions. This means that the conventional Yukawa couplings of π and K are phenomenological manifestations of (6)–(8). In 1949 Fermi and Yang (*37*) showed that if there exists a four-nucleon current-current interaction similar to the four-fermion Fermi-type coupling in weak interactions, it is possible to construct a pion out of a nucleon-antinucleon pair. What is more important, they showed that the pion constructed in this way is very much like the pion in the ordinary Yukawa theory as far as low energy phenomena are concerned. For instance, nucleons can still emit or absorb pions singly. In our case, the situation is more involved than the Fermi–Yang case because there are three current-current interactions. In the Fermi–Yang model it was possible to estimate both the pion mass and the phenomenological Yukawa constant from a single parameter which characterizes the four-nucleon coupling. This becomes practically impossible in our case. We can readily draw Feynman diagrams that produce Yukawa couplings, but calculations based on such diagrams are bound to be

[13] After the major part of the present investigations was completed, the author was informed that a theory somewhat similar to the present one had been contemplated by Feinberg and Gürsey, but that they gave it up mainly because of the B mass difficulties.

meaningless. In Section VI we discuss alternative mechanisms by which Yukawa-type couplings may emerge.

Although we do not know how to derive the exact form of a Yukawa-type coupling, it is correct to say that the low energy limit of the phenomenological Yukawa coupling of π to N has the familiar form $i\partial_k \phi_\pi u_N^+ \boldsymbol{\tau}\sigma_k u_N$ because this form depends only on charge independence and parity conservation. The question of up to what energies such an effective Hamiltonian is valid cannot be answered without detailed calculation. Our theory does not contradict the successful features of static, Chew–Low type meson theories (*38*, *39*).

The relation between the Chew–Low model and our fundamental theory is something like the relation between nuclear models such as the shell model and our knowledge about the basic nucleon-nucleon interaction. Most of us believe that the various nuclear models can be derived, in principle, from our knowledge of the nucleon-nucleon interaction even though this is a very formidable task. We use the various models decause they provide us with a simple way of looking at nuclear levels, etc. Similarly the derivation of the Chew–Low model from our three fundamental couplings is practically impossible at this stage; yet we may as well use the model because it provides us with a convenient framework by means of which we can understand p-wave πN scattering, low-energy photoproduction, and the tail end of the two-nucleon potential.

Relativistic dispersion relations, the derivations of which do not depend on the detailed structure of Lagrangians, are still expected to hold. In forward πN scattering there still is a simple pole at $\omega = -\mu_\pi^2/2M_N$, and if the Mandelstam conjecture turns out to be correct, the NN scattering amplitude regarded as a function of momentum transfer still has a singularity at $q^2 = -\mu_\pi^2$. The fact that we usually express the residues of such singularities in terms of the coupling constant that appears in the renormalized Yukawa-type Lagrangian does not mean that these dispersion relations rely heavily on the Lagrangian formalism in which Yukawa couplings are fundamental. From our points of view such a residue is nothing more than a phenomenological parameter that characterizes the strength of a very complicated process in which a nucleon emits a pion at some nonphysical energy where all three particles are on the mass shells.

On the other hand, the detailed predictions of our theory (if we could calculate them) are expected to be different from the detailed predictions based on the fully relativistic Yukawa-type Lagrangians especially when ω/M is comparable to unity. Already the much simplified calculation of Fermi and Yang (*37*) seems to indicate this point. If our theory turns out to be correct, the question of whether the *ps-ps* coupling or *ps-pv* coupling is right loses meaning, and any attempt to construct a nuclear potential from a fully relativistic Yukawa-type Lagrangian becomes a mere exercise.

Here our opponents may say, "What good is your theory if all you can show is that your theory is, *in principle*, compatible with the successful features of Chew–Low type meson theories and relativistic dispersion relations?" It turns out, much to our surprise, that there are more concrete and direct experimental indications in its favor. In the subsequent four sections we shall look for simple and direct tests of our theory and show that it offers explanations in precisely those areas where the conventional Yukawa-type theories give no simple answers. Of course, we never know how our predictions become affected by the "phenomenological" Yukawa couplings, and in most cases what we can predict are qualitative "yes-no" propositions. For instance, we can tell only whether the sign of a certain phase shift is positive or negative, or whether or not the theory can offer a qualitative explanation for a certain "mystery." Yet, if the theory makes correct yes-no type predictions ten times, the probability that this agreement is fortuitous is one part in 1024.

III

One of the puzzles that still remain in low-energy pion-physics is the peculiar isospin dependence of s-wave pion-nucleon scattering. Experimentally the $T = \frac{1}{2}$ s-state interaction is attractive whereas the $T = \frac{3}{2}$ interaction is repulsive with

$$\begin{aligned} \delta_1 &\approx 0.16\ \eta, \\ \delta_3 &\approx -0.11\ \eta, \end{aligned} \tag{22}$$

where η stands for the center-of-mass pion momentum in units of $1/\mu c$ (*40*). All "honest" calculations based on the *ps-ps* coupling seem to reveal that such calculations give too little isospin dependence; the isospin dependent part is expected to be smaller than the isospin independent part by a factor of ω/M in contradiction with experiments.[14] One may argue that the relativistic dispersion theory "explains" this puzzle because it is capable of expressing the charge-exchange scattering length in terms of the Yukawa constant determined from p-wave scattering and an integral over total cross sections (*41*). This argument is fallacious. To check the relation in question we must insert not only the p-wave data but also the observed s-wave data and data at much higher energies to the right hand side of the equation; hence we are not "explaining" anything about s-wave scattering.

Our theory offers an immediate explanation for this long-standing puzzle. The s-state "potential" between the pion and the nucleon is proportional to $\mathbf{T}_\pi \cdot \boldsymbol{\tau}_N/2$ times a positive quantity. Since $\mathbf{T}_\pi \cdot \boldsymbol{\tau}_N/2$ is -1 for $T = \frac{1}{2}$ (isospin antiparallel) and $\frac{1}{2}$ for $T = \frac{3}{2}$ (isospin parallel), the $T = \frac{1}{2}$ interaction is attractive and

[14] Within the framework of the γ_5 theory, it has been customary to explain the weakness of the s-state interaction by inventing various "pair suppression" mechanisms. These approaches are not satisfactory; they fail to explain why the isospin dependent part of s-wave scattering is not made smaller to the same degree as the isospin independent part.

the $T = \frac{3}{2}$ interaction is repulsive in agreement with observation. To be more quantitative $f_T^2/4\pi\mu_T^2$ which appears in our isospin current-current interaction

$$H = \frac{f_T^2}{4\pi\mu_T^2}\left(\boldsymbol{\phi}_\pi \times \frac{\partial \boldsymbol{\phi}_\pi}{\partial x_0}\right) \cdot u_N^+ \frac{\boldsymbol{\tau}}{2} u_N \tag{23}$$

turns out to be of the order of $0.1/\mu_\pi^2$ according to estimates of Klein (*42*) and of Drell *et al.* (*43*). Assuming the B_T mass of the order of $4\mu_\pi$, we obtain

$$\frac{f_T^2}{4\pi} \approx 1.5. \tag{24}$$

It is to be pointed out that (23) alone will make the $T = \frac{1}{2}$ state too attractive, and the $T = \frac{3}{2}$ state too little repulsive; an additional term of the form $\lambda\phi_\pi^2 u_N^+ u_N$ with $\lambda > 0$ is necessary. Within the framework of our theory we may be able to obtain such an isospin independent effective Hamiltonian by iterating the phenomenological Yukawa coupling. At this stage we should be satisfied with reproducing the major qualitative feature of s-wave pion-nucleon scattering, namely, the signs of δ_1 and δ_3, directly from our isospin current coupling.

The effects of the isospin current-current interaction (18) on p-wave scattering have been estimated in the Born approximation. They have been found to be negligible. For instance, in the neighborhood of the 3-3 resonance δ_{31} and δ_{13} are split by 5° at most. Hence our considerations do not invalidate the predictions of the usual static theories which have been so successful in reproducing the major features of p-wave scattering.

It is expected that s-wave $\pi\Sigma$ scattering is very different from s-wave πN scattering solely because the Σ isospin is unity in contrast to the N isospin of $\frac{1}{2}$. $\mathbf{T}_\pi \cdot \mathbf{T}_\Sigma$ is -2 for $T = 0$, -1 for $T = 1$, and 1 for $T = 2$. This means that the $T = 0$ $\pi\Sigma$ state is very attractive in comparison with the $T = \frac{1}{2}\pi N$ state. It is generally true that an attractive state is made more attractive than simple Born-type calculations indicate. We have derived an effective range formula for s-wave $\pi\Sigma$ scattering using a scattering formalism developed by Edwards and Matthews (*44*). The nucleon is treated statically, and a one-meson approximation is made. After the scattering matrix is obtained, the crossing symmetry is explicitly taken into account with respect to the initial and the final meson line. We have

$$\begin{aligned} p \cot \delta_\alpha &= -\frac{2\pi}{(f_T^2/4\pi\mu_T^2)n_\alpha\omega}\left(1 - \frac{\omega}{\omega_\alpha}\right), \\ \omega_\alpha &= -\frac{\pi^2}{(f_T^2/4\pi\mu_T^2)N_\alpha\Lambda}, \end{aligned} \tag{25}$$

with

$$
\begin{aligned}
n_\alpha &= -2 \text{ for } T = 0, \\
n_\alpha &= -1 \text{ for } T = 1, \\
N_\alpha &= -3/2 \text{ for } T = 0, \\
N_\alpha &= -1/2 \text{ for } T = 1.
\end{aligned} \tag{26}
$$

This s-wave effective range formula is peculiar in that the leading term is $1/\omega$ rather than a constant. This is the direct consequence of the fact that our effective potential expected from the current-current interaction goes like ω. Because of this peculiarity there definitely exists the possibility of an s-wave resonance in the $T = 0$ state (whereas no such resonance would be predicted for the usual $\phi_\pi^2 u_N^+ u_N$ type interaction, which leads to an effective range relation of the form $p \cot \delta = \text{const}$). With the cutoff energy $\Lambda \approx 4\mu_\pi$ and with $f_T^2/4\pi\mu_B^2$ determined from πN scattering, the $T = 0$ resonance is expected to occur at energies in the neighborhood of the $K^- p$ threshold. The $T = 1$ phase shift is still small ($\approx 30°$) at such energies so that the phase-shift difference between the $T = 0$ $\pi\Sigma$ state and the $T = 1$ $\pi\Sigma$ state is as large as 60°. This situation is to be contrasted with the global symmetry case where the phase-shift difference (derived from the π baryon Hamiltonian alone) must necessarily be small.

It turns out that this phase shift difference is "measurable" in the reaction $K^- + p \rightarrow \Sigma^{\pm,0} + \pi^{\mp,0}$ at low energies provided that the relative $K\Sigma$ parity is odd for which there is some evidence (*20, 45*). It is inferred from the most recent analysis that the phase shift difference in question is of the order of 60° (*46*). One may naively argue that this fits nicely with our theory. However, we have to be extremely careful; Dalitz and Tuan (*47*) have shown that the very fact that the $\bar{K}N$ channel is open can have a strong effect on $\pi\Sigma$ scattering. In fact, if the $\bar{K}N$ interaction turns out to be repulsive, on the basis of unitarity and analyticity alone, we can predict a resonance in the $\pi\Sigma$ system below the $K^- p$ threshold which bears no relevance whatsoever to a resonance one may obtain from the $\pi\Sigma$ Hamiltonian with the assumption that the $\bar{K}N$ channel does not exist.[15] Fortunately recent experiments seem to indicate that the $\bar{K}N$ interaction is attractive (*20*), which leads to the conclusion that the Dalitz–Tuan resonance does not exist. Moreover, it can be shown that if the so-called $b+$ (attractive) solution of Dalitz (*48*) turns out to be correct, for which we shall give some theoretical argument in Section IV, the "ideal" phase-shift difference obtained with the assumption that the $\bar{K}N$ channel does not exist bears some resemblance

[15] The Dalitz–Tuan resonance in $\pi\Sigma$ scattering should not be confused with our resonance. In fact our calculations leading to the $T = 0$ $\pi\Sigma$ resonance are meaningful only for those sets of the Dalitz solutions which do not predict any $\pi\Sigma$ resonance of the Dalitz–Tuan type. The author is indebted to Dr. S. F. Tuan for interesting discussions.

to the "actual" phase-shift difference.[16] A more detailed and quantitative discussion along this line will appear elsewhere when more reliable effective range parameters become available.

We now turn to a brief discussion on the existence of a π-π resonance. The very fact that the Yang–Mills B_T quantum with $J = 1$, $T = 1$ can "decay strongly" into two pions implies that the 2π system exhibits a resonant behavior in the $T = 1$ (p-wave) state. Frazer and Fulco (*49*) have shown that the isovector part of the electromagnetic structure of the nucleon can be readily understood by assuming such a resonance at the center-of-mass energy of the order of $4\mu_\pi$ to $5\mu_\pi$. Thus if our theory turns out to be correct, the existence of the desired resonance can be trivially explained.[17] Similarly the three pion system is expected to exhibit two resonances in the $T = 0$, $J = 1$ state corresponding to B_B and B_Y.*

It is tempting to speculate whether the so-called "higher resonances" in the πN interactions can be understood from the 2π and 3π resonances associated with our B quanta.[18] Various wild conjectures are possible as to how the observed resonances might emerge in our theory, but we cannot yet be too specific about detailed mechanisms. The following three points seem highly pertinent.

(1) There are two higher resonances in the $T = \frac{1}{2}$ state and one in the $T = \frac{3}{2}$ state. This may be related to the fact that in our theory there are two kinds of B quanta with $T = 0$ and one with $T = 1$.

(2) The π^+p and π^-p total cross sections above the three higher resonances are amazingly flat, and we do not seem to have a "rich spectroscopy" of the $S = 0$, $B = 1$ system above 2 Bev. If the observed resonances could be produced simply by piling up familiar 3-3 resonances, we should expect more higher resonances.

(3) The width of every one of the three higher resonances seems uncomfortably narrow to be accounted for by conventional mechanisms.

Turning now to multiple-pion production, we expect, on general grounds, that multiple-pion production cross sections are larger than what we would expect from Yukawa pictures or from statistical considerations even at energies below the B quanta thresholds; this seems to be the case experimentally. Consider the

* *Note added in proof:* A three pion resonance in the $T = 0$, $J = 1$ system has also been discussed by G. F. Chew [*Phys. Rev. Letters*, **4**, 142 (1960)]. The differences between Chew's approach and our approach are emphasized in J. J. Sakurai, *Nuovo cimento* [10] **16**, 388 (1960).

[16] This point was first pointed out to the author by Professor M. Ross and Dr. G. L. Shaw.

[17] The author is indebted to Professor Y. Nambu for pointing out this connection between the Frazer–Fulco resonance and the Yang–Mills B_T quantum.

[18] For the current experimental status of these "higher resonances" see Refs. *50* and *51*. These papers contain references to the earlier works.

production mechanism for two pions, for instance. Since the nucleon is surrounded by a B_T field, there exist continuous creations and annihilations of virtual pion pairs in $T = 1$ states. If there is sufficient energy available, such pairs become materialized rather readily. We expect that the pion pair created this way is in a relative p-state in the center-of-mass system of the two pions. If photoproduction of pion pairs also proceeds in this manner, we expect that in γp collisions $(\pi^+\pi^-)$ pairs are much more frequent than $(2\pi^0)$ pairs (since the $2\pi^0$ state is not accessible for $T = 1$). It would be interesting to check this point experimentally.

IV

Let us now turn our attention to various K particle phenomena.[19] The situation here is a little more involved than the π baryon interactions discussed in the previous section since K has hypercharge as well as isospin whereas π has no hypercharge.

We first consider the effects of the hypercharge current coupling on the $K^{\pm}N$ interactions. In complete analogy with Coulomb's law we have a repulsion (attraction) between two particles with nonvanishing hypercharges when their hypercharges are like (opposite). Let us recall that $K(K^+$ and $K^0)$ and $N(p$ and $n)$ bear positive hypercharges while $\bar{K}(K^-$ and $\bar{K}^0)$ and $\Xi^{0,-}$ bear negative hypercharges. Thus the KN interaction is repulsive whereas the $\bar{K}N$ interaction is attractive provided that the phenomenological Yukawa couplings play unimportant roles. Recent experiments show that the relative ΣK parity is likely to be odd (*20*, *45*) and that the relative ΛK parity is also likely to be odd (*52*). This means that the phenomenological Yukawa couplings of K particles are likely to be more important for p-state interactions, and that our approach may approximate reality for s-state interactions.

We can write down an effective Hamiltonian of the form

$$H = \frac{f_Y^2}{4\pi\mu_Y^2}\, i\left(\phi_K^+ \frac{\partial\phi_K}{\partial x_0} - \frac{\partial\phi_K^+}{\partial x_0}\phi_K\right) u_N^+ u_N, \tag{27}$$

which follows immediately from (19). In the potential language (27) means that the s-state KN interaction is repulsive whereas the s-state $\bar{K}N$ interaction is attractive. Equation (27) also implies that the KN and $\bar{K}N$ interactions are isospin independent, or equivalently there is no charge exchange scattering in either strangeness state.

It is amusing that in 1957 Christy (*53*) proposed KN and $\bar{K}N$ potentials which have precisely these features, in connection with his compound model.[20]

[19] The author is indebted to Professor R. H. Dalitz for discussions on the subject of K-particle physics.

[20] The origin of the Christy potentials can be traced back to earlier works of Goldhaber (*54*, *55*). Such potentials have been further considered in an unpublished work of B. T. Feld and collaborators.

At that time his potentials were criticized by Wentzel on the ground that such potentials are unreasonable from the point of view of field theory which requires K and $\bar{K}$ to be antiparticles of each other, and Christy was forced to admit that his potentials have not much to do with field theory. Clearly what Wentzel had in mind was the Wentzel-type pair interaction

$$H = \lambda \phi_K{}^{+} \phi_K u_N{}^{+} u_N , \tag{28}$$

which exhibits an exact symmetry between K and $\bar{K}$, and this leads to the same signs of potentials for both the KN and $\bar{K}N$ interactions. In our theory we have (27) rather than (28), so the Christy potentials have everything to do with field theory. It might be mentioned that the conventional *ps-ps* Yukawa-type couplings with $G(K\Lambda N) = \pm G(K\Sigma N)$ lead to (28) rather than to (27).

Experimentally, from K^+p data the $T = 1$ phase shift is definitely negative. Moreover, the charge-exchange scattering of K^+ on n is known to be small for s states. Thus for the s wave KN interaction both the $T = 1$ state and the $T = 0$ states are repulsive as our theory requires. Recent experiments show that there exists a constructive interference between the Coulomb potential and the K^-p "potential" (*20*). The charge exchange scattering of K^- on p is known to be small. A natural conclusion is that the $\bar{K}N$ interaction is attractive both in $T = 1$ and in $T = 0$. (Actually we have to be a little more careful in arguing that the smallness of the charge-exchange scattering of K^- implies that the potentials in the two isospin states have the same sign. The reason is that charge-exchange scattering is bound to be small if there are strong absorptions due to $K^- + p \rightarrow \Sigma$ (or Λ) $+ \pi$ in both isospin states; the very extreme case of *total* absorptions in both $T = 1$ and $T = 0$ leads to a null charge-exchange cross section. However, the fact that the real parts of the scattering lengths for the two isospin states in the $\bar{K}N$ interaction have to have the same sign is borne out by a more elaborate analysis of Jackson *et al.* (*56*) and Dalitz (*48*)). It is rather remarkable that our very simple considerations based on the hypercharge current coupling reproduce the qualitative features of both the KN and the $\bar{K}N$ interaction at low energies.

Because the K particle has isospin, we may naturally ask: What about the effects expected from the isospin current coupling? In addition to (27) we expect an effective Hamiltonian of the form

$$H = \frac{if_T{}^2}{4\pi\mu_T{}^2}\left(\phi_K{}^{+}\frac{\boldsymbol{\tau}}{2}\frac{\partial\phi_K}{\partial x_0} - \frac{\partial\phi_K{}^{+}}{\partial x_0}\frac{\boldsymbol{\tau}}{2}\phi_K\right)\cdot u_N{}^{+}\frac{\boldsymbol{\tau}}{2}u_N . \tag{29}$$

Note that $(\boldsymbol{\tau}/2)\cdot(\boldsymbol{\tau}_{KN}/2)$ is $\frac{1}{4}$ for $T = 1$ and $-\frac{3}{4}$ for $T = 0$. This means that in order that the qualitative agreement mentioned earlier obtained from the hypercharge current coupling alone be not spoiled by (29) $f_Y{}^2/4\pi\mu_Y{}^2$ is most likely to be larger than $f_T{}^2/4\pi\mu_T{}^2$. Once we assume this, we can make a more detailed prediction in KN scattering; the $T = 1$ state has to be more repulsive than the $T = 0$ state. This agrees with the phase shift analysis of Price *et al.* (*57*), who have obtained at 125 Mev

$$\begin{aligned} \delta &= -20^\circ \quad \text{for } T = 1, \\ \delta &= -7^\circ \quad \text{for } T = 0. \end{aligned} \tag{30}$$

A coupling constant relation such as $f_Y^2/4\pi\mu_Y^2 \approx 3f_T^2/4\pi\mu_T^2$ is probably reasonable to give this effect. Similarly we are led to the view that in the s-wave $\bar{K}N$ interaction the $T = 0$ state is more attractive than the $T = 1$ state. This seems to favor the b type attractive solution ($b+$) of Dalitz (*48*) where the real part of the $T = 0$ scattering length is larger than that of $T = 1$.

The fact that the major qualitative features of K^+ scattering and K^- scattering at low energies can be understood from (27) rather than from (29) indicates that the Pais doublet symmetry (S_1-S_2 rule) which forbids charge-exchange scattering has some domain of validity (*3*). It is easily seen that the hypercharge current and the baryonic current coupling respect the Pais doublet symmetry while the isospin current coupling destroys it. We shall come back to this point later in Section VI.

It is worth asking to what extent the qualitative features of $K^\pm$ particle interactions at low energies persist at higher energies. Experimentally the K^+p and the K^+n total cross section remain small ($\approx$ 15 mb–20 mb) and roughly constant up to Bev regions, and there does not seem to be any marked peak. (An increase in the charge exchange scattering of K^+ on n, which seems to be a pure p-wave effect, may be attributed to the phenomenological Yukawa couplings of K and π to baryons.) This qualitative feature can be understood if the KN potential is repulsive and short-ranged, in which case the cross section, being determined solely by the radius of the short-range potential, is expected to be constant. In contrast, it seems fairly well established that not only the total K^-p cross section but also the K^-p elastic cross section is larger than the K^+p elastic cross section at all energies. Here the apparent K^-p elastic cross section may be large because of strong absorption, but it is not inconceivable that the $\bar{K}N$ cross section would be large even in the absence of absorptive channels. For instance, at 400 Mev/c where $\pi\lambda\!\!\!^{-2} = 20$ mb the $T = 1$ absorption cross section is about 15 mb and the $T = 0$ cross section is about 20 mb whereas the K^-p elastic cross section is as large as 50 mb (*20*). Again using the potential language, we note that the large elastic cross section is still compatible with a short-range interaction, provided that the potential is strongly attractive. To sum up then, even at higher energies it is not entirely impossible to explain both the KN and $\bar{K}N$ interactions by Christy-type potentials, i.e., by potentials of the same range and type but of opposite signs (corresponding to the opposite signs of hypercharges), which is precisely the sort of behavior one expects from our current-current interaction brought about by the fundamental hypercharge current coupling. Another striking feature is that the charge-exchange scattering of K^- seems to remain small at all energies investigated so far. This may provide another piece of evidence in favor of the idea that the hypercharge current coupling which respects the Pais doublet symmetry is mainly responsible for $\bar{K}N$ scattering at all energies (even though strong absorptive effects may trick us).

If this idea that the gross features of $K^{\pm}N$ interactions can be understood from the hypercharge current coupling turns out to be correct, it becomes rather difficult to obtain information on the conventional coupling constants $G^2(K\Lambda N)$ and $G^2(K\Sigma N)$ from relativistic dispersion relations unless there exist very accurate data which are really sensitive to the poles associated with the one -Λ and the one -Σ state. Our case here is somewhat similar to the π^+ photoproduction case where the sign and magnitude of the pole associated with the photoelectric term at $\beta_\pi \cos\theta = 1$ would be difficult to obtain if it were not for very accurate data at forward angles since the major features of the reaction $\gamma + p \rightarrow n + \pi^+$ can be understood from the catastrophic term $(i\boldsymbol{\sigma}\cdot\mathbf{A}\phi_{\pi^-})$ and the 3-3 resonance (*58*). Our present inability to determine even the signs of the Λ and Σ pole terms (which are directly related to the relative $K\Lambda$ parity and the relative $K\Sigma$ parity) from KN dispersion relations may have its origin in the fact that the major features of KN and $\bar{K}N$ reactions bear little relevance to the phenomenological Yukawa couplings of K to the ΛN and ΣN systems (*59*).

We now consider

$$K^- + p \rightarrow \Xi + K. \tag{31}$$

It has been observed that the production cross section for this reaction is anomalously small—of the order of 25 μb for Ξ^0K^0 and less than 15 μb for Ξ^-K^+ at 1.7 Bev/c K^- (*20*). If we believe in the usual Yukawa picture, the sort of diagram responsible for K^+ and K^- scattering is also responsible for ΞK production, and this statement holds not just for lowest order diagrams but also for higher order diagrams. Although not much is known about K^+ and K^- scattering at such high energies, they must be of the order of 5 mb for K^+ and at least 10 mb for K^-. (For $K^- + p \rightarrow K^- + p + n\pi^0$ with $n = 0$ inclusive, the cross section is 24.2 $\pm$ 4.6 mb (*20*).) The phase space ratio for KN to $K\Xi$ is only 2:1 at this energy. So we conclude that (31) is suppressed by a factor of a few hundred. One may argue that the unitarity requirement suppresses the ΞK channel when there are several other competing channels, but still it is hard to understand why (31) is so rare. One answer would be that $G^2(K\Xi\Lambda)$ and $G^2(K\Xi\Sigma)$ are much smaller than $G^2(K\Lambda N)$ and $G^2(K\Sigma N)$. But if we compare the ΞK cross sections with associated production cross sections of ΛK and ΣK in πp collisions, they are not too small; at comparable final K momenta we have $\sigma(\pi^- + p \rightarrow \Sigma^+ +$

$K^+) \approx 200\ \mu$b. Note also that associated production cross sections for ΣK in πp collisions rise rapidly near threshold but then stay constant up to highest energies investigated so far.

In the following we propose a possible explanation for these anomalies. Perhaps both the smallness and the flatness of associated production cross sections are due to the fact that the phenomenological Yukawa couplings of K have "built-in cutoffs" at high energies. Processes such as K^-p scattering and K^+p scattering go via non-Yukawa type interactions of the hypercharge current of the nucleon with that of the K particle, and are not subject to cutoff limitations.

Even more interesting anomalies center around the ratio of $K\Lambda N$ to $K\bar{K}2N$ in NN collisions. According to recent experiments at Dubna the cross section for $N + N \rightarrow K + \bar{K} + 2N + n\pi$ is estimated to be larger than the cross section for $N + N \rightarrow K + \Lambda + N + n\pi$ by a factor of about two at 6.2 Bev/c p (with n average ≈ 2) while statistical calculations of Cerulus and Hagedorn give a factor of 1/10 (*60, 61*). What is perhaps more alarming is that such a tendency seems to persist at lower energies near the $K\bar{K}$ threshold (2.5 Bev p). For 3 Bev p (3.8 Bev/c) the ratio of $2NK\bar{K}$ to $(\Lambda NK) + (\Sigma NK)$ given by the Fialho–Serber statistical theory is as small as 1/30 (*62, 63*). Experimentally the cross section for ΛNK is anomalously small while a number of $K\bar{K}$ events have been identified at Cosmotron energies; it has been estimated that $K\bar{K}$ production competes favorably with ΛK production even near the $K\bar{K}$ threshold (*64, 65*).

This kind of anomaly is hard to understand in terms of the conventional Yukawa picture. One usually argues, following Gell-Mann, that the smallness of associated production is due to the smallness of $G^2(K\Lambda N)$ and $G^2(K\Sigma N)$, but then it is impossible to understand why $K\bar{K}$ production which involves $G^2(K\Lambda N)$ or $G^2(K\Sigma N)$ *twice* is so large. In our theory the mechanism for $K\bar{K}$ production is somewhat unconventional. Because of the strong B_Y field surrounding the nucleon, there are continuous creations and annihilations of virtual $K\bar{K}$ pairs, just as in the pion pair case discussed in Section IV. In NN collisions, if there is enough energy available, such virtual $K\bar{K}$ pairs readily become materialized. In contrast, ordinary associated production processes take place via a phenomenological Yukawa coupling, in which one unit of hypercharge must be transferred from a baryon to a meson, and which may well have a "built-in cutoff" at higher energies.

To sum up, processes which involve hypercharge transfers between baryons and mesons; e.g. associated production of ΛK and ΣK in πp and NN collisions and ΞK production in K^-p collisions, seem suppressed. On the other hand, processes in which there are no such hypercharge transfers; e.g., K^+ and K^- scattering and $K\bar{K}$ production, seem to be enhanced. (The only exception to this rule is the reaction $K^- + p \rightarrow \Sigma(\Lambda) + n\pi$, and here the very strong attractive well which tends to draw K^- into the absorption "black hole" and fuse together opposite hypercharges might be responsible for the largeness of the absorption cross section.) According to our theory the two types of processes go via entirely different mechanisms; the first type goes via phenomenological Yukawa couplings which seem to be damped at higher energies whereas the second type goes via hyper-charge current-current interactions which, from our point of view, are more fundamental. Hence the qualitative difference between the two does not seem too surprising even though we are very far from being able to make quantitative estimates.[21]

V

Nucleon-nucleon and nucleon-antinucleon phenomena are complicated by the fact that the three fundamental couplings all participate directly. We start our investigations of NN interactions by considering the baryonic current coupling and the hypercharge current coupling only.

Because the nucleon bears both hypercharge and baryonic charge, the effects expected from the hypercharge current coupling are hard to disentangle from those expected from the baryonic current coupling unless μ_Y is very different from μ_B. Assuming the relation $\mu_Y = \mu_B$, we obtain a static central potential between two nucleons brought about by the longitudinal components of the B_B and B_Y fields of the following form[22]

$$V = \frac{f_B^2 + f_Y^2}{4\pi} \frac{\exp(-\mu_B r)}{r}. \tag{32}$$

The force between two nucleons is strongly repulsive for distances less than $1/\mu_B$ in both the $T = 1$ state and the $T = 0$ state *regardless of angular momentum and parity.* We should like to suggest that this be the origin of the so-called "repulsive core." (Note that, if the repulsive core is due to the singular part of the one-pion exchange potential as first suggested by Levy (*67*), simple considerations show that, although we have repulsive cores for even L's $- (\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) = 3$, we must have deep attractive wells for odd L's $- (\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) = -9$ or -1. Needless to say, this is highly unsatisfactory; such attractive wells may lead to unphysical bound states.[23])

[21] One might argue that a *very strong* $\bar{K}K\pi\pi$ interaction, such as the one considered by Barshay (*66*), explains this qualitative difference. However, such an interaction fails to give a simple explanation as to why the KN potential is repulsive while the $\bar{K}N$ potential is attractive.

[22] There may be, in addition, an $f_B f_Y$ interference term.

[23] In some potential calculations which are alleged to be based on field theory, the strong attractive well in triplet odd states that appears in Levy–Gartenhaus type approaches is replaced by an infinite repulsive wall in an *ad hoc* manner. Such a procedure has no field-theoretic justification whatsoever within the framework of pion physics.

A very straightforward Thomas-type calculation based on our theory leads to the following spin-orbit potential*

$$V_{LS} = \frac{f_B^2 + f_Y^2}{4\pi} \frac{1}{2M_N^2} \frac{1}{r} \frac{d}{dr} \frac{\exp(-\mu_B r)}{r} \mathbf{S} \cdot \mathbf{L}. \tag{33}$$

First of all we note that the sign of the spin-orbit force is given correctly. We may compare this with the spin-orbit force of Signell *et al.* (*68*) who used the same Thomas–Yukawa type potential with range corresponding to $\frac{1}{2}\mu_\pi$. Assuming our μ_B is $2\mu_\pi$, we obtain $(f_B^2 + f_Y^2)/4\pi \approx 20$. The actual value may well be larger than this since our μ_B is most likely to be of the order of $4\mu_\pi$.† A spin-orbit force of a shorter range has been investigated by Gammel and Thaler (*69*), but unfortunately their results cannot be compared with ours since they have used a potential of the Yukawa-type rather than of the Thomas–Yukawa type. It is a curious, amusing (but possibly fortuitous) fact that one can obtain $f_B^2/4\pi \approx 30$ by assuming that the pion is a system of an N and an $\bar{N}$ bound by the B_B field, as previously pointed out by Fujii (*29*). The relation $f_B^2/4\pi \gg f_Y^2/4\pi$ has an interesting consequence on the origin of the $N\Xi$ mass difference as we shall discuss in Section VI.

Note that what we have done is extremely simple. No physicists *really* believe that the repulsive core is made up of an infinitely hard wall at some core radius. As soon as you make the core a little "softer" (as in the case of our theory where the core effect arises from a strong, short-range, repulsive, Yukawa-type potential), we can immediately obtain a spin-orbit force just by taking the Thomas derivative of the potential responsible for the core effect.[24] The author believes that simple effects such as the repulsive core and the spin-orbit force should have simple origins.[25]

* *Note added in proof:* The possibility that both the repulsive core and the spin-orbit force might be understood by postulating a neutral vector meson was also discussed by G. Breit [*Proc. Natl. Acad. Sci.* **46**, 746 (1960)].

† *Note added in proof:* In this paper only the Thomas-type spin-orbit force arising from the repulsive static potential has been considered. The spin-orbit force arising from the "radiation" field (the so-called Breit term) is twice as large as the Thomas term (but fortunately of the same sign). This seems to imply that $(f_B^2 + f_Y^2)/4\pi \approx 7$ for $\mu_B, \mu_Y \sim 4\mu_\pi$ as shown in J. J. Sakurai, *Phys. Rev.* (to be published). See also G. Breit, *Phys. Rev.* **34,** 55 (1929); **51,** 248 (1937); **51,** 778 (1937); **53,** 153 (1938).

[24] A similar suggestion seems to have been made by G. E. Brown. The author is indebted to Dr. J. M. Charap and Professor R. Oehme for informing him of Brown's work.

[25] It is easy to convince oneself by elementary arguments that, if the repulsive cores are to exist in all angular momentum, parity and isospin states, and if they have *simple* origins, they must arise from a neutral vector field with an effective vector-type coupling. A neutral scalar field gives the wrong sign. A pseudovector, a pseudoscalar, and a tensor field each give rise to both repulsions and attractions depending on spin states. $T = 1$ fields lead to both repulsions and attractions depending on isospin states.

Let us now switch on the isospin current-coupling. Because $f_B^2/4\pi$ is much greater than $f_T^2/4\pi$, the isospin current coupling does not have too much influence on the two-nucleon force as long as $\mu_T \gtrsim \mu_B$, μ_Y. If μ_T is smaller than μ_B (or μ_Y), there may be an isospin-dependent modification to (32), which makes the $T = 1$ state more repulsive and the $T = 0$ state less repulsive at distances between $1/\mu_{B,Y}$ and $1/\mu_T$. This means that the $T = 1$ state starts feeling the effect of repulsion before the $T = 0$ state as energies are raised in N-N scattering. Similarly, if $\mu_T < \mu_{B,Y}$, the $T = 1$ spin-orbit force gets enhanced whereas the $T = 0$ spin-orbit force gets weakened. So far there does not seem to be any experimental evidence for a spin-orbit force in the $T = 0$ state; we predict spin-orbit forces for both $T = 1$ and $T = 0$ with the sign given by the shell-model, even though the $T = 0$ spin-orbit force may be weaker.

Our theory points out a new direction for attacking the high-energy nuclear force problem. Instead of assuming an infinite wall at some distance and postulating an $\mathbf{L} \cdot \mathbf{S}$ potential outside the wall in an *ad hoc* manner, we should use a strong, short-range, repulsive central potential of the Yukawa-type (depending only weakly on isospin and not at all on parity) together with the $\mathbf{L} \cdot \mathbf{S}$ potential which is nothing more than the Thomas derivative of the strongly repulsive central potential. Needless to say, the "tail" of the two-nucleon potential should be given by the noncontroversial part of the Taketani–Gartenhaus–and-many-others potential. Such an analysis may not be a pure waste of time even if any analysis based on a static (plus simple $\mathbf{L} \cdot \mathbf{S}$) potential must fail eventually at high energies.

ΛN and ΣN forces at short distances should be given by (32) with $f_B^2 + f_Y^2$ replaced by just f_B^2 because neither Λ nor Σ has hypercharge. This means that ΛN or ΣN forces should be less repulsive at short distances than NN forces. The core property of Λ or Σ should be different from that of N but not too different if $f_B^2 \gg f_Y^2$.

In complete analogy with Coulomb's law the $N\bar{N}$ potential at short distances is given by the negative of (32). There is a very deep attractive well of radius roughly equal to $1/\mu_B$. Other baryon-antibaryon interactions are also expected to be very attractive at short distances though $\bar{\Lambda}N(\bar{\Sigma}N)$ forces may not be as strongly attractive as $\bar{N}N$ forces.

The very strong short-range attraction between N and $\bar{N}$ is capable of drawing an antinucleon into the deep attractive well of the nucleon even at high energies. The annihilation cross section is expected to be large even in Bev regions, which is not the case in the Ball–Chew theory where only the long-range attraction due to the G conjugated one-pion exchange potential is responsible for drawing the antinucleon into the annihilation region (*70*).[26] (It is to be emphasized that in

[26] A vector field somewhat similar to ours has been considered by Duerr (*71*) and Teller (*72*). However, our theory differs drastically from the Duerr–Teller theory in three important respects. First of all, our potential (32) and the corresponding spin-orbit potential (33)

our theory the long-range G conjugated potential of Ball and Chew does exist in addition to the deep, short-range attractive well characteristic of our theory.) We expect the $p\bar{p}$ and $n\bar{p}$ cross sections to decrease up to about 400 Mev in conformity with the Ball–Chew theory, but then stay fairly large all the way up to Bev regions. If the Ball–Chew mechanism were the only mechanism for "catching" the antinucleon, the annihilation cross section would dwindle down in Bev regions to something like 10 mb characteristic of the geometric cross section of the "black hole." Because of the very deep attractive well, this does not happen in our case. Note also that $p\bar{p}$ cross sections (both scattering and annihilation) are expected to be slightly larger than $n\bar{p}$ cross sections since there is an added attraction in the $T = 0$ case brought about by the isospin current coupling. Qualitative as they are, all these features expected from our theory seem to be in good agreement with experiments (*73*).

One of the mysteries in antinucleon physics is that the reaction

$$p + \bar{p} \rightarrow \pi^+ + \pi^- \tag{34}$$

seems to be spectacularly infrequent. Out of 3000 annihilations events with an antiproton beam at ≈ 1 Bev/c no event corresponding to (34) has been reported by the Berkeley propane chamber group; a similar tendency seems to have been observed by the hydrogen chamber group.[27] For the sake of discussion we take the frequency of (34) compared to all other processes to be of the order of 1/10,000. We may naturally ask: Why should an analytic continuation of the $\pi^{\pm}p$ scattering amplitude in the sense of Mandelstam dwindle down to such a fantastically small value? One of the most likely explanations is that the Yukawa concept which works very well for low-energy $\pi^{\pm}p$ scattering fails completely if the pion momentum is of the order of 1 Bev/c or larger. This is not surprising within the framework of our theory in which the Yukawa coupling of the pion is not fundamental.[28]

are short-ranged in comparison with theirs. Secondly Duerr and Teller have considered, in addition, a scalar field which leads to long-ranged *attractions* for *both* NN and $N\bar{N}$; this has no counterpart in our theory. Thirdly their scalar and vector fields are directly responsible for the major properties of nuclear matter such as the saturation condition and the effective mass. This last point is in complete disagreement with our philosophy; we believe that the major properties of nuclear matter can be understood eventually from our knowledge about the basic nucleon-nucleon interaction, the long-range part of which is due to the conventional Yukawa coupling of the pion to the nucleon. Also note that Feynman's criticisms that the Duerr–Teller Hamiltonian has no lowest state of energy and that "the universe must fall through a hole somehow" is not applicable to our theory since this difficulty has arisen only from the scalar field of the Duerr–Teller theory.

[27] The author is indebted to Professor B. T. Feld for calling attention to this problem, and to Professor S. Goldhaber and Miss J. Button for informative conversations on the current experimental status.

[28] If this view turns out to be correct, $K^- + p \rightarrow \Sigma + \pi$ should be much less frequent than $K^- + p \rightarrow \Sigma + n\pi$ with $n > 1$ in Bev regions.

Together with the above-mentioned mystery we must explain simultaneously the following second mystery in antinucleon physics. It has been observed that the average pion multiplicity in $p\bar{p}$ annihilations is about five, and this is too high to be expected from statistical considerations with an interaction volume of reasonable size; to save the statistical theory one is forced to assume a sphere with radius $2.2/\mu_\pi$ (*74*). But even with such an unreasonably large interaction radius, the $\pi^+\pi^-$ channel seems to be off by a factor of at least five.[29]

We should like to propose the following as a new model of $N\bar{N}$ annihilations. Just as in electron-positron annihilations two or three photons are produced, in $p\bar{p}$ annihilations two or three B quanta are produced. This is reasonable in our theory because the effective couplings of the B fields to the nucleon-antinucleon pair are expected to be much stronger than the phenomenological Yukawa coupling of the pion field. Some of the likely reactions in s states compatible with the selection rules are as follows:

$$\begin{aligned}
&{}^1S_0{}^1 \to 2B^0_{B,Y} \to (\pi^+\pi^-\pi^0) + (\pi^+\pi^-\pi^0),\\
&{}^1S_0{}^3 \to B^0_{B,Y} + B_T{}^0 \to (\pi^+\pi^-\pi^0) + (\pi^+\pi^-),\\
&{}^3S_1{}^1 \to B^0_{B,Y} + B_T{}^+ + B_T{}^- \to (\pi^+\pi^-\pi^0) + (\pi^+\pi^0) + (\pi^-\pi^0),\\
&{}^3S_1{}^3 \to B_T{}^+ + B_T{}^- \to (\pi^+\pi^0) + (\pi^-\pi^0).
\end{aligned} \tag{35}$$

The "spectroscopic" notation ${}^{(2S+1)}L_J^{(2T+1)}$ has been used. An annihilation into $\pi^+\pi^-$ would be possible only if p and $\bar{p}$ produce a single $B_T{}^0$, but this is unlikely because such a $B_T{}^0$ must necessarily be virtual. The observed large average pion multiplicity of five fits nicely with our model.

If one could see π^0's, it would be possible directly to test our model by plotting Q values of pions in various combinations. Unfortunately in bubble chamber experiments carried out up to now one can study correlations among charged pions only. Experimentally, angular correlations of like pairs ($\pi^+\pi^+$ or $\pi^-\pi^-$) turn out to be rather different from angular correlations of unlike pairs ($\pi^+\pi^-$) (*76*). From (35) we do expect that correlation effects of like pairs are different from those of unlike pairs, but it is impossible to decide whether the observed experimental tendency is in the right direction expected from our model. Since

[29] An alternative, interesting proposal has been made by Koba and Takeda (*75*). They use a much smaller interaction volume in which the annihilation takes place, but they emphasize that we should also count pions in the "clouds" which are, so to speak, left over. According to their estimates about two to three pions should be produced in the core-core annihilation and about two pions should be produced from the "clouds." However, to fit the $\pi^+\pi^-$ data we conclude that the probability of the "clouds" emitting no pion should be about 0.03% (assuming the probability of 2π emissions in core-core annihilations is of the order of 30%), which is to be compared with their estimate of 12% based on intermediate coupling calculations.

the observed correlations are likely to be explained in many ways (e.g., Bose statistics), direct measurements of energies and directions of π^0's together with those of $\pi^{\pm}$'s are urgently needed.

We now come back to the subject of nucleon-nucleon interactions to discuss the distribution of pions in high energy jets. In conventional theories of NN collisions at extreme high energies, as proposed by Fermi (*77*), Heisenberg (*78*), Landau (*79*), and others, two colliding nucleons form a *single* "fire ball" which subsequently emits various particles as some kind of equilibrium is reached. We should like to argue that this is impossible. The colliding nucleons have baryonic charges of the same sign, and because of the strong repulsion between like baryonic charges, they cannot fuse together to form a single "fire ball" with baryon number two. A "fire ball" with baryon number two cannot exist for the same reason as a "superbaryon" with baryon number greater than unity cannot exist, a point we shall discuss more fully in Section VI. The crux of the argument is that a repulsive energy associated with a system with $B = 2$ is too great for such a system to exist at all. Instead of having only one "fire ball" as in Fermi–Landau–Heisenberg type theories, what we have are two "fire balls," each of which has baryon number unity. It is expected that each "fire ball" emits pions directly or more likely as disintegration products of various B quanta isotropically with respect to the center of each "fire ball" and not with respect to the center of mass of the colliding system.

The existing high-energy jets with $E > 10^3$ Bev have been analyzed by Ciok *et al.* (*80*), Cocconi (*81*), and Niu (*82*), who all have arrived independently at the following interesting points.[30]

(A) Simple Fermi–Landau–Heisenberg type mechanisms with one center fail to explain the observed data unless unreasonably complicated angular distributions are assumed.

(B) There are *two* centers which move in opposite directions with the same velocities, and pions are emitted independently and isotropically with respect to each of the two centers.

Thus our theory seems to provide some theoretical justification for the "two-fire-ball model" previously proposed on purely phenomenological grounds.

It is expected that the "one-fire-ball" model should still work for $N\bar{N}$ collisions where two opposite baryonic charges can fuse together. It would be interesting to test this point experimentally.

VI

The view that the Yukawa interaction might not be "fundamental" has been previously expressed on a number of occasions. As mentioned in Section II,

[30] The author is indebted to Professor J. Nishimura for interesting discussions on the subject of high-energy jets.

already in 1949 Fermi and Yang (*37*) have shown that a theory based on a four-nucleon Fermi-type coupling results in a theory very similar to the conventional Yukawa theory as far as low-energy pion physics is concerned. The Fermi–Yang model has not been universally accepted mainly on the ground that it requires "a glue to explain the glue." The basic question is: How can we justify the existence of the glue necessary to bind an N and an $\bar{N}$? Models of Sakata (*83*) and Okun (*84*) are natural generalizations of the Fermi–Yang model to strange particles. (Similar models have been proposed by Levy and Marshak (*85*) and by Markov (*86*).) Here it is trivial to write down Lorentz invariant four-baryon couplings in which only Λ and N appear, and count the number of Λ's and $\bar{\Lambda}$'s to see which strange particles correspond to which bound states. What is more difficult, and at the same time much more important, is to justify the *dynamics* of the model in question on theoretical grounds. Similar criticisms apply to the Goldhaber–Christy model (*55, 53*) in which only K particles, pions and nucleons are "elementary." It is easy to count the number of K's and $\bar{K}$'s but hard to find a dynamical principle that tells us why the KN system should have no bound states while the $\bar{K}N$ system has two bound states.

It is amusing that our theory provides the dynamical bases of the various compound models in a very natural manner. In the Fermi–Yang–Sakata–Okun model the baryon-baryon interaction at short distances must be repulsive in order that a "super-baryon" with baryon number greater than unity does not exist, whereas the baryon-antibaryon interaction must be strongly attractive in order that the baryon-antibaryon pair is capable of forming a meson. Our baryonic current coupling does precisely that. The "glue to glue the glue" appears naturally in our attempt to localize the concept of baryon conservation, as already pointed out by Fujii (*29*). The fact that the pion is lighter than the K particle can be explained, since the $N\bar{N}$ force is more attractive than the $N\bar{\Lambda}$ force because of the additional attraction due to the hypercharge current coupling. In the Goldhaber–Christy model both the repulsion of the KN interaction and the attraction of the $\bar{K}N$ interaction have their common origins in the hypercharge current coupling. The smallness of the $\Lambda\Sigma$ mass difference is a direct consequence of the fact that the isospin current coupling is effectively weaker than the hypercharge current coupling, and Λ should be lighter than Σ because, according to our theory, the $T = 0$ (isospin antiparallel) $\bar{K}N$ interaction should be more attractive than the $T = 1$ (isospin parallel) $\bar{K}N$ interaction. Thus all the *ad hoc* assumptions that must be made in order that the various compound models work at all can be explained trivially from first principles once our theory is accepted.

In spite of all these, we should like to suggest that the questions of which elementary particles are "more elementary than others" and of which compound model is right have not much meaning. The reason is that, if the binding energy is so large as to be comparable to the sum of the rest energies of the constituents, the "elementary" constituents completely lose their original identities. Given a pion, which is supposed to be a bound system of an N and an $\bar{N}$ in the Fermi–Yang model, we cannot even tell where the nucleon or antinucleon is located. Even if we could locate the nucleon, the nucleon in this bound system would be very different from that in free space. Even if the pion were made up of an N and an $\bar{N}$, the "structure" of the pion is (according to the conjecture of the dispersion experts) chiefly determined by the lowest mass states into which the pion can disintegrate; namely, a 3π state, a 5π state, etc., but not an $N\bar{N}$ state. This situation should be contrasted with the deuteron case where by performing electron-deuteron scattering we can locate where the proton is, and we can, to a very good approximation, understand the electromagnetic structure of the deuteron once the electromagnetic structures of the proton and the neutron are known. One may argue that we should regard stable particles as elementary particles as much as possible, but this argument is fallacious because the deuteron is stable while the neutron is unstable. It is easy to convince oneself that the Sakata–Okun model works just as well even if we regard, instead of N and Λ, Ξ and Λ as "elementary," a point already noted by Okun (*84*).

After all, what are elementary particles? They are nothing more than systems with radius less than 10^{-13} cm which are specified by certain internal properties. To characterize a strongly interacting particle, we need only specify its internal attributes, such as baryon number, hypercharge, and isospin. If we could examine the "structure" of a baryon closely enough, we would be able to talk meaningfully about the mean-square baryonic-charge radius in the same way as we can talk meaningfully about the mean-square electric-charge radius of the proton, but we would never be able to tell whether a Ξ hyperon is made up of two Λ's and one $\bar{N}$ as in the Sakata–Okun model or two $\bar{K}$'s and one N as in the Goldhaber–Christy model.

Let us now visualize three kinds of fluid-like substance, which we may call "Urschmiere,"[31] corresponding to the three kinds of internal attributes. To create a Λ hyperon, for instance, we bring together bits of baryonic "Urschmiere" until we have one unit of baryonic charge, keeping in mind that the total spin of the system must be one-half. Although the total hypercharge of the Λ must be zero, the hypercharge density of the physical Λ need not identically vanish since there are virtual processes such as $\Lambda \rightleftarrows N + \bar{K}$ and $\Lambda \rightleftarrows \Xi + K$. The rest energy of the Λ to the first approximation is the energy required to create a "pure baryon" by bringing together bits of baryonic "Urschmiere." This "Urschmiere" approach to the self-energy of a particle is reminiscent of an interesting work of Huang (*87*) (motivated by an earlier work of Weisskopf (*88*)) in which he tried to inter-

[31] The author is indebted to Professor R. Oehme for suggesting this term. The term "Urschmiere" has been previously used by Heisenberg in a slightly different connection.

pret the electromagnetic self-energy diagrams of the nucleon in the conventional formalism in terms of "semiclassical" energies required for the nucleon to acquire its electromagnetic properties.

It has to be admitted that it is much more difficult to compute phenomenological-Yukawa-coupling constants in our "Urschmiere" approach than in the Fermi–Yang compound approach. This is because we have no reliable formalism to attack the question of how various kinds of "Urschmiere" rearrange themselves when a Yukawa process takes place. In the Fermi–Yang case the conventional field theory is sufficient to enable us to estimate $G^2(\pi NN)$ from a single constant no matter how crude such an estimate may be. This is not so in our case. We do not even have an adequate language to describe the very complicated process of the nucleon emitting a pion.

We now discuss the mass spectrum of strongly interacting particles under the assumption that the mass of an elementary particle is the energy required to bring together bits of various kinds of "Urschmiere." The baryonic current coupling is the strongest of the three fundamental couplings of strong interactions. Hence we expect that the mass of any baryon is roughly determined by the baryonic current coupling alone, and is equal to the energy necessary to bring together bits of baryonic "Urschmiere" until the total baryonic charge is unity. The empirical fact that the various baryon masses do not differ by an order of magnitude is compatible with our way of thinking that the two other couplings, which presumably disturb the complete baryon degeneracy, are weaker. As we have seen in Section V, $f_B^2/4\pi$ determined from the spin-orbit coupling in the pp interaction is about 2000 times larger than 1/137. This might have something to do with the observed fact that a typical baryon, for instance a Λ, is about 2000 times as heavy as the electron. But we are again reminded of the "μ problem"; so we cannot be too confident about this speculation.

In reality there are two other strong couplings in addition to the baryonic current coupling. The baryon degeneracy is removed as we switch on the hypercharge current coupling and the isospin current coupling. One may expect that the $N\Xi$ mass difference must remain zero since N and Ξ have the same $|Y|$ and the same isospin. This may not be true for the following reason. We have already remarked in Section II that the $B_\mu^{(Y)}$ field and the $B_\mu^{(B)}$ field have the same transformation properties under G. This means that there is, in general, an interference effect between the baryonic current coupling and the hypercharge current coupling. Specifically the B_Y quantum can convert itself into a system of three pions with $T = 0$, which can in turn become a B_B quantum. Thus an N (or a Ξ) may emit a B_Y quantum which is subsequently absorbed as a B_B quantum. This immediately implies that the $N\Xi$ degeneracy is removed. Since the couplings are strong such an argument should not be taken too literally. However, it is worth noting that the observed large mass difference between N and Ξ is not contradictory with our theory in view of the relation $f_B^2/4\pi \gg f_Y^2/4\pi$, which may imply that terms proportional to $f_B f_Y$ are more significant than terms proportional to f_Y^2 in accordance with the observed mass spectrum where the Y term is larger than the Y^2 term.[32] Needless to say the $\Lambda\Sigma$ mass difference is a direct consequence of the fact that Σ can interact with the $B_\mu^{(T)}$ field while Λ cannot. We expect the Σ mass to be larger than the Λ mass in agreement with observation. The fact that the $\Lambda\Sigma$ mass difference is relatively small is also anticipated because the isospin current coupling responsible for the $\Lambda\Sigma$ split is the weakest of the three fundamental strong couplings.

To sum up, with the baryonic current coupling alone all baryons are degenerate. The hypercharge current coupling together with the baryonic current coupling depresses the rest energy of one of the $|Y| = 1$ baryons, namely N, and raises that of the other, namely Ξ, leaving the $\Lambda\Sigma$ degeneracy untouched. The isospin current coupling splits Σ from Λ and further influences the rest energies of N and Ξ.

The main point we should like to emphasize is that, although our fundamental couplings are highly universal and symmetric, it is possible for the three couplings *alone* to account for the observed mass spectrum. This is not true with a theory based on Yukawa couplings. If all G_π's are equal and all G_K's are equal, all baryons, if degenerate in the absence of interactions, are still degenerate in the presence of interactions. Various attempts have been made to assign different intrinsic parities to various particles leaving the equality relations of the coupling constants unchanged. Such approaches can hardly be compatible with the spirit of a universal theory of strong interactions; no *physically interesting* relations emerge by equating an unrenormalized *ps-ps* constant to an unrenormalized *s-s* constant.

We now turn our attention to the boson mass spectrum. Recall that the $T = 1$ π has no hypercharge while the $T = \frac{1}{2}$ K does bear hypercharge. It is not surprising that the pion is lighter than the K particle because the isospin current coupling is weaker than the hypercharge current coupling. Also note that, although the baryonic charge density of the pion must be strictly zero, the baryonic charge density of the K particle need not vanish identically. This may be another reason why K is more massive. A fictitious charge-singlet, $Y = 0$ boson, which

[32] The actual situation may be more complex. If there exists a complete symmetry between N and Ξ to begin with as in our "Urschmiere" approach (but not in the Okun–Sakata approach) any $B_Y R_B$ interference must necessarily vanish in the absence of the electromagnetic coupling, provided that the $B_\mu^{(Y)}$ and $B_\mu^{(B)}$ fields are stable. This follows from the invariance of the total strong interaction Lagrangian under the transformation (*A*) of Feinberg and Behrends (*89*): $N \leftrightarrow \Xi$, $K \leftrightarrow K_G$, $B_Y \leftrightarrow -B_Y$, $B_B \leftrightarrow B_B$. But the fact that neither B_Y nor B_B is forbidden to decay strongly into three pions *may* invalidate this argument based on such substitutional invariance. The author is indebted to Professor G. Feinberg for pointing out the possible $N\Xi$ degeneracy in our "Urschmiere" approach.

we may denote by $\pi^{0\prime}$, has never been observed. (The relation between $\pi^{0\prime}$ and $\pi^{\pm,0}$ would be entirely analogous to the relation between Λ and $\Sigma^{\pm,0}$; the $\pi^{0\prime}$ should not be identified with our charge-singlet B quanta ($B_Y{}^0$ and $B_B{}^0$), which play entirely different roles in the physics of strong interactions.) In our theory there is no reason why the $\pi^{0\prime}$ should exist because no "Urschmiere" would be associated with such a particle. The $\pi^{0\prime}$ would have no internal attributes, *ergo* no self-energy. Perhaps, if it is spinless, it might be identified with the vacuum state. We mention in passing that within the framework of the Fermi–Yang–Sakata–Okun model it may be difficult to explain why the $\pi^{0\prime}$ does not exist.

We may ask if there is any place for more "elementary" particles within the framework of our theory. Recently the possible existence of a charge-singlet $S = \pm 2$ particle, denoted by $D^{\pm}$, has been discussed (*90*). Such a particle would correspond to a doubly hypercharged particle. All our theory can say is that the $D^{\pm}$, if it exists, has to be more massive than the K particle. This is because the D, having two units of hypercharge, would have more energy than the $Y = 1$ K particle for the same reason as a sphere with two units of electric charge has a greater electrostatic energy than a sphere with only one unit of electric charge. In our theory the question of whether or not the $D^{\pm}$ exists is a *dynamical* problem, not a group-theoretic problem. If the hypercharge current coupling is sufficiently strong, an elementary particle with two units of hypercharge can never be formed for the same reason as a soap bubble can accommodate only a finite amount of electric charge.

A similar consideration applies to the existence or nonexistence of a "superbaryon," an *elementary* particle with baryon number greater than unity. The empirical fact that there does not seem to be a particle with baryon number two is not any more mysterious than the fact that there are no superheavy nuclei. Just as the Coulomb repulsion prevents the formation of a nucleus with $Z > 100$, the formation of an elementary particle with baryon number two cannot take place if the baryonic current coupling is sufficiently strong. What *is* mysterious centers around the question of why the values of $f_B{}^2/4\pi$ and the "fundamental length" (which is presumably related to μ_B and the masses of other elementary particles) are arranged in such a manner as to make the existence of a "superbaryon" impossible.

We have seen from experimental data and also from the mass spectrum that the relation

$$\frac{f_B{}^2}{4\pi} > \frac{f_Y{}^2}{4\pi} > \frac{f_T{}^2}{4\pi} \tag{36}$$

must hold. This reminds us once again of the question of whether Pais' concept of a hierarchy of interactions with different symmetry properties (*91*) is realized in nature within the realm of strong interactions. When only the baryonic current coupling is "on," all baryons are on the same footing. There exists a permutation symmetry among eight baryons, which we may call "octet symmetry." (This should not be confused with global symmetry.) When we switch on the hypercharge current coupling, the octet symmetry is destroyed, but we can readily verify that the Pais doublet symmetry (*3*, *4*) still holds. (Recall that the Pais doublet symmetry implies that Λ and Σ can be treated on the same footing as two doublets as well as one singlet and one triplet, and that K^+ and K^0 can be regarded as two singlets as well as one doublet. We should like to emphasize here that the basic concept of the Pais doublet symmetry has its origin in the group properties of a four-dimensional isospin space and has nothing to do with the existence of Yukawa-type Lagrangians just as the basic idea behind charge independence is entirely independent of any Lagrangian formalism.[33]) It is noteworthy that the Pais doublet symmetry which is the weakest symmetry stronger than charge independence is satisfied by the stronger two of our three fundamental couplings. Also note that we did not postulate the Pais doublet symmetry in the beginning, but we have obtained it as a kind of gift when the isospin current coupling is "off."

The conjecture that the Pais doublet symmetry might work has originated in the recognition that the $\Lambda\Sigma$ mass difference is fractionwise the smallest nonelectromagnetic mass difference between any pair of strongly interacting particles (*3*). It is most natural to argue that there exists a connection between the very characteristic that distinguishes Σ from Λ and the coupling that destroys the Pais doublet symmetry. Needless to say, the basic difference between Σ and Λ is that Σ has isospin but Λ has no isospin. Our isospin current coupling that distinguishes Σ from Λ destroys the Pais doublet symmetry as anticipated.

Although the hypercharge current coupling destroys the "octet symmetry" of the baryonic current coupling, it does not destroy the universality of the baryonic current coupling. Similarly, although the isospin current coupling destroys the Pais doublet symmetry of the baryonic current coupling and of the hypercharge current coupling, it destroys neither the universality of the baryonic current coupling nor that of the hypercharge coupling. Hence we still have only three universal constants in the presence of all strong couplings. By banishing the idea that the Yukawa couplings of π and K are fundamental, we have succeeded, for the first time, in realizing both the Pais principle of a hierarchy of interactions (*91*) and the Pais principle of economy of constants (*92*) in a natural and elegant manner.[34]

The conservation law of baryons associated with the strongest of the three strong couplings is absolute in so far as we are concerned with the time scale

[33] Thanks are due to Professor A. Pais for repeatedly reminding the author of this point

[34] All previous attempts along this line have failed miserably, leading to nothing but ugliness and inconsistencies.

which is at most of the order of 10^{23} years. The conservation law of hypercharge associated with the second strongest coupling is "respected" by the minimal electromagnetic coupling, but it is broken by the "weak" interactions, which are weaker by many orders of magnitude. The conservation law of isospin associated with the weakest coupling of the three strong couplings is broken by both the electromagnetic interactions and the weak interactions. We are led to speculate that there may be a connection between the limits of the validity of a conservation law and the strength of the corresponding coupling. Along similar lines several people have conjectured that the stronger the couplings, the more symmetries they admit (*93*, *94*). Such conjectures and speculations may have far more profound implications in our theory than in any other theory now that the very existence of a coupling is deeply rooted in the corresponding conservation law.[35]

VII

If the present theory turns out to be correct, one may naturally ask whether *all* fundamental couplings that exist in nature are rooted in the conservation laws of internal attributes.[36] In addition to the three conservation laws of the strong interactions and the conservation law of electric charge, there is the conservation law of leptons. But, because the conservation law of baryons is absolute as far as elementary particle physics goes, lepton conservation is equivalent to fermion conservation. Let us note that baryons as well as leptons interact weakly, and that there exist no bosons which interact *only* weakly. So we are led to the idea that there is a deep connection between the origin of weak interactions and the law of fermion conservation.[37]

We assume that all masses are due to strong and electromagnetic interactions. One may find this objectionable for two reasons. First of all, if one believes in the conventional field theory, the self-energy δm is always proportional to the bare mass. But the conventional field theory should not be trusted in details, and we may hope that something like our "Urschmiere" approach leads to a theory in which masses can be produced out of nothing once a "fundamental length" is given. Secondly, the "μ problem" exists. But let us assume that this mystery is solved somehow.

[35] The author had been skeptical about such conjectures until the formulation of the present theory because in previous theories it has been impossible to understand why there are such good reasons for the particular form(s) of the electromagnetic (and possibly weak) coupling(s) while the same could not be said about the strong couplings.

[36] The author is indebted to Professor G. Wentzel for asking the right question which has led to the investigations in this section.

[37] For previous attempts along this line, see works of Bludman (*95*) and Salam and Ward (*96*). The possible connection between the gauge principle and the vector nature of the weak couplings was first discussed by Yang (*97*) when the A^{35} recoil experiment of Allen and collaborators was still an unconfirmed rumor.

The question now is: How can we distinguish a fermion from an antifermion? Pauli (*98*) pointed out that for the massless neutrino the concept of particle versus antiparticle is ill-defined because the particle-antiparticle-mixing transformation

$$\begin{aligned} \psi &\rightarrow a\psi + b\gamma_5 C\bar{\psi}^T, \\ C\gamma_\mu{}^T C^{-1} &= -\gamma_\mu, \\ |a|^2 + |b|^2 &= 1 \end{aligned} \tag{37}$$

carries the Hamiltonian into some equivalent Hamiltonian without any observable change. In our theory the new point is that we can not distinguish a fermion from an antifermion even for baryons, electrons and muons as the strong and electromagnetic couplings are switched off. This is because, as the coupling constants for the strong and electromagnetic couplings go to zero, the internal attributes such as baryon number and electric charge which would otherwise distinguish a fermion from an antifermion all disappear. So all fermions become neutrino-like in the absence of the strong and electromagnetic couplings.

If we are to write down the conserved current for fermionic charge when $m = 0$, $f_B = f_Y = f_T = e = 0$, we must first project the "true fermion" state. The fermionic charge operator Q_F has the following property:

$$CQ_F\,|\Psi> = -Q_F C\,|\Psi>, \tag{38}$$

where C is the charge conjugation operator. We would like to believe that the concept of fermionic charge is related to some kind of internal degree of freedom of the Dirac spinor. So we look for Q_F which is of the form of a Dirac matrix. In terms of the field operator we try to find Γ_F such that

$$C(\overline{\Gamma_F\psi})^T = -\Gamma_F C\bar{\psi}^T, \tag{39}$$

where Γ_F is a linear combination of the sixteen independent Dirac matrices. One can readily show that Γ_F that satisfies (39) and that does not depend on the orientation of space-time axes is $ai + b\gamma_5$ with a and b real. The ai term is not suitable for the fermionic charge matrix whose diagonal elements must be real. Since the eigenvalues of the charge matrix must be ± 1 we are led to the only possibility $a = 0$, $b = \pm 1$. Without loss of generality we can define the true fermion state in such a way that $b = 1$, and with this convention leptons and baryons in the usual theory are true fermions and not antifermions. The fact that γ_5 diagonalizes the fermionic charge operator means that as the strong and electromagnetic couplings disappear, the differentation between fermion and antifermion or between matter and antimatter can be made only via the sign of γ_5 or equivalently only by differentiating "right" from "left." (Note that states of positive helicities and states of negative helicities for $m = 0$ particles are never mixed up by the Pauli transformation (37).)

The conserved current for fermionic charge reads

$$J_\mu = \tfrac{1}{2}\bar{\psi}\gamma_\mu(1 + \gamma_5)\psi. \tag{40}$$

The current-current interaction constructed out of (40) is the now familiar four-fermion interaction in the universal V-A theory. Although the author has no ideas as to why neutral currents such as $(\bar{\nu}\nu)$, $(\bar{e}\mu)$ do not appear in weak interactions, why the observed coupling between $(\bar{\Lambda}p)$ and $(\bar{e}\nu)$ is considerably smaller than that between $(\bar{n}p)$ and $(\bar{e}\nu)$, why the muon cannot decay into an electron and a photon, why the muon exists at all and is 207 times heavier than the electron, nor does the author know how to calculate, from the basic couplings of strong and weak interactions, quantities such as the ratio of C_A to C_V in nuclear β decay and the asymmetry parameters for the various decay modes of Σ hyperons, it is not too difficult to imagine that a chain of arguments similar to the one presented here may appear in the future correct theory. In any case, it is gratifying that the points of view presented in this paper lead to some unified understanding of parity conservation in strong interactions, parity conservation in electromagnetic interactions, and parity nonconservation in weak interactions from the *common* principle of generalized gauge invariance. Previously we had to rely on the structure on the Yukawa-type Lagrangians to "explain" parity conservation in strong interactions (*99–101*, *10*, *11*), on gauge invariance in the case of parity-conserving electromagnetic interactions (*100*), and on arguments based on chirality invariance (*102*) or mass reversal invariance (*103*) to "deduce" parity nonconservation of the V-A weak coupling.

Having discussed weak interactions, we naturally wonder how gravitational interactions fit into our general scheme.[38] According to the Eötvös experiment, the strength of the coupling of the gravitational field to matter is proportional to the inertial mass which is essentially the rest energy. Thus we are led to speculate that there exists a deep connection between energy conservation and the very existence of the gravitational coupling. The gravitational field, being the dynamical manifestation of energy, is to be coupled to energy-momentum density. Now there is energy associated with the gravitational field itself, hence the gravitational field can interact with itself in the same way as the $T = 1$ Yang-Mills $B_\mu^{(T)}$ field (which is the dynamical manifestation of isospin) can interact with itself. Assuming that such a nonlinear self-coupling produces a mass (which point is highly controversial), we can estimate the mass of the graviton from the mass of the B_T quantum and the coupling constants in question, provided that there is only one fundamental length. The graviton mass must be about 10^{39} times as small as the B_T mass because the dimensionless gravitational constant GM_N^2 is 10^{39} times smaller than $f_T^2/4\pi$.

[38] Many experts seem to regard the remaining part of this section as completely nonsensical.

If the graviton is massive, we expect that the gravitational potential is of the Yukawa type rather than of the Newton type, and the range of the gravitational potential is given by the graviton Compton wavelength. Should this Compton wavelength turn out to be of the order of the radius of the solar system, our speculation would be completely worthless because we know that Newton's law works well for calculating the orbits of the various planets such as Neptune and Pluto and, more recently, of artificial satellites. It so happens that our simple calculation with $\mu_T = 4\mu_\pi$ gives 3×10^8 light years for the range of the gravitational potential. (It is to be mentioned that the reciprocal of the square of the graviton Compton wavelength might be related to the Einstein cosmological constant Λ introduced in a purely *ad hoc* manner in conventional theories of cosmology.) This value is somewhat smaller (but not much smaller) than the radius of Hubble's universe, which is of the order of 5×10^9 light years. Our speculation may have some cosmological signific nce. In the massless graviton case, unless the matter density falls off faster than $1/r^2$ at large distances, the properties of space here are determined by distant galactic matter, the integral $G \int dr d\Omega \, \rho r^2/r$ being badly divergent. In our theory, however, the gravitational potential is screened, and galaxies considerably more remote than 3×10^8 light years produce no effect.

Our discussions on the cosmological implications of elementary particle interactions would be incomplete without pondering over the puzzling preponderance of positive baryonic charges in the universe as we know. Would it be that some "antigalaxies" really exist? (*104*) If the anwser to this question is to be negative, we may naturally ask: Is there any baryon-nonconserving interaction characterized by a time scale much longer than 4×10^{23} years (which is the present lower limit on the proton lifetime (*14*))? This kind of interaction, if it also violates energy conservation, may well be the very interaction responsible for the creation of the universe. The observed preponderance of nucleons over antinucleons can be explained as the direct consequence of vacuum fluctuations produced by this very, very weak interaction which conserves neither baryon number nor energy.

The ultimate physical theory must explain everything that happens in the universe—from s-wave πN scattering to cosmology.

VIII

There is one question that greatly puzzles the author: Why has nobody tried this kind of approach before? Perhaps our theory might have been tried a long time ago if it were not for the fact that the conventional Yukawa-type explanations of low-energy pion phenomena and low-energy nucleon-nucleon interactions have been so successful.

Let us imagine a beginning student (or an experimentalist who scoffs at highbrow theories) who is imaginative but has no training in the so-called meson theory. He looks at elementary particle phenomena for the first time without any theoretical prejudice, and marvels at the regularities of nature that can be understood from the conservation laws of isospin, hypercharge and baryon number. He tries to visualize these conserved quantities in classical manners which are more familiar to him. For instance, he imagines isospin as a kind of classical dipole (current-loop), and says there should be a repulsion (attraction) between two isospins when they are parallel (antiparallel). He looks at πN scattering and says, "Although I can't explain the 3–3 resonance, I *can* explain why *s*-wave scattering is repulsive in $T = 3/2$ (isospins parallel) and attractive in $T = 1/2$ (isospin antiparallel)." He looks at nucleon-nucleon scattering at high energies, and argues that the repulsive core at short distances is due to a repulsion between two baryonic charges of the same sign in analogy with electrostatics. He doesn't really understand the Dirac equation, but he figures out from a formula in Schiff's book that the short-ranged spin-orbit potential in *pp* scattering has the right sign. He looks at the $K^{\pm}N$ interaction and notes that his simple idea, based on an analogy with Coulomb's law, that two like hypercharges repel and two opposite hypercharges attract works out perfectly. He becomes more ambitious and asks whether a meson can be built up of a baryon and an antibaryon. This, he figures, is possible because particles with opposite baryonic charges must attract each other.

Is there any element of truth in what this beginning student is doing? He is, at least, offering very simple explanations of strong interaction phenomena precisely in those areas in which there are no simple explanations based on conventional Yukawa-type theories. This student reminds us of Feynman's "dope" who has found a simple and idiotic rule that works. This "dope" is trying to do what more learned and sophisticated theoreticians should do but have somehow forgotten to do. The ultimate task of elementary particle physics should be not just to locate all the singularities in the complex plane corresponding to each scattering or production process, nor to argue endlessly over whether the present field theory is consistent or inconsistent, but to "elaborate," in Schwinger's words, "a complete dynamical theory of elementary particles from a few general concepts" and thereby obtain "a convenient frame of reference in seeking a more coherent account of natural phenomena" (*6*).

It is assuming that our theory satisfies simultaneously almost all the principles that have been proposed on simple theoretical grounds by various deep thinkers of elementary particle physics. The theory is, in a certain sense, founded on Heisenberg's conviction that besides the selection rules and the invariance principles the only other guiding principle should be simplicity (*105*). It exploits the profound idea anticipated by Schwinger (*5*, *9*) that internal attributes such as baryonic charge (= his nucleonic charge) and hypercharge should have "dynamical manifestations." It fulfills the dream of Wigner and Gell-Mann that there ought to exist a universal coupling related to baryon conservation (*7, 8, 5*). It answers Pauli's question (*106*) in January, 1957, "Why does the Lord still appear to be right-left symmetric when he expresses himself strongly?" and, at the same time, satisfies Lee and Yang who argue that a satisfactory answer to Pauli's question should not depend on the detailed structure of the interaction Lagrangian (e.g., restrictions to nonderivative-type Yukawa couplings only). It realizes both the Pais principle of economy of constants (*92*) and the Pais principle of a hierarchy of interactions (*91*) in a natural and elegant manner, and it somehow reminds us of Feynman's remark that one should generate new ideas by asking what would have happened if history were different (*107*). These theoretical arguments, together with the experimental indications mentioned earlier, seem to the author to be strong enough to suggest that this theory might not be complete nonsense and that, even if the theory turns out to be wrong in the end, it is at least worth trying to work out various consequences of it.

There are a number of new experimental and theoretical directions to be explored.

(i) Every conceivable attempt should be made to detect experimentally direct quantum manifestations of the three kinds of vector fields introduced in our theory. Recently an attempt has been made by Bernardini *et al.* (*108*) at Frascoti to establish the possible existence of a neutral boson X^0 with mass less than $3.5\mu_\pi$ in the reaction $\gamma + p \rightarrow X^0 + p$, and the answer has turned out to be negative. Because of the extreme importance of its implications, we suggest that the Frascoti experiment be repeated even though it is very likely that our B quanta are too massive to be seen by that experiment. Unfortunately at higher energies it becomes more difficult to detect such particles by Frascoti-type experiments in which only recoil protons are detected because of the many possible multiple pion channels.[39] A more fruitful way might be to study Q values of two pions $(\pi^+\pi^-, \pi^\pm\pi^0)$ and three ipons $(\pi^+\pi^-\pi^0)$ in nucleon-antinucleon annihilation processes (see Section V) and in multiple-pion events in γp, πp and pp collisions.[40] This might be feasible in a heavy liquid chamber or, as suggested by Glaser (*109*), by means of a very elaborate mosaic of counters feeding into a computer which is programmed to search for kinematical correlations among various pions.

(ii) From a theoretical point of view, an attack should be made on the possible origins of the masses of the various B quanta. Especially we must understand why the B_Y and B_B quanta are massive while the photon is massless.

(iii) A quantum theory of "metastable fields" should be investigated in view

[39] The author is indebted to Dr. K. Berkelman for this remark.

[40] It is interesting to observe that high-energy experiments on multiple pion production favor rather than rule out the possibility of the existence of particles which immediately decay into pions, as previously discussed by Gupta (*110*).

of the fact that our B quanta decay immediately into 2π and 3π via strong interactions. For all practical purposes these B quanta appear as "resonances." The question of to what extent such an unstable quantum can carry the desired properties required by the theory is an open one.[41]

(iv) More than ever before we need methods of calculations that are suitable for fully relativistic strong couplings. In the present paper we have treated only static effects, and even there we have been able to make only crude estimates. If we are interested in quantitative results at higher energies, dynamic effects should be properly taken into account. Once a suitable calculational method is invented, we may, for instance, be able to tackle the π-π problem from first principles. Eventually we should be able to express the constant λ that appears in the Chew–Mandelstam equation *(111)* in terms of the fundamental constants that appear in our theory.

(v) Attempts should be made to understand the two $T = \frac{1}{2}$ higher resonances and one $T = \frac{3}{2}$ high resonance in the high energy πN interactions in terms of the two $T = 0$ (3π) and one $T = 1$ (2π) "fundamental resonances" that arise in our theory.

(vi) The major features of the electromagnetic properties of the nucleon may eventually be understood from our three "fundamental resonances." In particular the two $T = 0$, $J = 1$ three-pion resonances are expected to be important for the isoscalar properties of the nucleon. It is hoped that our theory will throw light on the question of why the isovector charge radius and the isoscalar charge radius are both large while the isoscalar moment is much smaller than the isovector moment.

(vii) Certain phenomenological parameters in K-particle physics, such as the scattering lengths in K^-N reactions and the s-wave phase shifts in K^+N scattering, are more closely related to the fundamental constants that appear in our theory than $G^2(K\Lambda N)$ and $G^2(K\Sigma N)$; hence they deserve more attention. The Dalitz ambiguity in K^-N reactions should be resolved.

(viii) A fresh attack should be made on high-energy pp scattering. The repulsive core is not made up of an infinitely hard wall; such an unphysical wall should be replaced by a very strong short-ranged, parity-independent Yukawa-type potential. The $\mathbf{L} \cdot \mathbf{S}$ force that arises naturally by taking the Thomas derivatime of the short-ranged repulsive potential should be investigated. Similar analyses should be done for the $T = 0$ case.

(ix) There seem to be a number of mysteries in antinucleon physics. Why are annihilation cross sections so large not only at 100 Mev but also in Bev regions? Why is the average pion multiplicity in annihilation processes so high? Why is the reaction $p + \bar{p} \rightarrow \pi^+ + \pi^-$ so sensationally rare? These questions are as challenging to us as the $\tau\theta$ puzzle was four years ago. Apart from the question of

[41] The author is indebted to Professor R. Oehme for calling attention to this problem.

whether or not the possible solutions given in Section V contain elements of truth, it is likely that satisfactory clues to these mysteries will mark the dawning of a new epoch in our understanding of strong interactions in Bev regions.

(x) Possible deviations from predictions based on statistical theories should be looked for. As mentioned earlier, the statistical theory gives insane answers for the ratio of $(K\bar{K}2N)$ to $(K\Lambda(\Sigma)N)$ in NN collisions, the numbers of pions in $N\bar{N}$ annihilations, (and possibly in πN and NN collisions) and the angular distribution of pions in high-energy jets. Perhaps there are many more spectacular deviations in store for us. Explorations into high energies are exciting because they reveal so many challenging surprises which are completely unexpected. But for such surprises there would be no justification for building expensive machines.

(xi) Weak decays of strongly interacting particles into strongly interacting particles such as $K \rightarrow 2\pi$ or 3π and $\Lambda \rightarrow N + \pi$ should be studied especially to see whether simple phenomenological rules (e.g., the $\Delta T = \frac{1}{2}$ rule) emerge naturally from our theory. The reason for $|C_A/C_V| > 1$ in nuclear β decay should be investigated.

(xii) Finally there are difficult, burning questions. Why these fundamental coupling constants? Why is there a wide gap in strength between the weak coupling and the other couplings? Why is one unit of baryonic charge associated with a half unit of spin? Why are electric charge and baryonic charge "quantized?" Why does the fermionic charge coupling (i.e., the weak coupling) destroy hypercharge conservation and isospin conservation but neither baryon conservation nor electric charge conservation? And many more. Perhaps it is appropriate to close the paper by the following remark made by Yukawa whose insight and vision have influenced all of us who have worked in the now twenty-five-year-old meson theory (*112*). "If you look at the whole body of elementary particles, including all the new particles, then the photon, which is most familiar to us, is in a sense the strangest of all. One of the mysteries of the photon is related to the concept of charge independence, which seems to work quite well in the cases of meson-nucleon and nucleon-nucleon interactions and also in a more comprehensive theory of particles. However, the introduction of electromagnetic interactions destroys the isotropy of isotopic spin space. This seems quite strange to me. I have no way of understanding this strange situation so far, and I cannot be very confident about the notion of isospin space, until there appears a good idea of explaining the peculiarity of electromagnetic interactions."

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REFERENCES

1. M. GELL-MANN, *Phys. Rev.* **92,** 833 (1953).
2. T. NAKANO AND K. NISHIJIMA, *Progr. Theoret. Phys. (Kyoto)* **10,** 581 (1953).
3. A. PAIS, *Phys. Rev.* **110,** 574 (1958).
4. A. PAIS, *Phys. Rev.* **110,** 1480 (1958).
5. M. GELL-MANN, *Phys. Rev.* **106,** 1296 (1957).
6. J. SCHWINGER, *Annals of Physics* **2,** 407 (1957).
7. E. P. WIGNER, *Proc. Am. Phil. Soc.* **93,** 521 (1949).
8. E. P. WIGNER, *Proc. Natl. Acad. Sci. U.S.* **38,** 449 (1952).
9. J. SCHWINGER, *Phys. Rev.* **104,** 1164 (1956).
10. J. J. SAKURAI, *Phys. Rev.* **113,** 1679 (1959).
11. G. FEINBERG AND F. GÜRSEY, *Phys. Rev.* **114,** 1153 (1959).
12. M. GELL-MANN, *in* "Hearings before the Subcommittee on Legislation of the Joint Committee on Atomic Energy." Congress of the United States, Washington, D. C., 1958.
13. E. C. G. STUECKELBERG, *Helv. Phys. Acta* **11,** 299 (1938).
14. F. REINES, C. L. COWAN, JR., AND H. W. KRUSE, *Phys. Rev.* **109,** 609 (1958).
15. E. P. WIGNER, *Phys. Rev.* **51,** 106 (1937).
16. W. HEISENBERG, *Z. Physik* **77,** 1 (1932).
17. R. H. HILDEBRAND, *Phys. Rev.* **89,** 1090 (1953).
18. B. PONTECORVO's Kiev report (unpublished).
19. F. S. CRAWFORD *et al.*, *Phys. Rev. Letters* **3,** 394 (1959).
20. L. ALVAREZ' Kiev report (unpublished).
21. L. MICHEL, *Nuovo cimento* [9], **10,** 319 (1953).
22. T. D. LEE AND C. N. YANG, *Nuovo cimento* [10], **3,** 749 (1956).
23. G. C. WICK, A. S. WIGHTMAN, AND E. P. WIGNER, *Phys. Rev.* **88,** 101 (1952).
24. B. D'ESPAGNAT AND J. PRENTKI, *Nuclear Physics* **1,** 33 (1956).
25. C. N. YANG AND R. L. MILLS, *Phys. Rev.* **96,** 191 (1954).
26. W. PAULI, *Revs. Modern Phys.* **13,** 203 (1941).
27. W. PAULI, "Encyclopedia of Physics," Vol. 5, p. 143. Springer, Berlin, 1958.
28. T. D. LEE AND C. N. YANG, *Phys. Rev.* **98,** 1501 (1955).
29. Y. FUJII, *Progr. Theoret. Phys. (Kyoto)* **21,** 232 (1959).
30. S. S. GERSHTEIN AND J. B. ZEL'DOVICH, *J. Exptl. Theoret. Phys. U.S.S.R.* **29,** 698 (1955); translation: *Soviet Phys. JETP* **2,** 576 (1957).
31. R. P. FEYNMAN AND M. GELL-MANN, *Phys. Rev.* **109,** 193 (1958).
32. Y. NAMBU, *Phys. Rev.* **106,** 1366 (1957).
33. E. C. G. STUECKELBERG, *Helv. Phys. Acta* **11,** 225 (1938).
34. J. SCHWINGER, *Phys. Rev.* **76,** 790 (1949).
35. Y. AHARONOV AND D. BOHM, *Phys. Rev.* **115,** 485 (1959).
36. J. TIOMNO, *Nuovo cimento* [10], **6,** 69 (1957).
37. E. FERMI AND C. N. YANG, *Phys. Rev.* **76,** 1739 (1949).
38. G. F. CHEW AND F. E. LOW, *Phys. Rev.* **101,** 1570 (1956).

39. G. F. Chew and F. E. Low, *Phys. Rev.* **101**, 1579 (1956).
40. J. Orear, *Phys. Rev.* **100**, 288 (1955).
41. M. D. Goldberger, H. Miyazawa, and R. Oehme, *Phys. Rev.* **99**, 986 (1955).
42. A. Klein, *Phys. Rev.* **99**, 938 (1955).
43. S. D. Drell, M. H. Friedman, and F. Zachariasen, *Phys. Rev.* **104**, 236 (1956).
44. S. F. Edwards and P. T. Matthews, *Phil. Mag.* [8] **2**, 176 (1957).
45. R. Levi-Setti, private communication based on world-wide attempts to detect $(\Sigma^- n)$ compounds.
46. S. F. Tuan, private communication.
47. R. H. Dalitz and S. F. Tuan, *Phys. Rev. Letters* **2**, 425 (1959).
48. R. H. Dalitz, *in* "Proceedings of 1958 Annual International Conference on High-Energy Physics at CERN," p. 187. CERN, Geneva, 1958.
49. W. R. Frazer and J. R. Fulco, *Phys. Rev. Letters* **2**, 365 (1959).
50. J. C. Brisson, J. Detoef, P. Falk-Vairant, L. van Rossum, G. Valladas, and L. C. L. Yuan, *Phys. Rev. Letters* **3**, 561 (1959).
51. M. J. Longo, J. A. Helland, W. N. Hess, B. J. Moyer, and V. Perez-Mendez. *Phys. Rev. Letters* **3**, 568 (1959).
52. M. M. Block, E. B. Bruecker, I. S. Hughes, T. Kikuchi, C. Meltzer, F. Anderson, A. Pevsner, E. M. Harth, J. Leitner, and H. O. Cohn, *Phys. Rev. Letters* **3**, 291 (1959).
53. R. F. Christy, *in* "Proceedings of the Seventh Annual Rochester Conference on High-Energy Physics," Chapter IX, p. 1. Interscience, New York, 1956.
54. M. Goldhaber, *Phys. Rev.* **92**, 1279 (1953).
55. M. Goldhaber, *Phys. Rev.* **101**, 433 (1956).
56. J. D. Jackson, D. G. Ravenhall, and H. W. Wyld, Jr., *Nuovo cimento* [10], **9**, 834 (1958).
57. O. R. Price, D. H. Stork, and H. K. Ticho, *Phys. Rev. Letters* **1**, 212 (1958).
58. J. G. Taylor, M. J. Moravcsik, and J. L. Uretsky, *Phys. Rev.* **113**, 689 (1959).
59. R. Karplus, L. Kerth, and T. Kycia, *Phys. Rev. Letters* **2**, 510 (1959).
60. J. Steinberger, Kiev report (unpublished).
61. F. Cerulus and R. Hagedorn (to be published).
62. G. E. Failho, *Phys. Rev.* **105**, 328 (1957).
63. R. Serber, *in* "Proceedings of the Seventh Annaul Rochester Conference on High-Energy Physics," Chapter V, p. 1. Interscience, New York, 1956.
64. C. O. Dechand, *Phys. Rev.* **115**, 1730 (1959).
65. D. O. Caldwell, private communication based on an unpublished work of the M.I.T. cloud-chamber group.
66. S. Barshay, *Phys. Rev.* **109**, 2160 (1958).
67. M. M. Levy, *Phys. Rev.* **88**, 725 (1952).
68. P. S. Signell, R. Zinn, and R. E. Marshak, *Phys. Rev. Letters* **1**, 416 (1958).
69. J. Gammel and R. Thaler, *Phys. Rev.* **107**, 291 (1957).
70. J. S. Ball and G. F. Chew, *Phys. Rev.* **109**, 1385 (1958).
71. H. P. Duerr, *Phys. Rev.* **103**, 469 (1956).
72. E. Teller, *in* "Proceedings of the Sixth Annual Rochester Conference on High-Energy Physics," Chapter VII, p. 18. Interscience, New York, 1956.
73. E. Segré, Kiev report (unpublished).
74. N. Horwitz, D. Miller, J. Murray, and R. Tripp, *Phys. Rev.* **115**, 472 (1959).
75. Z. Koba and G. Takeda, *Progr. Theoret. Phys. (Kyoto)* **19**, 269 (1958).
76. G. Goldhaber, W. B. Fowler, S. Goldhaber, T. F. Hoang, T. E. Kalogeropoulos, and W. M. Powell, *Phys. Rev. Letters* **3**, 181 (1959).

77. E. Fermi, *Phys. Rev.* **81,** 683 (1951).
78. W. Heisenberg, *in* "Vorträge über Kosmische Strahlung," p. 148. Springer-Verlag, Berlin, 1953.
79. S. S. Belenki and L. D. Landau, *Nuovo cimento* [10], Suppl. 1, **3,** 15 (1956).
80. P. Ciok, T. Coghen, J. Gierula, R. Holyński, A. Jurak, M. Miesowicz, and T. Saniewska, *Nuovo cimento* **10,** 741 (1958).
81. G. Cocconi, *Phys. Rev.* **111,** 1699 (1958).
82. K. Niu, *Nuovo cimento* **10,** 994 (1958).
83. S. Sakata, *Progr. Theoret. Phys. (Kyoto).*
84. L. B. Okun, *in* "Proceedings of the 1958 Annual International Conference on High-Energy Physics at CERN" p. 223. CERN, Geneva, 1958.
85. M. M. Levy and R. E. Marshak, *Nuovo cimento* **11,** 366 (1954).
86. M. A. Markov, *in* "Proceedings of the Sixth Annual Rochester Conference on High-Energy Physics," Chapter VIII, p. 30. Interscience, New York, 1956.
87. K. Huang, *Phys. Rev.* **101,** 1173 (1956).
88. V. F. Weisskopf, *Phys. Rev.* **56,** 72 (1939).
89. G. Feinberg and R. E. Behrends, *Phys. Rev.* **115,** 745 (1959).
90. T. Yamanouchi, *Phys. Rev. Letters* **3,** 480 (1959).
91. A. Pais, *Proc. Natl. Acad. Sci. U.S.* **40,** 484 (1954).
92. A. Pais, *Phys. Rev.* **110,** 574 (1958).
93. T. D. Lee, *in* "Conservation Laws in Weak Interactions," Nevis-50, Columbia University (unpublished).
94. R. Oppenheimer, *in* "Proceedings of the Seventh Annual Rochester Conference on High-Energy Physics," Chapter IX, p. 27. Interscience, New York, 1957.
95. S. A. Bludman, *Nuovo cimento* [10], **9,** 433 (1958).
96. A. Salam and J. C. Ward, *Nuovo cimento* **11,** 568 (1959).
97. C. N. Yang, *in* "Proceedings of the Seventh Annual Rochester Conference on High-Energy Physics," Chapter IX, p. 25. Interscience, New York, 1956.
98. W. Pauli, *Nuovo cimento* [10], **6,** 204 (1957).
99. G. Feinberg, *Phys. Rev.* **108,** 878 (1957).
100. S. N. Gupta, *Can. J. Phys.* **35,** 1309 (1957).
101. V. G. Solov'ev, *J. Exptl. Theoret. Phys. U.S.S.R.* **33,** 537, 796 (1957); translation: *Soviet Phys. JETP* **6**(33), 419, 613 (1957).
102. E. C. G. Sudarshan and R. E. Marshak, *Phys. Rev.* **109,** 1860 (1958).
103. J. J. Sakurai, *Nuovo cimento* [10], **7,** 649 (1958).
104. G. R. Burbidge and F. Hoyle, *Nuovo cimento* [10], **4,** 558 (1956).
105. W. Heisenberg, *Revs. Modern Phys.* **29,** 269 (1957).
106. W. Pauli, "private" communication to V. F. Weisskopf.
107. R. P. Feynman, *in* "Proceedings of the Seventh Annual Rochester Conference on High-Energy Physics," Chapter IX, p. 42. Interscience, New York, 1957.
108. C. Bernardini, R. Querzoli, G. Salvini, A. Silverman, and G. Stoppini, *Nuovo cimento* [10], **14,** 268 (1959).
109. D. A. Glaser, *in* "Proceedings of Argonne Accelerator Users Croup Meeting," June, 1959 (unpublished).
110. S. J. Gupta, *Phys. Rev.* **111,** 1698 (1958).
111. G. F. Chew and S. Mandelstam (to be published).
112. H. Yukawa, *Revs. Modern Phys.* **29,** 213 (1957).